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International Journal of Solids and Structures 49 (2012) 1877–1883

Contents lists available at SciVerse ScienceDirect

International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr

Analysis of fracture initiation angle in some cracked ceramics using the generalized maximum tangential stress criterion

M.R.M. Aliha [a,b], M.R. Ayatollahi [b,*]

[a] Welding and Joining Research Center, School of Industrial Engineering, Iran University of Science and Technology, Narmak, Tehran 16846-13114, Iran
[b] Fatigue and Fracture Laboratory, Center of Excellence in Experimental Solid Mechanics and Dynamics, School of Mechanical Engineering, Iran University of Science and Technology (IUST), Tehran 16846-13114, Iran

ARTICLE INFO

Article history:
Received 1 June 2011
Received in revised form 18 March 2012
Available online 3 April 2012

Keywords:
Fracture initiation angle
Mixed mode fracture
Ceramics
Brazilian disc specimen

ABSTRACT

Brittle fracture in ceramics sometimes occurs under combined opening-sliding (or mixed mode I/II) crack deformation. In this paper, a generalized maximum tangential stress criterion is employed for predicting the fracture initiation angle under mixed mode I/II loading in some brittle ceramics including alumina, zirconia, soda lime glass and three silicon based ceramics. The experimental results reported for the fracture angles in these ceramics have been obtained from fracture tests on the centrally cracked circular disc (often called the Brazilian disc). Very good agreement is shown to exist between the experimental results and the theoretical predictions. According to the fracture model, the mixed mode fracture angle is strongly dependent on the elastic *T*-stress in the tested ceramics. The negative *T*-stress that exists in the Brazilian disc specimen can be the main influencing parameter for decreasing the fracture initiation angle in the investigated ceramics.

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1. Introduction

Ceramics are extensively used in various industrial applications like electrical devices, turbine or compressor blades, advanced composite materials and refractory products because of their favourable physical properties such as high hardness, low weight, high wear and corrosion resistance and high temperature strength. However, ceramics are vulnerable to brittle fracture particularly in the presence of cracks. The cracks are often generated in ceramics during the manufacturing or machining processes or due to the application of mechanical and thermal loads during the service life. The evaluation of both fracture toughness and the crack growth direction are important issues for designing and manufacturing advanced components made from ceramic materials. In general, the cracks generated inside the engineering components can be subjected to three basic modes of deformation namely mode I (opening), mode II (in-plane sliding) and mode III (out-off plane tearing). Any combinations of these modes are known as mixed mode deformation. Most of the previous studies on fracture of ceramics have focused on mode I crack deformation (e.g., Barker, 1978; Daividge, 1979; Dalgleish et al., 1980; Lawn, 1981; Sorensen et al., 1996). However, flaws in real ceramic components are seldom subjected to pure mode I loading. Indeed, catastrophic fracture can be initiated from mixed mode cracks or pre-existing flaws which are oriented at arbitrary angles relative to the far field loading directions or subjected to multiaxial stresses. While for mode I loading, the fracture trajectory is stable and self similar, the crack extension under mixed mode loading initiates at an angle other than the pre existing crack. Hence, the evaluation of fracture behaviour and its path under mixed mode deformation is more complicated than the simple pure mode I case.

The direction of fracture initiation angle is an important stage in estimating the path of crack growth. There are some theories for evaluating the direction of fracture initiation for mixed mode I/II crack problems. The maximum tangential stress (MTS) criterion (Erdogan and Sih, 1963), the minimum strain energy density (SED) criterion (Sih, 1974) and the maximum energy release rate (*G*) criterion (Hussain et al., 1974) are some of the well-known mixed mode fracture criteria. Based on the MTS criterion, a mixed mode crack extends from the crack tip along the direction of maximum tangential stress, θ_0. According to this criterion, the direction of fracture initiation is determined from the following equation (Erdogan and Sih, 1963):

$$\sin\theta_0 = \frac{K_{II}}{K_I}(1 - 3\cos\theta_0)$$

$$\therefore \quad \theta_0 = \tan^{-1}\left(\frac{-3\left(\sqrt{8K_{II}^2 + K_I^2} + K_I\right)K_{II}}{3K_{II}^2 + K_I\sqrt{8K_{II}^2 + K_I^2}}\right) \qquad (1)$$

* Corresponding author. Tel.: +98 21 77240201; fax: +98 21 77240488.
E-mail addresses: mrm_aliha@iust.ac.ir, m.ayat@iust.ac.ir (M.R. Ayatollahi).

