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# A Simplified Posteriori Estimator to Cascade Channel based on Amplify-and-Forward Multi-Relaying Systems

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**Abstract**

This paper analyzes the cascade channel estimation of Amplify-and-Forward (AF) multi-relaying systems. To improve the accuracy, a maximum a posteriori (MAP) estimator is obtained with the prior probability distribution function (PDF) of the cascade channel. For lessening the computational amount of the MAP method, the complicated generalized- $K$ distribution is approximated with a Gamma PDF based on the moment-matching method. Moreover, a closed-form and simplified MAP (sMAP) estimator is derived. Numerical simulation shows that the proposed sMAP estimator decreases the computational duration sharply than the MAP method with marginal performance loss, and outperforms its least square (LS) counterpart in the low SNR region.


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*Keywords:* Amplify-and-Forward (AF); multi-relaying; cascade channel estimation; bayesian theory

## 1. Introduction

In recent years, much attention has been attracted on Amplify-and-Forward (AF) relay networks due to its prominent performance and low complexity[1-3]. By exploiting cooperative diversity, the relay networks connect remote communication users and improve the link quality. To explore the system performance, some emerging techniques, such as relay beamforming[4], relay selection[5], relay antenna selection[6] and network space-time coding[7], require the knowledge of current channel state information (CSI). For this purpose, training signals are often utilized in channel estimation. Therefore, it is critical to design a channel estimator with low computational complexity and high accuracy.

Channel estimation for AF relay networks is recently considered. In reference 8, a channel estimation scheme was proposed in two phases. The scheme requires the relay to produce and broadcast training signals to the destination

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in the first phase, the known training signals were identical to what the relay receives from the source in the second phase. The shortcoming is that the estimation processing consumed extra wireless resource. In reference 9, the least square (LS) estimator on the relaying channel were presented. Both the researches could belief that the relaying channel estimation follows the same approach of the ordinary transmitters and receivers scenario. However, in a relaying system, the two independent disintegrate channels between source and destination via relaying node can be connected as a cascade channel. This cascade channel is quite different from a typical Rayleigh or Rice channel. At the meanwhile, the relay only amplifies and forwards the mixed signals, which simplifies the maintenance and the control of the system. To minimize the MSE of the channel estimator, the authors[10, 11] obtained the training sequence by solving the energy-constrained optimization problem. With the Rayleigh channel assumption, a maximum a posteriori (MAP) estimator was proposed[12], based on the expectation-maximization (EM) method. A closed-form expression of Bayesian Cramer-Rao bound was derived as well. However, the undermining drawback of this scheme is the huge computational amount of the non-convex optimization in the maximization part.

In this paper, a MAP estimator for the cascade channel is proposed. The accuracy of the estimator is improved with the prior knowledge of the cascade channel. Based on the Rayleigh model, the cascade channel can be represented as the inner product of two complex Gaussian random vectors with zero-mean. Furthermore, it is proved that this random variable (RV) subjects to the generalized-K distribution[13]. However, it is not straightforward to derive a closed-form MAP estimator because of the Bessel function in the prior distribution. Meanwhile, numerical approach, which would cost enormous computation, is unacceptable for the relay networks. Therefore, a feasible solution is to approximate the original probability distribution function (PDF) with a Gamma distribution through matching their moments[14]. This approximation is verified by an introduced parameter, normalized relative PDF error, and a simplified MAP estimator is obtained consequently. Besides, for comparing the performance of the estimators, approximated Bayesian Cramer-Rao bound (ABCRB) is also derived according to the maximum entropy theory[15]. Numerical simulation demonstrates that this MAP substitution reduces the computational amount remarkably, and shows the superiority to its LS counterpart in the low SNR region especially.

The remainder of this paper is organized as follows: section II describes the system model; section III derives the MAP estimators of the cascade channel, meanwhile, its simplified substitution is explored based on the moment-matching method; in section IV, the ABCRB of the posteriori estimators are deduced according to the maximum entropy theory; numerical simulation in section V verifies the advantage of the simplified estimator; while section VI gives some concluding remarks.

Notation: we use lower case letter, upper and lower case boldface to denote scalar, matrix and vector, respectively. The conjugation, transpose, and Hermitian of matrix $\boldsymbol{A}$ are represented as $\boldsymbol{A}^*$, $\boldsymbol{A}^T$ and $\boldsymbol{A}^H$, respectively. Symbol $\boldsymbol{I}$ is the identity matrix. $\otimes$ and $\odot$ means the Hadamard and Kronecker products, respectively, and $E(\cdot)$ stands for expectation of a RV.

