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Harmless Delays and Global Attractivity for Nonautonomous Predator-Prey System with Dispersion

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Abstract—In this paper, we consider a nonautonomous predator-prey model with dispersion and a finite number of discrete delays. The system consists of two Lotka-Volterra patches and has two species: one can disperse between two patches, but the other is confined to one patch and cannot disperse. Our purpose is to demonstrate that the time delays are harmless for uniform persistence of the solutions of the system. Furthermore, we establish conditions under which the system admits a positive periodic solution which attracts all solutions.

Keywords—Harmless delays, Global attractivity, Dispersion.

1. INTRODUCTION

A number of models in ecology can be formulated as systems of differential equations with time delays. One of the most important problems for this type of system is to analyze the effect of time delays on the stability of the system. May [1] has shown that if a time delay is incorporated into the resource limitation of the logistic equation, then it has destabilizing effect on the stability of the system. For some systems the stability switches can happen many times and the systems will eventually become unstable when time delays increase (see [2,3]). Gopalsamy and Aggarwala [4] has shown that for certain values of the delay, there occurs an unstable equilibrium with periodic oscillation. Freedman and Wu [5] discussed persistence in a delayed system by using the monotone dynamical systems theory developed by Smith [6]. By constructing suitable

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persistence functionals, Wang and Ma [7] obtained uniform persistence conditions for Lotka-Volterra predator-prey systems with a finite number of discrete delays. Their results suggest that delays are "harmless" for uniform persistence. Similar phenomena were observed by Zanolin [8] in delayed Kolmogorov competing species systems. By utilizing the results in [9], Cao *et al.* [10] studied uniform persistence for Kolmogorov-type predator-prey and competition models with per capita net growth rates that are dependent on time-delayed population densities. Kuang and Tang [11] also established sufficient conditions for uniform persistence in nonautonomous Kolmogorov-type delayed population models. See also [12–14]. For other related work, we refer to [15,16].

For general ODEs with dispersion, Levin [17] first established this kind of model for the autonomous Lotka-Volterra system. Kishimoto [18] and Takeuchi [19] also studied these kinds of models, but all the coefficients in the system they studied are constants. Song and Chen [20] extended the autonomous Lotka-Volterra system to a two species nonautonomous dispersion Lotka-Volterra system. Reference [21] extended the results of [20] to the continuous time delay, and investigated persistence of the populations and periodic behavior of the system.

In this paper, we consider a nonautonomous system consisted of two species predator-prey system with dispersion and a finite number of discrete delays. Thus, the model includes not only the dispersal processes, but also some of the past states of the system. Our purpose is to demonstrate that the time delays are harmless for uniform persistence of the solution of the system. Furthermore, we establish conditions under which the system admits a positive periodic solution which attracts all solutions.

2. ANALYSIS OF UNIFORM PERSISTENCE

In this paper, we consider the following Lotka-Volterra population model:

$$\begin{aligned} \dot{x}_1 &= x_1 \left[r_1(t) - \sum_{j=1}^{m} a_{1j}(t) x_1(t-\tau_{1j}(t)) - \sum_{j=1}^{m} b_{1j}(t) y(t-\rho_{1j}(t)) \right] + D_1(t)(x_2 - x_1), \\ \dot{x}_2 &= x_2 \left[r_2(t) - \sum_{j=1}^{m} a_{2j}(t) x_2(t-\tau_{2j}(t)) \right] + D_2(t)(x_1 - x_2), \\ \dot{y} &= y \left[-r_3(t) + \sum_{j=1}^{m} a_{3j}(t) x_1(t-\tau_{3j}(t)) - \sum_{j=1}^{m} b_{2j}(t) y(t-\rho_{2j}(t)) \right], \end{aligned} \tag{2.1}$$

with initial conditions

$$\begin{aligned} x_1(s) &= \varphi_1(s) \ge 0, & s &\in [-\tau, 0]; & \varphi_1(0) &> 0, \\ x_2(s) &= \varphi_2(s) \ge 0, & s &\in [-\tau, 0]; & \varphi_2(0) &> 0, \\ y(s) &= \psi(s) \ge 0, & s &\in [-\tau, 0]; & \psi(0) &> 0, \end{aligned} \tag{2.2}$$