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http://dx.doi.org/10.1016/j.ijsolstr.2012.03.029

where K_I and K_{II} are the mode I and mode II stress intensity factors, respectively. When both K_I and K_{II} are positive, Eq. (1) gives a negative solution for θ_0 ranging from zero (for pure mode I) to $-70.5°$ (for pure mode II). However, it is important to validate the theoretical predictions of a fracture criterion using the experimental results. There are some experimental data in the literature for the fracture initiation angle in ceramics under mixed mode I/II loading obtained from different test specimens (Shetty et al., 1987; Singh and Shetty, 1989a,b; Suresh et al., 1990; Machida and Isobe, 1995; Li and Sakai, 1996; Tikare and Choi, 1997; Awaji and Kato, 1998; Margevicius et al., 1999; Choi et al., 2005). For example, some researchers have performed mixed mode fracture tests on alumina, polycrystalline tungsten, plasma-sprayed ZrO_2–8wt%Y_2O_3 thermal barrier coating and Macor ceramic using cracked or V notched asymmetric four-point bend specimen (Suresh et al., 1990; Li and Sakai, 1996;Tikare and Choi, 1997; Margevicius et al., 1999; Choi et al., 2005). Some other researchers have utilized a centrally cracked circular disc often called the Brazilian disc (BD) subjected to diametral compression for investigating mixed mode fracture initiation angle in some ceramics such as alumina and zirconia (Singh and Shetty, 1989a, 1989b), soda lime glass (Shetty et al., 1987; Awaji and Kato, 1998) and silicon based ceramics such as sialon, mullite and SiC (Machida and Isobe, 1995). A review of literature shows that among the different available test specimens, the BD specimen has been used more frequently by researchers for investigating mixed mode fracture in ceramics mainly because of its convenience in introducing different combinations of mode I and mode II. The simple geometry, the easy test set up and the application of compressive loads are some other advantages of the BD specimen in fracture tests on ceramics. However, the experimental results reported for the fracture initiation angle in ceramics obtained from the BD specimen are not consistent with the theoretical predictions of the conventional fracture criteria such as the MTS criterion. For example, the mixed mode fracture initiation angles of different ceramics tested with the BD specimen have been compared in Fig. 1 with the curve of the MTS criterion. The experimental results have been directly extracted from (Shetty et al., 1987; Singh and Shetty, 1989a, 1989b; Machida and Isobe, 1995; Awaji and Kato, 1998). In Fig. 1, M^e is the mixity parameter showing the ratio of mode I and mode II stress intensity factors as:

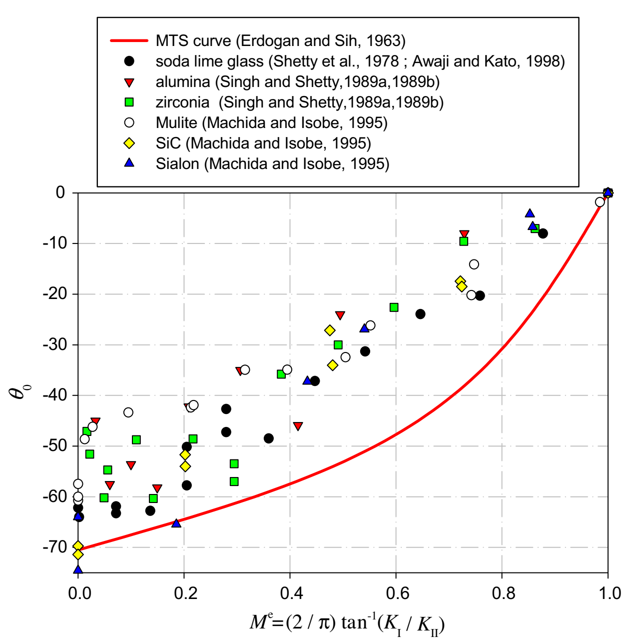

Fig. 1. Comparison between the fracture initiation angles for some ceramics obtained from the Brazilian disc specimen and predictions of the conventional MTS criterion.

$$M^e = \frac{2}{\pi}\tan^{-1}\left(\frac{K_I}{K_{II}}\right) \tag{2}$$

M^e varies from 0 for pure mode II to 1 for pure mode I. It is seen from Fig. 1 that for each mode mixity, there is a significant discrepancy (typically about 10–30°) not only between the reported test data and the predictions of the MTS criterion but also between the different sets of experimental results whereas, Eq. (1) gives a unique value of θ_0 for any fixed K_I/K_{II}. Therefore, the aim of this paper is to use a generalized MTS criterion and to provide more precise predictions for the fracture initiation angles in ceramics tested with the Brazilian disc specimen.