## 2. System model

As shown in Fig.1, a single-antenna user A needs to send messages to another single-antenna user B. Owning to the heavily fading channel between them, relays are indispensable for the communication. The signal transmission is fulfilled in two phases. During the first phase, A transmits training symbols to $m$ potential relays; in the second phase, the relays amplify the received signal and broadcast the replicas to B from different paths. Here, we define the cascade channel as the integral effect of the signal propagation from A to B via multi-relaying.

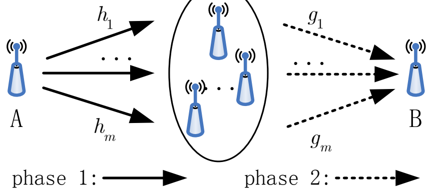



Fig.1 multi-relaying AF communication system

During the cascade channel estimation, the channels are assumed to be quasi-static flat fading, which means they do not change within one training frame. We also assume that one training frame contains $N$ slots, therefore, the received training signals of multi-relaying at the slot $j$, $j=1,\ldots,N$, and the corresponding replicas of B can be denoted as

$$\begin{cases} \boldsymbol{r}_j = \boldsymbol{T}_j \boldsymbol{h} + \boldsymbol{n}_{1j} \\ y_j = \alpha \boldsymbol{g}^{\mathrm{H}} \boldsymbol{T}_j \boldsymbol{h} + \alpha \boldsymbol{g}^{\mathrm{H}} \boldsymbol{n}_{1j} + n_{2j} \end{cases} \tag{1}$$

where the vectors $\boldsymbol{h}=(h_1,h_2,\ldots,h_m)^T$ and $\boldsymbol{g}=(g_1,g_2,\ldots,g_m)$ stand for the disintegrated channels A-relays and those of relays-B, respectively. Besides, both $\boldsymbol{h}$ and $\boldsymbol{g}$ are assumed as zero-mean circularly symmetric complex Gaussian random vectors. Therefore, we have $h_i \sim \mathcal{CN}(0,\sigma_h^2)$ and $g_i \sim \mathcal{CN}(0,\sigma_g^2)$, $i=1,\ldots,m$, where $\sigma_h^2$ and $\sigma_g^2$ are the variances, respectively. Diagonal matrix $\boldsymbol{T}_j = \mathrm{diag}(t_j \otimes \boldsymbol{1}_{1\times N})$ represents the training signals with its diagonal elements $|t_j|^2 = P$. $\boldsymbol{n}_{1j}$ and $n_{2j}$ are additive white Gaussian noise (AWGN) with variance $\sigma_n^2$ which can be denoted as vectors $\boldsymbol{n}_1 = (n_{11},\ldots,n_{1m})^T$ and $\boldsymbol{n}_2 = (n_{21},\ldots,n_{2m})^T$. Factor $\alpha$ is a scale related to the relay power $P_r$, typically defined as $\alpha = \sqrt{P_r / (\sigma_h^2 P + \sigma_n^2)}$. Since each element of sequence shares the same power, (1) can be rewritten as

$$\frac{\boldsymbol{t}^H \odot \boldsymbol{y}}{\alpha P} = \boldsymbol{g}^T \boldsymbol{h} \cdot \boldsymbol{1} + \frac{\boldsymbol{t}^H}{P} \odot \left( \boldsymbol{g}^T \boldsymbol{N}_1 + \frac{\boldsymbol{n}_2}{\alpha} \right) \tag{2}$$

with $\boldsymbol{t} = (t_1,\ldots,t_N)^{\mathrm{T}}$, $\mathbf{y} = (y_1,\ldots,y_N)^T$ and $\boldsymbol{N}_1 = \boldsymbol{n}_1 \otimes \boldsymbol{1}_{1\times N}$. $b \triangleq \boldsymbol{g}^T \boldsymbol{h}$ is the cascade channel to estimate, and the second term of the right-hand side of (2) represents the zero-mean interference, with variance $\sigma^2 = (m\sigma_h^2 + \alpha^{-2}) \cdot \sigma_n^2 / P$ [16]. For clarity, we define $\boldsymbol{z} = \boldsymbol{t}^H \odot \mathbf{y} / \alpha P$.