where x_1 and y are the population density of prey species x and predator species y in patch 1, and x_2 is the density of prey species x in patch 2. Predator species y is confined to patch 1, while the prey species x can disperse between two patches. $D_i(t)$, $(i = 1, 2)$ are dispersion coefficients of species x. $r_i(t)$, $a_{ij}(t)$, $\tau_{ij}(t)$, $(i = 1, 2, 3)$; $b_{ij}(t)$, $\rho_{ij}(t)$, $(i = 1, 2;\ j = 1, 2, \ldots, m)$ are nonnegative bounded continuous functions. Not all of $a_{ij}(t)$ and not all of $b_{2j}(t)$ are zero. $\varphi_1(s)$, $\varphi_2(s)$, and $\psi(s)$ are continuous on the interval $[-\tau, 0]$ in which

$$\tau = \sup_{t \ge 0} \{\tau_{ij}(t),\ (i = 1, 2, 3),\ \rho_{ij}(t)(i = 1, 2;\ j = 1, 2, \ldots, m)\}.$$

We introduce the following notation: for any strictly positive function defined on $[t_0, \infty)$, we let

$$0 < f^l = \inf_{t \geq t_0} f(t) \leq f(t) \leq \sup_{t \geq t_0} f(t) = f^u < +\infty.$$

Let $z(t) = (x_1(t), x_2(t), y(t))$ denote the solution of system (2.1) corresponding to the initial conditions (2.2).

DEFINITION 2.1. *System (2.1) is said to be uniformly persistent if there exists a compact region $K_1 \subset \text{int } R^3_+$ such that every solution $z(t)$ of (2.1) with the initial conditions (2.2) eventually enters and remains in the region K_1.*

LEMMA 2.1. *Every solution $z(t)$ of system (2.1) with initial conditions (2.2) exists in the interval $[0, +\infty)$ and remains positive for all $t \geq 0$.*

PROOF. It is true because

$$\dot{x}_1|_{x_1=0} = D_1(t)x_2 > 0, \quad \text{for } x_2 > 0, \qquad \dot{x}_2|_{x_2=0} = D_2(t)x_1 > 0, \quad \text{for } x_1 > 0,$$
$$y(t) = y(0) \exp\left\{\int_0^t \left[-r_3(s) + \sum_{j=1}^m a_{3j}(s)x_1(s - \tau_{3j}(s)) - \sum_{j=1}^m b_{2j}(s)y(s - \rho_{2j}(s))\right] ds\right\}, \quad \text{for } y(0) > 0.$$

LEMMA 2.2. *Every solution $z(t)$ of system (2.1) with initial conditions (2.2) is bounded for all $t \geq 0$ and all these solutions are ultimately bounded.*

PROOF. We define

$$V(t) = \max\{x_1(t), x_2(t)\}.$$

Calculating the upper-right derivative of V along the positive solution of system (2.1), we have the following.

(P_1) If $x_1(t) > x_2(t)$ or $x_1(t) = x_2(t)$ and $\dot{x}_1(t) \geq \dot{x}_2(t)$,

$$\begin{aligned} D^+V(t) &= x_1(t)\left[r_1(t) - \sum_{j=1}^m a_{1j}(t)x_1(t - \tau_{1j}(t)) - \sum_{j=1}^m b_{1j}(t)y(t - \rho_{1j}(t))\right] \\ &\quad + D_1(t)(x_2 - x_1) \\ &\leq x_1(t)[r_1(t) - a_{1i}(t)x_1(t - \tau_{1i}(t))] \\ &\leq x_1(t)\left[r_1^u - a_{1i}^l \exp(-r_1^u\tau)x_1(t)\right]. \end{aligned}$$

(P_2) If $x_1(t) < x_2(t)$ or $x_1(t) = x_2(t)$ and $\dot{x}_1(t) \leq \dot{x}_2(t)$,

$$\begin{aligned} D^+V(t) &= x_2(t)\left[r_2(t) - \sum_{j=1}^m a_{2j}(t)x_2(t - \tau_{2j}(t))\right] + D_2(t)(x_1 - x_2) \\ &\leq x_2(t)[r_2(t) - a_{2i}(t)x_2(t - \tau_{2i}(t))] \\ &\leq x_2(t)\left[r_2^u - a_{2i}^l \exp(-r_2^u\tau)x_2(t)\right]. \end{aligned}$$

We let

$$M = \max\left\{\frac{(r_1^u + 1)\exp(r_1^u\tau)}{a_{1i}^l}, \frac{(r_2^u + 1)\exp(r_2^u\tau)}{a_{2i}^l}\right\}.$$

From (P_1) and (P_2), we derive

(I) if $\max\{x_1(0), x_2(0)\} \leq M$, then $\max\{x_1(t), x_2(t)\} \leq M \quad t \geq 0$,

(II) if $\max\{x_1(0), x_2(0)\} > M$, and let $-\alpha = \max_{j=1,2}\{M(r_j^u - a_{ji}^l M \exp(-r_j^u\tau))\}$ $(\alpha > 0)$, we consider the following three possibilities:

(i) $V(0) = x_1(0) > M, \quad (x_1(0) > x_2(0))$,

(ii) $V(0) = x_2(0) > M, \quad (x_1(0) < x_2(0))$,

(iii) $V(0) = x_1(0) = x_2(0) > M$.