2. Generalized MTS theory

The generalized form of the MTS criterion is described here for predicting the crack initiation angle under mixed mode loading. In comparison with the conventional MTS criterion (Erdogan and Sih, 1963) the generalized criterion (which was first developed by Smith et al., 2001) makes use of a more accurate description for the crack tip stresses. Williams (1957) showed that the tangential stress around the crack tip can be written as an infinite series expansion:

$$\sigma_{\theta\theta} = \frac{1}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left[K_I\cos^2\frac{\theta}{2} - \frac{3}{2}K_{II}\sin\theta\right] + T\sin^2\theta + O(r^{1/2}) \tag{3}$$

where r and θ are the conventional crack tip co-ordinates in the polar system. The T-stress in the first non-singular term of series expansion is a constant stress term which is independent of distance from the crack tip. The higher order terms $O(r^{1/2})$ are assumed to be negligible near the crack tip. While in the conventional MTS criterion the effects of singular term alone are considered, the generalized MTS criterion takes into account the influence of T term in addition to the term containing K_I and K_{II}. The T-stress effect in mixed mode brittle fracture was first shown by Williams and Ewing (1972) and Ueda et al. (1983) but only for the simple case of center crack plate which has closed form solutions for K_I, K_{II} and T. Later Smith et al. (2001) proposed the generalized MTS criterion which is able to take into account the effects of T-stress for any arbitrary mixed mode crack problem. According to this criterion, mixed mode crack growth initiates radially from the crack tip along the direction of maximum tangential stress. Hence, based on the generalized MTS criterion the direction of fracture initiation angle (θ_0) is determined from:

$$\frac{\partial\sigma_{\theta\theta}(r,\theta)}{\partial\theta}\Big|_{\theta=\theta_0} = 0, \quad \frac{\partial^2\sigma_{\theta\theta}(r,\theta)}{\partial\theta^2} < 0 \tag{4}$$

which gives (Smith et al., 2001):

$$[K_I\sin\theta_0 + K_{II}(3\cos\theta_0 - 1)] - \frac{16T}{3}\sqrt{2\pi r_c}\cos\theta_0\sin\frac{\theta_0}{2} = 0 \tag{5}$$

In Eq. (5), r_c is the critical distance from the crack tip and is often considered as a constant material property. If the T-stress and r_c are normalized as:

$$B = \frac{T\sqrt{\pi a}}{\sqrt{K_I^2 + K_{II}^2}} \tag{6}$$

and

$$\alpha = \frac{\sqrt{2r_c}}{\sqrt{a}} \tag{7}$$

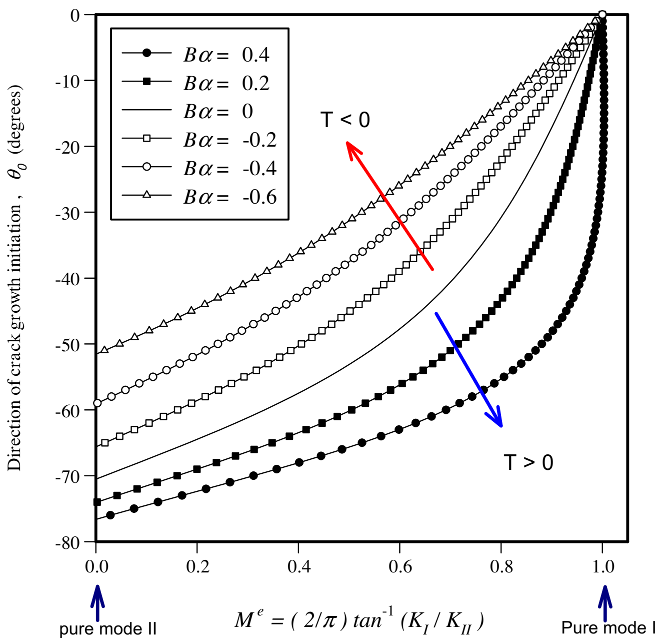

Fig. 2. The influence of T-stress on mixed mode fracture initiation angle.