Theoretically, more relays introduced in cooperation would achieve the better performance in the systems. However, scenario with large $m$ is not considered for the following reasons:

1. With the arising number of cooperative relays, the synchronization and management of the communication results in an upsurge of scheduler complexity.
2. The relaying nodes, also users have to consume large amount of power and frames to cooperative with others, which impair their own communication demands.
3. It is pointed that the computational complexity of relay selection is exponential with relaying number $m$ [17], which would be burdensome for the networks otherwise.

Based on all these facts, the relaying number $m$ beyond 3 is not recommended in this paper.

## 3. Cascade channel estimators

In this section, the maximum a posteriori (MAP) estimator of $b$ are proposed. The MAP method deems $b$ as a RV which can be utilized as prior knowledge. The accuracy of estimation is improved through this knowledge. the cascade channel $b$ is seen to be equivalent to inner product of two zero-mean independent complex Gaussian vectors. Therefore, we have the following theorem.

**Theorem 1:** The inner product of two $m$-length zero-mean independent complex Gaussian vectors, with variances $\sigma_h^2$ and $\sigma_g^2$, is

$$p(b)=\frac{2|b|^m}{\pi\cdot\Gamma(m)\cdot(\sigma_h\sigma_g)^{m+1}}\cdot K_{m-1}\left(\frac{2|b|}{\sigma_h\sigma_g}\right) \tag{3}$$

where $\Gamma(\cdot)$ is Gamma function, and $K_{m-1}(\cdot)$ is the second kind modified Bessel function with order $m-1$.

Proof: a detailed proof is given in reference 18.

Since the prior PDF of $b$ is obtained according to the Theorem 1, the MAP estimator of $b$ can be listed as

$$\hat{b}_{MAP}=\arg\max_{b} p(z|b)p(b) \tag{4}$$

with the conditional PDF $p(z|b)$ as

$$p(z|b)=\frac{1}{\pi\sigma^2}\exp\left(-\frac{1}{\sigma^2}\sum_{i=1}^{N}|z_i-b|^2\right) \tag{5}$$

Substituting (3) and (5) into (4), the MAP estimator for $b$ can be obtained by maximizing the following expression.

$$\begin{cases}\left(|\hat{b}|\right)_{\mathrm{MAP}}=\arg\max_{r\in\mathbb{R}} r^m K_{m-1}\left(\frac{2r}{\sigma_h\sigma_g}\right)\exp\left(-\frac{Nr^2}{\sigma^2}+\frac{2r}{\sigma^2}\sum_{i=1}^{N}|z_i|\right)\\ \left(\angle\hat{b}\right)_{\mathrm{MAP}}=\angle\left(\sum_{i=1}^{N}z_i\right)\end{cases} \tag{6}$$

The MAP estimator derived in (6) utilizes the prior PDF of $b$ and the approximation $\sum_{i=1}^{N}|z_i-b|^2 \doteq \sum_{i=1}^{N}(|z_i|-b)^2$. However, it is not straightforward to obtain analytical result due to the arising Bessel functions, which would cost much computation. Here, an approximation of the prior PDF by a more tractable Gamma distribution through moment-matching method is studied. This manipulation achieves considerable calculation reduction only in the price of marginal performance loss. Since the cascade channel $b$ mainly gathers near the peak region of the PDF, we can conclude that the moments of interest are the first two positive ones[14].

According to (3), the first and second moments of RV $|b|$ can be obtained as

$$\begin{cases}E(|b|)=\frac{\sqrt{\pi}}{2}\sigma_h\sigma_g\frac{\Gamma(m+1/2)}{\Gamma(q)}\\ E\left(|b|^2\right)=m\sigma_h^2\sigma_g^2\end{cases} \tag{7}$$

At the meanwhile, the PDF of a Gamma distributed RV $r$, with shape parameter $k$ and scale parameter $\theta$, can be denoted as

$$p_\Gamma(r)=\frac{\theta^{-k}}{\Gamma(k)}r^{k-1}\exp\left(-\frac{r}{\theta}\right)\quad r\geq 0 \tag{8}$$

The corresponding positive integer moments are

$$E_{\Gamma}\left(r^{n}\right)=\Gamma\left(k+n\right)\theta^{n}/\Gamma\left(k\right) \tag{9}$$