If (i) holds, then there exists $\epsilon > 0$, such that if $t \in [0,\epsilon), V(t) > M$, and we have

$$D^+V(x_1(t),x_2(t)) = \dot{x}_1(t) < -\alpha < 0.$$

If (ii) holds, then there exists $\epsilon > 0$, such that if $t \in [0,\epsilon), V(t) = x_2(t) > M$ and also we have

$$D^+V(x_1(t),x_2(t)) = \dot{x}_2(t) < -\alpha < 0.$$

If (iii) holds, then there exists $\epsilon > 0$, such that if $t \in [0,\epsilon), V(t) = x_1(t) > M$ or $V(t) = x_2(t) > M$. Similar to (i) and (ii), we have

$$D^+V(x_1(t),x_2(t)) = \dot{x}_i(t) < -\alpha < 0, \qquad (i = 1 \text{ or } 2).$$

From investigating the above (i), (ii), and (iii), we can conclude that if $V(0) > M$, then $V(t)$ is strictly monotone decreasing with speed at least α, so there exists $T_1 > 0$ if $t \geq T_1$, we have

$$V(t) = \max\{x_1(t),\ x_2(t)\} \leq M.$$

Using the inequality

$$\dot{y}(t) \leq y(t)\left[\sum_{j=1}^{m} a_{3j}^u M - b_{2i}^l y(t-\rho_{2i}(t))\right], \qquad t \geq T_1,$$

it is found that there exists a constant $N > 0$, and a $T_2 \geq T_1 + \tau$ such that $y(t) \leq N$ for all $t \geq T_2$. Consequently, $z(t) = (x_1(t), x_2(t), y(t))$ is bounded and

$$0 < x_i(t) \leq M, \quad (i = 1,2); \qquad 0 < y(t) \leq N, \quad \text{for } t \geq T_2.$$

This completes the proof.

We let

$$\eta_1(t) = [r_1(t) - D_1(t)]\sum_{j=1}^{m} a_{3j}^l - r_3(t)\sum_{j=1}^{m} a_{1j}^u, \qquad \eta_2(t) = r_2(t) - D_2(t).$$

THEOREM 2.1. *If $\eta_i(t) > 0$, $(i = 1,2)$, then system (2.1) is uniformly persistent.*

PROOF. Construct the first continuous functional

$$V_1(t) = V_1(t,x_1,y) = (x_1(t))^{\alpha_1}(y(t))^{\beta_1}\exp[f_1(t)], \qquad \text{for } t > 0, \tag{2.3}$$

where

$$\alpha_1 = \sum_{j=1}^{m} a_{3j}^l, \qquad \beta_1 = \sum_{j=1}^{m} a_{1j}^u,$$

$$f_1(t) = -\sum_{j,k=1}^{m} a_{3j}^l b_{1k}^u \int_{t-\rho_{1k}(t)}^{t} y(s)\,ds - \sum_{j,k=1}^{m} a_{3j}^l a_{1k}^u \int_{t-\tau_{1k}(t)}^{t} x_1(s)\,ds$$
$$+ \sum_{j,k=1}^{m} a_{1j}^u a_{3k}^l \int_{t-\tau_{3k}(t)}^{t} x_1(s)\,ds - \sum_{j,k=1}^{m} a_{1j}^u b_{2k}^u \int_{t-\rho_{2k}(t)}^{t} y(s)\,ds.$$

Calculating the derivative of V_1 with respect to t along the solution of system (2.1), we have

$$\dot{V}_1(t) > V_1(t)\left[\alpha_1(r_1(t) - D_1(t)) - r_3(t)\beta_1 - \sum_{j,k=1}^{m}\left(a_{3j}^l b_{1k}^u + a_{1j}^u b_{2k}^u\right)y(t)\right].$$