Eq. (5) can also be written in terms of $B\alpha$:

$$[K_{\rm I}\sin\theta_0 + K_{\rm II}(3\cos\theta_0 - 1)] - \frac{16B\alpha\sqrt{K_{\rm I}^2 + K_{\rm II}{}^2}}{3}\cos\theta_0\sin\frac{\theta_0}{2} = 0 \tag{8}$$

In Eq. (7), a is the crack length for edge cracks and semi-crack length for center cracks. Either of Eqs. (5) and (8) can be used to estimate the mixed mode fracture initiation angle θ_0 from the generalized MTS criterion. It is seen from these equations that θ_0 depends not only on the stress intensity factors but also on the magnitude and the sign of T. A negative T term decreases the absolute value of θ_0 and conversely the fracture initiation angle increases in specimen having positive values of T. Fig. 2 presents the curves related to Eq. (8) for different values of $B\alpha$ showing the influence of T-stress on fracture initiation angle.

In order to determine θ_0 for any mode mixity, three fracture parameters namely $K_{\rm I}$, $K_{\rm II}$ and T should be known for the cracked specimen. These parameters are functions of specimen geometry and the type of loading. In the next section, following a brief description of the Brazilian disc specimen, its mixed mode fracture parameters are presented.

3. Fracture parameters for BD specimen

The geometry and loading configuration of the BD specimen is shown in Fig. 3. The specimen is a circular disc of radius R and thickness t having a centre crack of length $2a$. The state of mode mixity varies in this specimen by changing the crack angle β, i.e., the angle between the crack line and the direction of diametral compressive load F. The BD specimen is able to introduce the complete mode mixities ranging from pure mode I to pure mode II and various intermediate combinations of mode I and mode II (Ayatollahi and Aliha, 2007). Also shown in Fig. 3 are the fracture initiation angle θ_0 and the typical crack growth trajectory usually observed in this specimen. The fracture parameters $K_{\rm I}$, $K_{\rm II}$ and T for the BD specimen are written as:

$$K_{\rm I} = Y_{\rm I}(a/R,\beta)\frac{F}{Rt}\sqrt{\frac{a}{\pi}} \tag{9}$$

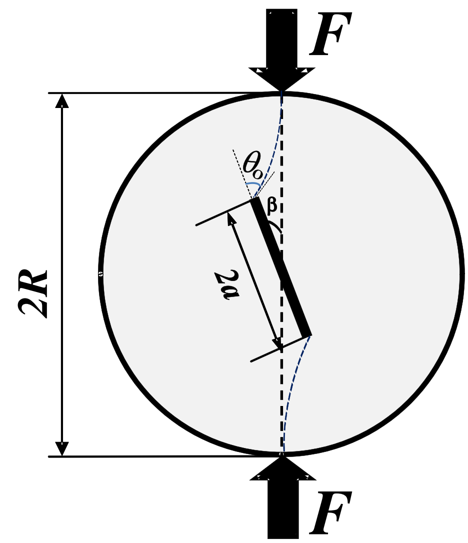

Fig. 3. Centrally cracked Brazilian disc (BD) specimen subjected to mixed mode loading, its typical fracture trajectory and the fracture initiation angle θ_0.

$$K_{\rm II} = Y_{\rm II}(a/R,\beta)\frac{F}{Rt}\sqrt{\frac{a}{\pi}} \tag{10}$$

$$T = T^*(a/R,\beta)\frac{F}{\pi t(R-a)} \tag{11}$$

where, $Y_{\rm I}$, $Y_{\rm II}$ and T^* are the normalized forms of fracture parameters which depend on the crack orientation angle (β) and crack length ratio (a/R).

These fracture parameters have already been computed for the BD specimen by numerical or analytical methods (e.g., Atkinson et al., 1982; Fett, 2001; Ayatollahi and Aliha, 2007).

4. Analysis of fracture angle for ceramics

In order to predict the fracture initiation angle in the ceramics tested by the BD specimen, first Eq. (8) is rewritten in terms of $Y_{\rm I}$, $Y_{\rm II}$ and T^* as:

$$[Y_{\rm I}\sin\theta_0 + Y_{\rm II}(3\cos\theta_0 - 1)] - \frac{16}{3}B\alpha\sqrt{Y_{\rm I}^2 + Y_{\rm II}^2}\cos\theta_0\sin\frac{\theta_0}{2} = 0 \tag{12}$$

Then the non-dimensional parameters $Y_{\rm I}$, $Y_{\rm II}$ and T^* are extracted from Ayatollahi and Aliha (2007) using the values of a/R reported for each BD test specimen and for different crack inclination angles β. Finally, the fracture angle θ_0 is determined by solving Eq. (12). By repeating the aforementioned steps for all crack inclination angles ranging from pure mode I to pure mode II, a theoretical curve can be plotted for the values of θ_0 calculated for each tested ceramic specimen.