Matching the first and second moment of $|b|$ and $r$, we have

$$\begin{cases} k\theta = \dfrac{\sqrt{\pi}}{2}\sigma_h\sigma_g \dfrac{\Gamma\left(m+1/2\right)}{\Gamma\left(q\right)} \\ \left(k+1\right)k\theta^2 = m\sigma_h^2\sigma_g^2 \end{cases} \tag{10}$$

Meanwhile, the two parameters $k$ and $\theta$ can be obtained

$$\begin{cases} \theta = \dfrac{2m\sigma_h\sigma_g\Gamma\left(q\right)}{\sqrt{\pi}\Gamma\left(m+1/2\right)} - \dfrac{\sqrt{\pi}}{2}\sigma_h\sigma_g \dfrac{\Gamma\left(m+1/2\right)}{\Gamma\left(m\right)} \\ k = \dfrac{\sqrt{\pi}}{2}\sigma_h\sigma_g \dfrac{\Gamma\left(m+1/2\right)}{\Gamma\left(m\right)} \cdot \dfrac{1}{\theta} \end{cases} \tag{11}$$

The prior PDF (3) is replaced by the Gamma distribution (8), with the calculated parameters $k$ and $\theta$ in (11). The simplified MAP (labelled as sMAP) estimator is derived after the similar algebraic manipulation of (4) and (6), which is

$$\begin{cases} \left(\left|\hat{b}\right|\right)_{sMAP} = \max\left(\dfrac{-c_2+\sqrt{c_2^2-4c_1c_3}}{2c_1},0\right) \\ \left(\angle\hat{b}\right)_{sMAP} = \angle\left(\sum_{i=1}^{N} z_i\right) \end{cases} \tag{12}$$

where the coefficients are $c_1 = \dfrac{2N}{\sigma^2}$, $c_2 = \dfrac{1}{\theta} - \dfrac{2}{\sigma^2}\left|\sum_{i=1}^{N} z_i\right|$ and $c_3 = 1-k$.

## 4. Performance bound of the proposed estimators

In Bayesian framework, the improved accuracy of a posteriori estimator is attributed to the prior PDF, whose the Bayesian Fisher Information Matrix (BFIM) is

$$J_b' = \underbrace{E\left[\left(\frac{\partial \ln p\left(z|b\right)}{\partial b^*}\right)\left(\frac{\partial \ln p\left(z|b\right)}{\partial b^*}\right)^{\mathrm{H}}\right]}_{J_b} + E\left[\left(\frac{\partial \ln p\left(b\right)}{\partial b^*}\right)\left(\frac{\partial \ln p\left(b\right)}{\partial b^*}\right)^{H}\right] \tag{13}$$

where $J_b = \sigma^2/N$ means the FIM (19) of scale $b$. However, the Bessel function in the prior PDF causes intractability for deriving $J_b'$ in closed form. An approximation of $J_b'$, denoted as $J_{b'}'$, can be obtained based on the maximum entropy theorem depicted as following: Gaussian PDF maximizes entropy among all distributions with certain mean and variance.

According to the variance of zero-mean RV $b$, the corresponding Gaussian PDF is

$$p(b') = \frac{1}{\pi m \sigma_h^2 \sigma_g^2} \exp\left(-\frac{|b'|}{\pi m \sigma_h^2 \sigma_g^2}\right) \tag{14}$$

Consequently, the ABCRB can be derived as

$$\operatorname{var}(b) \geq \frac{1}{J'_b} \approx \frac{1}{J'_{b'}} = \frac{m\sigma^2 \sigma_h^2 \sigma_g^2}{mN\sigma_h^2 \sigma_g^2 + \sigma^2} \tag{15}$$

## 5. Numerical results

In this section, the performance of cascade channel estimators given above is compared. The disintegrated channels $h$ and $g$, the noise are all zero-mean circularly symmetric complex Gaussian RV with unit variances. The parameter $N$ is set to 4, and the ratio of relay power $P_r$ to $P$ is 0.1, which means that the relaying nodes only spend one tenth of their transmission power in cooperation. The signal-to-noise-ratio (SNR) is defined as $P/\sigma_n^2$. The simulated cascade channel $b$ is generated as the inner product of random vectors $\boldsymbol{h}$ and $\boldsymbol{g}$ with the number of relays $m$, and stays unchanged during estimation. As a comparison merit, the mean square error (MSE) of estimators is calculated after about $10^5$ Monte Carlo runs.