We let

$$\Delta_1 = \sum_{j,k=1}^{m} \left(a_{3j}^l b_{1k}^u + a_{1j}^u b_{2k}^u\right).$$

Hence,

$$\dot{V}_1(t) > V_1(t)[\eta_1(t) - \Delta_1 y(t)].$$

Then $\eta_1(t) > 0$ by assumption. Choose $h_1 > 0$ so small such that $0 < h_1 \le (1/2)\min(N, \eta_1^l/\Delta_1)$, if $0 < y(t) \le h_1$, we have

$$\dot{V}_1(t) > \frac{\eta_1^l}{2} V_1(t). \tag{2.4}$$

Construct the second continuous functional

$$V_2(t) = V_2(t, x_2) = x_2 \exp[f_2(t)], \qquad \text{for } t > 0, \tag{2.5}$$

where

$$f_2(t) = -\sum_{j=1}^{m} a_{2j}^u \int_{t-\tau_{2j}(t)}^{t} x_2(s)\,ds.$$

Calculating the derivative of V_2 with respect to t along the solution of system (2.1), we have

$$\dot{V}_2(t) > V_2(t)\left[r_2(t) - D_2(t) - \sum_{j=1}^{m} a_{2j}^u x_2(t)\right].$$

We let

$$\Delta_2 = \sum_{j=1}^{m} a_{2j}^u.$$

Hence,

$$\dot{V}_2(t) > V_2(t)[\eta_2(t) - \Delta_2 x_2(t)].$$

Then $\eta_2(t) > 0$ by assumption. Choose $h_2 > 0$ so small such that $0 < h_2 \le (1/2)\min(M, \eta_2^l/\Delta_2)$, if $0 < x_2(t) \le h_2$ we have

$$\dot{V}_2(t) > \frac{\eta_2^l}{2} V_2(t). \tag{2.6}$$

Now construct the third continuous functional

$$V_3(t) = V_3(t, x_1, y) = (x_1(t))^{\alpha_2}(y(t))^{-\beta_2} \exp[f_3(t)], \qquad \text{for } t > 0, \tag{2.7}$$

where

$$\alpha_2 = \sum_{j=1}^{m} b_{2j}^l, \qquad \beta_2 = \sum_{j=1}^{m} b_{1j}^u,$$

$$\begin{aligned} f_3(t) = &-\sum_{j,k=1}^{m} b_{2j}^l a_{1k}^u \int_{t-\tau_{1k}(t)}^{t} x_1(s)\,ds - \sum_{j,k=1}^{m} b_{2j}^l b_{1k}^u \int_{t-\rho_{1k}(t)}^{t} y(s)\,ds \\ &- \sum_{j,k=1}^{m} b_{1j}^u a_{3k}^u \int_{t-\tau_{3k}(t)}^{t} x_1(s)\,ds + \sum_{j,k=1}^{m} b_{1j}^u b_{2k}^l \int_{t-\rho_{2k}(t)}^{t} y(s)\,ds. \end{aligned}$$

Calculating the derivative of V_3 with respect to t along the solution of system (2.1), we have

$$\dot{V}_3(t) > V_3(t)\left[\alpha_2\left(r_1(t) - D_1(t)\right) + r_3(t)\beta_2 - \sum_{j,k=1}^{m} \left(b_{2j}^l a_{1k}^u + b_{1j}^u a_{3k}^u\right) x_1(t)\right].$$

We let

$$\eta_3(t) = \alpha_2[r_1(t) - D_1(t)] + r_3(t)\beta_2 > 0, \qquad \Delta_3 = \sum_{j,k=1}^{m} \left(b_{2j}^l a_{1k}^u + b_{1j}^u a_{3k}^u\right).$$

Hence,

$$\dot{V}_3(t) > V_3(t)[\eta_3(t) - \Delta_3 x_1(t)].$$

Choose $h_3 > 0$ so small such that $0 < h_3 \le (1/2)\min(M, \eta_3^l/\Delta_3)$, if $0 < x_1(t) \le h_3$, we have

$$\dot{V}_3(t) > \frac{\eta_3^l}{2} V_3(t). \tag{2.8}$$

We can complete the proof by showing that system (2.1) is uniformly persistent under the hypotheses. In fact, if system (2.1) is not persistent, then there exists a solution $z(t) = (x_1(t), x_2(t), y(t))$ and a sequence $t_n : t_{n+1} > t_n, t_n \to \infty$, as $n \to \infty$, such that one of the following seven cases holds:

$$\text{(a)} \quad \lim_{n\to\infty} x_1(t_n) = 0; \quad \lim_{n\to\infty} x_2(t_n) > 0; \quad \lim_{n\to\infty} y(t_n) > 0; \tag{2.9}$$

$$\text{(b)} \quad \lim_{n\to\infty} x_1(t_n) > 0; \quad \lim_{n\to\infty} x_2(t_n) = 0; \quad \lim_{n\to\infty} y(t_n) > 0; \tag{2.10}$$

$$\text{(c)} \quad \lim_{n\to\infty} x_1(t_n) > 0; \quad \lim_{n\to\infty} x_2(t_n) > 0; \quad \lim_{n\to\infty} y(t_n) = 0; \tag{2.11}$$

$$\text{(d)} \quad \lim_{n\to\infty} x_1(t_n) = 0; \quad \lim_{n\to\infty} x_2(t_n) = 0; \quad \lim_{n\to\infty} y(t_n) > 0; \tag{2.12}$$

$$\text{(e)} \quad \lim_{n\to\infty} x_1(t_n) > 0; \quad \lim_{n\to\infty} x_2(t_n) = 0; \quad \lim_{n\to\infty} y(t_n) = 0; \tag{2.13}$$

$$\text{(f)} \quad \lim_{n\to\infty} x_1(t_n) = 0; \quad \lim_{n\to\infty} x_2(t_n) > 0; \quad \lim_{n\to\infty} y(t_n) = 0; \tag{2.14}$$

$$\text{(g)} \quad \lim_{n\to\infty} x_1(t_n) = 0; \quad \lim_{n\to\infty} x_2(t_n) = 0; \quad \lim_{n\to\infty} y(t_n) = 0. \tag{2.15}$$

If (2.9) holds, then it follows from (2.4) that

$$V_1(t_n) > V_1(t_m) > 0, \qquad \text{for } t_n > t_m, \tag{2.16}$$

such that $0 < y(t_n) < h_1$. On the other hand, we have from (2.3) that

$$V_1(t_n) = V_1(x_1, y)(t_n) = (x_1(t_n))^{\alpha_1} (y(t_n))^{\beta_1} \exp[f_1(t_n)] = (x_1(t_n))^{\alpha_1} g_1(t_n),$$

where

$$g_1(t_n) = (y_1(t_n))^{\beta_1} \exp[f_1(t_n)].$$

From the boundedness of the solutions of (2.1), one can see that $g_1(t_n)$ is also bounded. From (2.9), we derive that

$$\lim_{n\to\infty} V_1(t_n) = 0, \tag{2.17}$$

and (2.17) contradicts (2.16). Similarly using $V_1(t)$ or $V_2(t)$ or $V_3(t)$, we can show that (2.10) and (2.11) will also lead to a contradiction. Suppose now that (2.12) holds, by (2.4) and (2.6), it follows that

$$V_1(t_n) > V_1(t_m) > 0, \qquad V_2(t_n) > V_2(t_m) > 0, \quad \text{for } t_n > t_m, \tag{2.18}$$

such that $0 < y(t_n) \le h_1$, $0 < x_2(t_n) \le h_2$. Consider now $V_4(t)$ defined as follows:

$$V_4(t_n) = V_1(t_n)V_2(t_n) = (x_1(t_n))^{\alpha_1} x_2(t_n) (y(t_n))^{\beta_1} \exp[f_1(t_n) + f_2(t_n)]$$

in which $(y(t_n))^{\beta_1} \exp[f_1(t_n) + f_2(t_n)]$ is bounded, that

$$\lim_{n\to\infty} V_4(t_n) = 0. \tag{2.19}$$

But (2.18) implies that $V_4(t_n) > C_0$ for some $C_0 > 0$, $m > n$, and this contradicts (2.19). Similarly using $V_4(t)$, we can show that (2.13)–(2.15) will also lead to a contradiction. The proof of the uniform persistence of system (2.1) is now complete.

3. GLOBAL ATTRACTIVITY OF PERIODIC SOLUTION

In this section, we suppose that system (2.1) is a periodic system. We derive sufficient conditions for all positive solutions of (2.1) to converge to a periodic solution.