The generalized MTS criterion is employed here for predicting the mixed mode fracture angles reported earlier for some ceramics and obtained from the BD experiments. These ceramics are soda lime glass (Shetty et al., 1987; Awaji and Kato, 1998), polycrystalline alumina (Singh and Shetty, 1989a, 1989b), zirconia (CeO_2-TZP) (Singh and Shetty, 1989a, 1989b) and three silicon based ceramics namely SiC, mullite and sialon (Machida and Isobe, 1995). The dimensions of the BD specimens including the disc radius (R), the semi-crack length (a), the disc thickness (t) and the crack length ratio (a/R) for these ceramics are listed in Table 1.

Table 1
The dimensions of the BD specimens used for testing different ceramics.

Material	R (mm)	a (mm)	t (mm)	a/R
Sialon (Machida and Isobe, 1995)	10	3	2	0.3
Mullite (Machida and Isobe, 1995)	10	3	2	0.3
SiC (Machida and Isobe, 1995)	22.5	6.75	5	0.3
Soda-Lime Glass (Shetty et al., 1987)	25	6.25	2.3	0.25
Soda Lime float Glass (Awaji and Kato, 1998)	20	8	2	0.40
Polycrystalline Alumina (Singh and Shetty, 1989a,1989b)	10.8	5.18	2.56	0.48
Zirconia (CeO_2-TZP) (Singh and Shetty, 1989a,1989b)	11.18	4.7	2.54	0.42

Table 2
The size of critical distance (r_c) calculated for the tested ceramics.

Material	K_{Ic} (MPa m$^{0.5}$)	σ_t (MPa)	r_c (mm) (from Eq. (13))
Sialon	4.5	670	0.072
Mullite	2	317	0.068
SiC	3	450	0.07
Glass	0.7	58.5	0.23
Alumina	3.3	170	0.65
Zirconia	6.1	400	0.4

In order to determine θ_0 using Eq. (12), the value of r_c is another parameter which should be known for each ceramic. Recently, Ayatollahi and Aliha (2011) have proposed a model for estimating the size of r_c in ceramics. Based on their model, r_c can be determined from:

$$r_c = \frac{\pi}{2}\left(\frac{K_{Ic}}{\sigma_t}\right)^2 \tag{13}$$

where K_{Ic} and σ_t are the pure mode I fracture toughness and the tensile strength of ceramic, respectively. Table 2 presents the values of r_c calculated using Eq. (13) for the ceramics investigated in this paper. More details about the tensile strength and fracture toughness of the tested ceramics and the calculations of their r_c can be found in (Ayatollahi and Aliha, 2011).

The critical distance r_c calculated for each ceramic was used for obtaining θ_0 according to Eq. (12) and using the numerical values of Y_I, Y_{II} and T^* for each BD specimen.

5. Results and discussion

The theoretical curves of the generalized MTS criterion for the considered ceramics were obtained using the procedure described in the previous section. Figs. 4–9 show the curves of mixed mode fracture angle for glass, sialon, SiC, mullite, alumina and zirconia, respectively. Also shown in these figures are the curves fitted to the averages of test data and the curve related to the conventional MTS criterion. As shown in these figures, in all cases the generalized criterion which uses three fracture parameters (K_I, K_{II} and T), provides good predictions for the direction of fracture initiation in ceramics tested by the BD specimen. However, the conventional MTS curve which is based only on the stress intensity factors and ignores the influence of the T-stress, overestimates significantly the fracture initiation angles reported for the tested ceramics. This is mainly due to the effect of considerable negative T-stresses that exist in the BD specimen. It was shown earlier by Ayatollahi and Aliha (2007) that the sign of T^* in the BD specimen for all crack inclination angles (i.e., all mode mixities) is negative and its absolute value becomes more noticeable for dominantly mode I conditions. Based on the generalized MTS theory, the angle of fracture initiation in mixed mode loading decreases for specimens having a negative T-stress. This is consistent well with the data reported for the tested ceramics and investigated in this research. In other words, all the test data are above the conventional MTS curve.