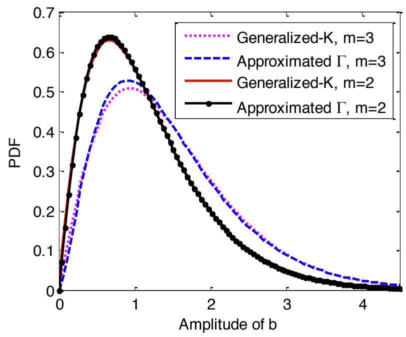



Fig.2 PDFs of Generalized-K and approximated Gamma

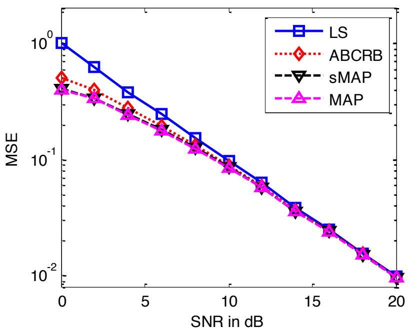
Fig.2 illustrates the curves of original generalized-K PDF and the approximated Gamma PDF where $m$ is set to 2 and 3. It is obvious that a good fit can be obtained through the moment-matching method. The single-peaked and long-tailed character of prior PDF is well portrayed by the Gamma PDF. The proposed Gamma approximation of $b$ can be validated by the normalized PDF relative error $\varepsilon$, which is defined as the average relative value of the absolute difference between the two distributions

$$\varepsilon = \frac{1}{R} \sum_{i=1}^{R} \frac{|p(r_i) - p_\Gamma(r_i)|}{p(r_i)}$$

where $1/R$ is the normalized coefficient. In the region of $[0, 4.5]$, the errors for $m = 2$ and $m = 3$ are 1.81% and 4.26%, respectively. This result demonstrates that the divergence of the two distributions is marginal, and the PDF approximation is feasible.

Fig.3 and Fig.4 display the MSE performances of the LS[9], MAP, simplified MAP (sMAP) estimators with $m = 2$ and $m = 3$. It can be seen that the two posteriori estimators outperform LS remarkably with low SNR, and all the performances coincide with high SNR. Besides, we may notice that the MSE performances of the MAP and the sMAP estimators break the ABCRB, this is because that both the entropies of the distributions (3) and (8) are less than that of (14) when they share the identical variances. The performance loss between the sMAP and the MAP estimators, which can be measured by the gap against the two posteriori MSE curves, is negligible. This phenomenon validates the PDF substitution through the proposed moment-matching method.

Low computational complexity is an important advantage of the proposed estimator. Fig.5 exhibits the average consumed time for the two methods. For the both estimators, it seems that the number of relaying nodes $m$ has nothing to do with the duration of the estimations. Besides, the estimation duration of the MAP estimator is almost $10^4$ times to that of the sMAP ones.



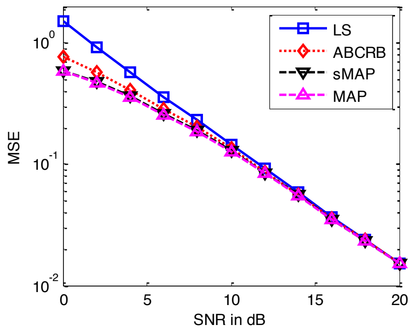




Fig.3 Comparison of estimators ( $m = 2$ ) Fig.4 Comparison of estimators ( $m = 3$ )

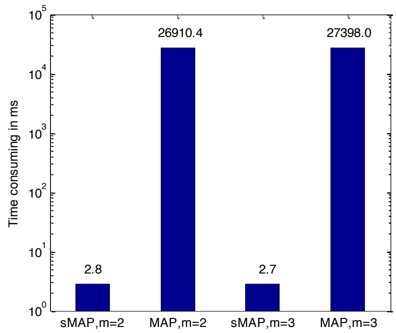



Fig.5 Time consuming results of MAP and sMAP estimators

## 6. Conclusions

In order to estimate the cascade channel of AF multi-relaying systems with single-antenna nodes, an MAP estimator was proposed. The MAP method improves the estimation accuracy through the prior PDF. Besides, with the prior distribution approximated by a Gamma PDF, a simplified MAP estimator was deduced which declines the computational complexity sharply. For comparison among estimators, the ABCRB of the posteriori estimators was also derived. Numerical simulation indicated that the proposed estimator outperform the LS counterpart, especially in the low SNR region.

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