We let the following denotes the unique solution of periodic system (2.1) for initial value $Z^0 = \{x_1^0, x_2^0, y^0\}$:

$$Z\left(t, Z^0\right) = \left\{x_1\left(t, Z^0\right), x_2\left(t, Z^0\right), y\left(t, Z^0\right)\right\}, \qquad \text{for } t > 0,$$
$$Z\left(0, Z^0\right) = Z^0.$$

Now define Poincare transformation $A: R_+^3 \to R_+^3$ is

$$A\left(Z^0\right) = Z\left(\omega, Z^0\right),$$

here, ω is the period of periodic system (2.1). In this way, the existence of periodic solution of system (2.1) will be equal to the existence of the fixed point of A.

THEOREM (BROUWER). *Suppose that the continuous operator A maps closed and bounded convex set $Q \subset R^n$ onto itself, then the operator A has at least one fixed point in set Q.*

THEOREM 3.1. *If periodic system (2.1) satisfies $\eta_i(t) > 0$, $(i = 1, 2)$, then there is at least one strictly positive periodic solution of system (2.1).*

PROOF. If $\eta_i(t) > 0$, $(i = 1, 2)$ is satisfied, then from Theorem 2.1 we know that there exist $m_1 > 0$, $m_2 > 0$, $m > 0$ such that

$$x_1(t) \ge m_1, \quad x_2(t) \ge m_2, \quad y(t) \ge m. \tag{3.1}$$

Let

$$K_1 = \{(x_1, x_2, y) \mid m_1 \le x_1 \le M,\ m_2 \le x_2 \le M,\ m \le y \le N\},$$

then the compact region $K_1 \subset R_+^3$ is a positive invariant set of system (2.1), and K_1 also is a closed bounded convex set. So we have $Z^0 \in K_1 \Rightarrow Z(t, Z^0) \in K_1$, also $Z(\omega, Z^0) \in K_1$, thus $AK_1 \subset K_1$. The operator A is continuous because the solution is continuous about the initial value. Using the fixed point theorem of Brouwer, we can obtain that A has at least one fixed point in K_1, then there exists at least one strictly positive ω-periodic solution of system (2.1). This completes the proof of Theorem 3.1.

Now we consider the global attractivity of periodic solution. If $(x_1(t), x_2(t), y(t))$ denotes any solution of system (2.1), then we define u_1, u_2, and u_3 as follows:

$$u_1(t) = \ln x_1(t), \qquad u_2(t) = \ln x_2(t), \qquad u_3(t) = \ln y(t). \tag{3.2}$$

It is found from (2.1) that u_1, u_2, u_3 are governed by

$$\begin{aligned}
\frac{du_1}{dt} &= r_1(t) - \sum_{j=1}^{m} a_{1j}(t) e^{u_1(t-\tau_{1j}(t))} - \sum_{j=1}^{m} b_{1j}(t) e^{u_3(t-\rho_{1j}(t))} \\
&\quad + D_1(t)(e^{u_2-u_1} - 1), \\
\frac{du_2}{dt} &= r_2(t) - \sum_{j=1}^{m} a_{2j}(t) e^{u_2(t-\tau_{2j}(t))} + D_2(t)(e^{u_1-u_2} - 1), \\
\frac{du_3}{dt} &= -r_3(t) + \sum_{j=1}^{m} a_{3j}(t) e^{u_1(t-\tau_{3j}(t))} - \sum_{j=1}^{m} b_{2j}(t) e^{u_3(t-\rho_{2j}(t))}.
\end{aligned} \tag{3.3}$$

If (u_1, u_2, u_3) and (v_1, v_2, v_3) are any two solution of (3.3), then

$$\begin{aligned}
\frac{d[u_1 - v_1]}{dt} &= -\sum_{j=1}^{m} \left\{ a_{1j}(t) \left[e^{u_1(t-\tau_{1j}(t))} - e^{v_1(t-\tau_{1j}(t))} \right] + b_{1j}(t) \left[e^{u_3(t-\rho_{1j}(t))} - e^{v_3(t-\rho_{1j}(t))} \right] \right\} \\
&\quad + D_1(t) \left(e^{u_2 - u_1} - e^{v_2 - v_1} \right), \\
\frac{d[u_2 - v_2]}{dt} &= -\sum_{j=1}^{m} a_{2j}(t) \left[e^{u_2(t-\tau_{2j}(t))} - e^{v_2(t-\tau_{2j}(t))} \right] + D_2(t) \left(e^{u_1 - u_2} - e^{v_1 - v_2} \right), \\
\frac{d[u_3 - v_3]}{dt} &= \sum_{j=1}^{m} \left\{ a_{3j}(t) \left[e^{u_1(t-\tau_{3j}(t))} - e^{v_1(t-\tau_{3j}(t))} \right] - b_{2j}(t) \left[e^{u_3(t-\rho_{2j}(t))} - e^{v_3(t-\rho_{2j}(t))} \right] \right\}.
\end{aligned} \tag{3.4}$$