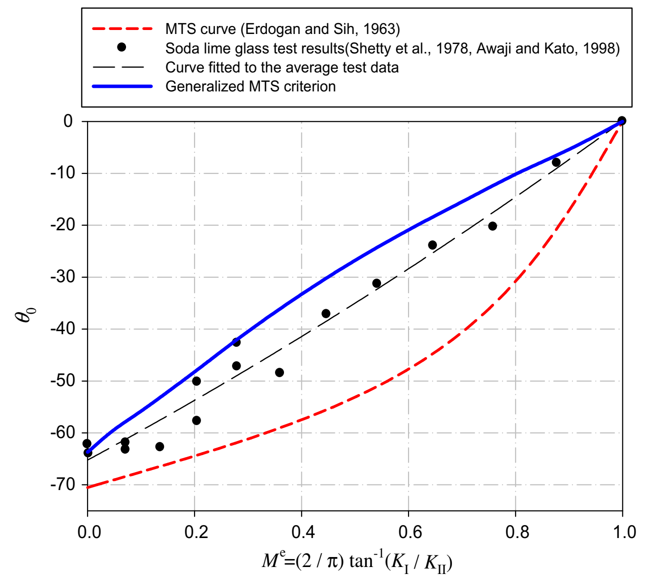

Fig. 4. Predictions of generalized MTS criterion for mixed mode fracture angle of glass tested using the BD specimen.

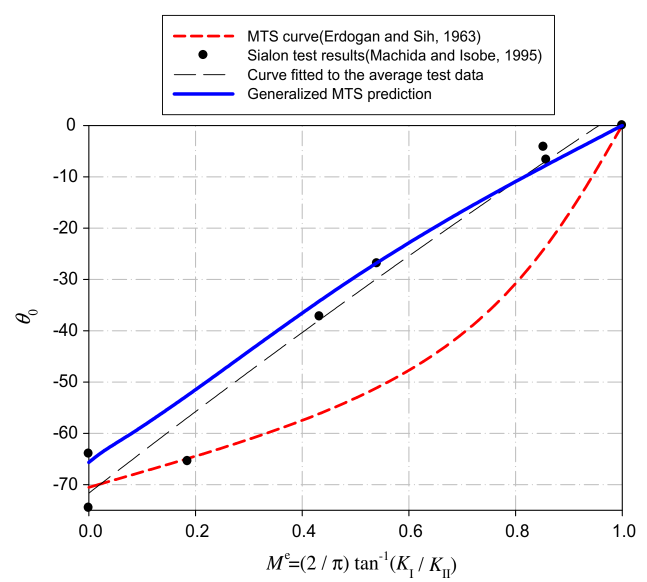

Fig. 5. Predictions of generalized MTS criterion for mixed mode fracture angle of sialon tested using the BD specimen.

The improved predictions of generalized MTS criterion suggest that the T-stress plays an important role in the fracture tests conducted by the BD specimen. Very close to the crack tip the elastic

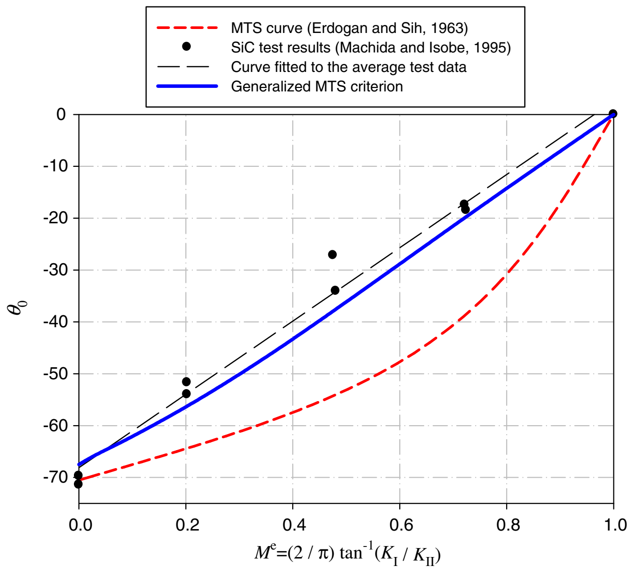

Fig. 6. Predictions of generalized MTS criterion for mixed mode fracture angle of SiC tested using the BD specimen.