We define y_1, y_2, and y_3 as follows:

$$y_i(t) = u_i(t) - v_i(t), \qquad (i = 1, 2, 3). \tag{3.5}$$

We derive from (3.1), (3.4), and (3.5) that

$$\begin{aligned}
\frac{dy_1}{dt} &\le -\sum_{j=1}^{m} A_{1j}(t) y_1(t - \tau_{1j}(t)) - \sum_{j=1}^{m} B_{1j}(t) y_3(t - \rho_{1j}(t)) + C_2(t) y_2(t), \\
\frac{dy_2}{dt} &\le -\sum_{j=1}^{m} A_{2j}(t) y_2(t - \tau_{2j}(t)) + C_1(t) y_1(t), \\
\frac{dy_3}{dt} &= \sum_{j=1}^{m} A_{3j}(t) y_1(t - \tau_{3j}(t)) - \sum_{j=1}^{m} B_{2j}(t) y_3(t - \rho_{2j}(t)),
\end{aligned} \tag{3.6}$$

where

$$\begin{aligned}
A_{ij}(t) &= a_{ij}(t) e^{\theta_{ij}(t)}, \quad (i = 1, 2, 3), \qquad & B_{ij}(t) &= b_{ij}(t) e^{\delta_{ij}(t)}, \quad (i = 1, 2), \\
C_1(t) &= \frac{D_2(t) e^{\gamma_1(t)}}{m_2}, & C_2(t) &= \frac{D_1(t) e^{\gamma_2(t)}}{m_1},
\end{aligned}$$

$\theta_{ij}(t)$ lies between $u_i(t - \tau_{ij}(t))$ and $v_i(t - \tau_{ij}(t))$, $(i = 1, 2)$; $\delta_{ij}(t)$ lies between $u_3(t - \rho_{ij}(t))$ and $v_3(t - \rho_{ij}(t))$, $(i = 1, 2)$; $\theta_{3j}(t)$ lies between $u_1(t - \tau_{3j}(t))$ and $v_1(t - \tau_{3j}(t))$; $\gamma_i(t)$ lies between $u_i(t)$ and $v_i(t)$, $(i = 1, 2;\ j = 1, 2, \dots, m)$.

We return now to a further analysis of (3.6); for $t > t_0 + 2\tau$, we rewrite (3.6) as follows:

$$\begin{aligned}
\frac{dy_1}{dt} &\le -\sum_{j=1}^{m} A_{1j}(t) y_1(t) + \sum_{j=1}^{m} A_{1j}(t) \int_{t-\tau_{1j}(t)}^{t} \dot{y}_1(s)\, ds - \sum_{j=1}^{m} B_{1j}(t) y_3(t - \rho_{1j}(t)) \\
&\quad + C_2(t) y_2(t), \\
\frac{dy_2}{dt} &\le -\sum_{j=1}^{m} A_{2j}(t) y_2(t) + \sum_{j=1}^{m} A_{2j}(t) \int_{t-\tau_{2j}(t)}^{t} \dot{y}_2(s)\, ds + C_1(t) y_1(t), \\
\frac{dy_3}{dt} &= -\sum_{j=1}^{m} B_{2j}(t) y_3(t) + \sum_{j=1}^{m} B_{2j}(t) \int_{t-\rho_{2j}(t)}^{t} \dot{y}_3(s)\, ds + \sum_{j=1}^{m} A_{3j}(t) y_1(t - \tau_{3j}(t)).
\end{aligned} \tag{3.7}$$

We define

$$\tilde{y}_i(t) = \sup_{s \in [t-2\tau, t]} |y_i(s)|, \qquad (i = 1, 2, 3), \quad t > t_0 + 2\tau. \tag{3.8}$$

We let $\frac{D}{Dt}$ denote the upper-right derivative and derive from above that for $t \geq t_0 + 2\tau$,

$$\begin{aligned}
\frac{D|y_1(t)|}{Dt} &\leq -A_1^l |y_1(t)| + C_2^u |y_2(t)| + (A_1^u)^2 \tau \tilde{y_1} + A_1^u C_2^u \tau \tilde{y_2} + (B_1^u + A_1^u B_1^u \tau)\, \tilde{y_3}, \\
\frac{D|y_2(t)|}{Dt} &\leq C_1^u |y_1(t)| - A_2^l |y_2(t)| + A_2^u C_1^u \tau \tilde{y_1} + (A_2^u)^2 \tau \tilde{y_2}, \\
\frac{D|y_3(t)|}{Dt} &\leq -B_2^l |y_3(t)| + (A_3^u + A_3^u B_2^u \tau)\, \tilde{y_1} + (B_2^u)^2 \tau \tilde{y_3},
\end{aligned} \tag{3.9}$$

where

$$A_i(t) = \sum_{j=1}^{m} A_{ij}(t), \qquad B_i(t) = \sum_{j=1}^{m} B_{ij}(t).$$

Equation (3.9) can be rewritten as follows:

$$\left(\frac{D}{Dt}\right) \begin{bmatrix} |y_1(t)| \\ |y_2(t)| \\ |y_3(t)| \end{bmatrix} \leq P \begin{bmatrix} |y_1(t)| \\ |y_2(t)| \\ |y_3(t)| \end{bmatrix} + Q \begin{bmatrix} \tilde{y_1}(t) \\ \tilde{y_2}(t) \\ \tilde{y_3}(t) \end{bmatrix},$$

where

$$P = \begin{pmatrix} -A_1^l & C_2^u & 0 \\ C_1^u & -A_2^l & 0 \\ 0 & 0 & -B_2^l \end{pmatrix}, \qquad Q = \begin{pmatrix} (A_1^u)^2 \tau & A_1^u C_2^u \tau & B_1^u + A_1^u B_1^u \tau \\ A_2^u C_1^u \tau & (A_2^u)^2 \tau & 0 \\ A_3^u + B_2^u A_3^u \tau & 0 & (B_2^u)^2 \tau \end{pmatrix}.$$

If we assume that the matrix $-(P+Q)$ is an M-matrix, then by the result of [22], it follows that there exist positive numbers k_1, k_2, k_3, and δ such that

$$|y_i(t)| \leq k_i e^{-\delta t}, \qquad \text{for } t \geq t_0.$$

According to the above, we obtain the following.

THEOREM 3.2. *Suppose the coefficients of (2.1) and (3.6) satisfy $\eta_i(t) > 0$, $(i = 1, 2)$ and $-(P+Q)$ is an M-matrix, then system (2.1) has a unique globally attractive positive periodic solution.*

4. DISCUSSION

For one patch case, Wang and Ma [7] considered the following autonomous predator-prey system with a finite number of discrete delays:

$$\begin{aligned}
\dot{x} &= x \left[r_1 - \sum_{j=1}^{m} a_{1j} x(t - \tau_{1j}) - \sum_{j=1}^{m} b_{1j} y(t - \rho_{1j}) \right], \\
\dot{y} &= y \left[-r_3 + \sum_{j=1}^{m} a_{3j} x(t - \tau_{3j}) - \sum_{j=1}^{m} b_{2j} y(t - \rho_{2j}) \right].
\end{aligned} \tag{4.1}$$

If $r_1 \sum_{j=1}^{m} a_{3j} > r_3 \sum_{j=1}^{m} a_{1j}$, then system (4.1) is uniformly persistent.

Similarly, we can obtain the following result: if all coefficients in system (4.1) are time dependent, and $r_1(t) \sum_{j=1}^{m} a_{3j}^l > r_3(t) \sum_{j=1}^{m} a_{1j}^u$, then system (4.1) is uniformly persistent.

In this paper, we consider a predator-prey system in which the prey population can disperse between two patches and there are time delays in the self-regulation terms in both species. Moreover, all coefficients in system (2.1) are time dependent. We first show that the system is persistent independent of the time delay by choosing a Liapunov-type function. In the second part, we assume that all the coefficients are indeed periodic and prove that all solutions converge to a periodic solution of the system.

Our uniform persistence condition in Theorem 2.1 is

$$r_1(t)\sum_{j=1}^{m} a_{3j}^{l} > D_1(t)\sum_{j=1}^{m} a_{3j}^{l} + r_3(t)\sum_{j=1}^{m} a_{1j}^{u}, \qquad r_2(t) > D_2(t).$$

Obviously, the time delays and the smaller dispersion rates do not change the property of persistence.

From this paper, we can find the time delays and the smaller dispersion rates also have no effect on the existence of a positive periodic solution, but the time delays and the dispersion rates have an effect on the global attractivity of periodic solution.

We expect a similar technique to work in higher-dimensional systems with time delays and dispersion. We leave this investigation for future work.

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