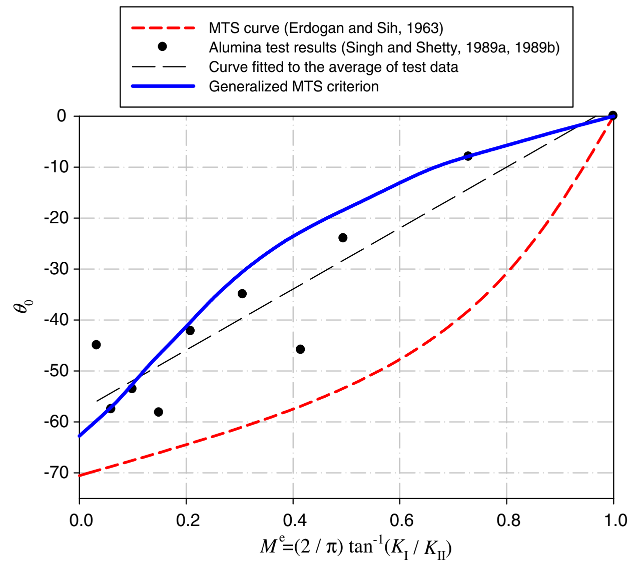

Fig. 8. Predictions of generalized MTS criterion for mixed mode fracture angle of alumina tested using the BD specimen.

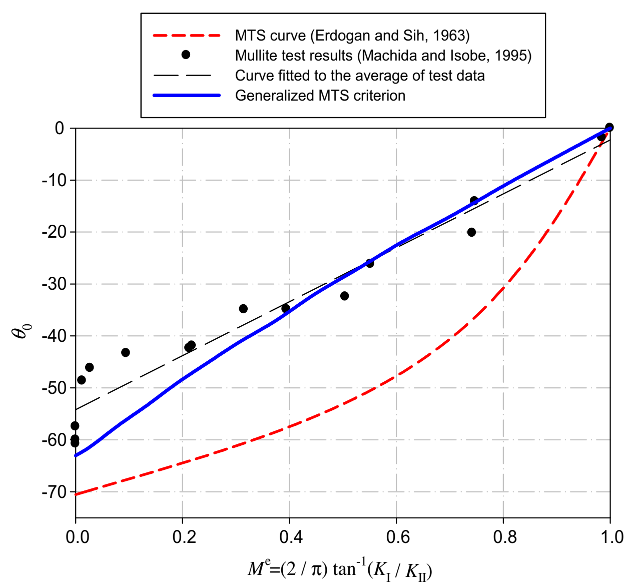

Fig. 7. Predictions of generalized MTS criterion for mixed mode fracture angle of mullite tested using the BD specimen.

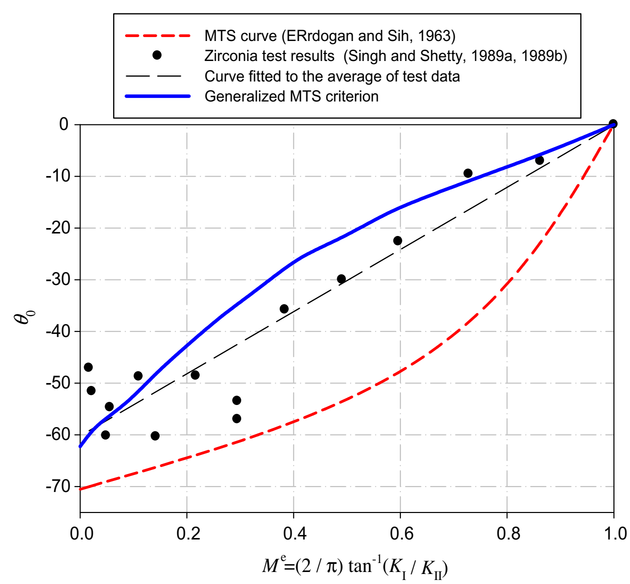

Fig. 9. Predictions of generalized MTS criterion for mixed mode fracture angle of zirconia tested using the BD specimen.

stresses are dominated by the singular stress term and hence the higher order terms in Eq. (3) can be ignored. However, at the critical distance r_c where brittle fracture is assumed to initiate, the singular stresses drop and the T-stress is no longer negligible. Hence, for those test configurations like BD specimen which has a significant value of T-stress, ignoring the effect of T-stress may introduce significant discrepancies between the experimental results and the theoretical predictions. To justify the improved predictions of the generalized MTS criterion, one may suggest that at a micro-structurally important distance from the crack tip like r_c, the singular term in Eq. (3) is not sufficient for describing the stresses that give rise to the initiation of crack growth. Therefore, the generalized MTS criterion that in addition to the conventional singular stress term takes the effect of T-stress into account is able to provide more accurate estimates for the experimentally determined fracture initiation angles.

It should be noted that in practical applications, the prediction of critical fracture load or fracture toughness is also an important objective in mixed mode crack problems. However, the estimated values of the mixed mode fracture toughness depend significantly on the accuracy of the fracture initiation angle θ_0. According to the second hypothesis of the generalized MTS criterion, the onset of mixed mode fracture can be determined from (Smith et al., 2001):

$$K_{Ic} = \cos\frac{\theta_0}{2}\left[K_{If}\cos^2\frac{\theta_0}{2} - \frac{3}{2}K_{IIf}\sin\theta_0\right] + \sqrt{2\pi r_c}T\sin^2\theta_0 \tag{14}$$

where K_{If} and K_{IIf} are the critical mode I and mode II stress intensity factors corresponding to the fracture load. In order to use Eq. (14), the angle θ_0 should be first determined from Eq. (5) or Eq. (8). This implies that the T-stress affects Eq. (14) not only directly thorough the last term in the right hand side of this equation but also indirectly through the angle θ_0. The fracture trajectory is also another important issue in many crack growth problems. It is clear that the path of crack growth in mixed mode fracture depends significantly on the initial direction of crack growth (θ_0). Thus, in order to provide suitable assessments for the fracture trajectory in mixed mode crack problems, it is necessary to consider the influence of major affecting parameters (i.e., the stress intensity factors together with the *T*-stress) through the generalized MTS criterion for the initial stage of crack growth.

Ayatollahi and Aliha (2009, 2011) have recently used the same values of K_I, K_{II}, T and r_c (given in this paper for each ceramic) and demonstrated that the generalized MTS criterion is able to predict very well the mixed mode fracture resistance of the investigated glass, sialon, mullite, SiC, alumina and zirconia ceramics. Therefore, it can be concluded that the generalized MTS criterion provides good procedures not only for estimating the mixed mode fracture angle but also for predicting the onset of mixed mode fracture for all the ceramics investigated in this paper and tested with the BD specimens.

The generalized MTS criterion can be considered as an extension of the critical distance theory when dealing with mixed mode crack problems. The concept of critical distance was first proposed by Peterson (1950) for investigating the fatigue behavior of notched components and by Novozhilov (1969) to estimate the onset of brittle fracture under static loading. Its application was later extended to mixed mode fracture by Williams and Ewing (1972) using the MTS criterion and by Sih and Kipp (1974) and Kipp and Sih (1975) using the SED criterion. Indeed, the concept of core region presented by Sih in the SED criterion is very similar to that of the critical distance in the stress based fracture theories. Many researchers have attempted in the past 40 years to suggest appropriate theoretical models to calculate the critical distance (sometimes called the characteristic length) for different brittle and quasi-brittle materials. For instance, Weiss (1971) and more recently Gomez and Elices (2006) employed the Hillerborg's model (Hillerborg et al., 1976 and Hillerborg, 1985) to propose mathematical relations for the size of critical distance, particularly for quasi-brittle materials.

The determination of critical distance from the crack tip r_c is an important issue in using the generalized MTS criterion too. Ayatollahi and Aliha (2011) made use of a Dugdale type model for the size of damage zone around the crack tip and proposed a simple relation for estimating the value of r_c for ceramics in terms of their mode I fracture toughness and tensile stress. The same relation was used in this paper (Eq. (13)) for estimating the angle of fracture initiation. It is useful to note that the values of critical distance obtained in this research for different ceramics are within the range of values given by Gomez and Elices (2006) for the characteristic length of similar ceramics. The results presented in Ayatollahi and Aliha (2011) and also in this paper show that based on the simple relation suggested for r_c the generalized MTS criterion provides very good estimates not only for the fracture angle but also for the mixed mode fracture resistance of various types of ceramics reported earlier in several independent papers. However, the proposed model still needs further investigation particularly from a microstructural point of view. This is because in the processes of crack growth, the microstructural properties of the damage zone developing around the crack tip are sometime not the same for different brittle and quasi-brittle materials. Another important point in mixed mode fracture testing of ceramics is the level of sharpness in the pre-cracks generated in the ceramic samples. While the crack tip radius in our analyses is always assumed to be zero, in practice the crack tips in the laboratory specimens are often slightly blunt. Blunt crack tips can also affect the test results to some extent.

6. Conclusions

1. The conventional fracture criteria fail to provide good estimates for the direction of fracture initiation in the BD specimens made of ceramics and subjected to mixed mode loading. Generally, the conventional fracture criteria overestimate the test data reported for the mixed mode fracture angle in the BD specimen.
2. The generalized MTS criterion which takes into account the influence of *T*-stress in addition to the stress intensity factors could estimate well the angle of fracture initiation for fracture tests conducted on glass, sialon, mullite, SiC, alumina and zirconia.
3. The negative *T*-stresses that exist in the BD specimen decrease noticeably the angle of fracture initiation.

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