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Radiative brane-mass terms in $D > 5$ orbifold gauge theories ☆

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Abstract

A gauge theory with gauge group $\mathcal{G}$ defined in $D > 4$ space–time dimensions can be broken to a subgroup $\mathcal{H}$ on four-dimensional fixed point branes, when compactified on an orbifold. Mass terms for extra-dimensional components of gauge fields A_i (brane scalars) might acquire (when allowed by the brane symmetries) quadratically divergent radiative masses and thus jeopardize the stability of the four-dimensional theory. We have analyzed $\mathbb{Z}_2$ compactifications and identified the brane symmetries remnants of the higher-dimensional gauge invariance. No mass term is allowed for $D = 5$ while for $D > 5$ a tadpole $\propto F_{ij}^{\alpha}$ can appear when there are $U_\alpha(1)$ factors in $\mathcal{H}$. A detailed calculation is done for the $D = 6$ case and it is established that the tadpole is related, although does not coincide, with the $U_\alpha(1)$ anomaly induced on the brane by the bulk fermions. In particular, no tadpole is generated from gauge bosons or fermions in real representations.

An important issue in any model of particle physics is how the mechanism of electroweak symmetry breaking is realized. In the Standard Model this is achieved by introducing the Higgs field. However, this phenomenologically well motivated mechanism comes with an undesired effect. The Higgs mass should be of the order of the electroweak symmetry breaking scale ($\sim 10^2$ GeV), which is unnaturally small compared to the ultraviolet (UV) cut-off of the Standard Model ($\sim 10^{19}$ GeV). In addition the hierarchy of scales is destroyed by radiative corrections and fine tuning is required to keep the Higgs light. This is commonly known as the hierarchy problem and one might wish to extend the Standard Model by supersymmetry to soften the UV sensitivity of the scalar sector.

However, introducing extra space–time dimensions opens new ways to solve the hierarchy problem. On the one hand, in the presence of transverse (as large as submillimeter) dimensions where only gravity propagates the scale of quantum gravity (string scale) can be lowered from the Planck scale to the TeV range [1,2], thus alleviating the problem of quadratic divergences. On the other hand, in the presence of (longitudinal) TeV extra dimensions [3] it is not necessary to introduce any fundamental scalars at all, but instead one could use the fact that the extra-dimensional components of gauge bosons are scalars under the four-dimensional (4D) Lorentz symmetry and transform non-trivially under the gauge symmetry they generate in the higher-dimensional theory. These

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scalars can be used to break electroweak symmetry spontaneously [4–8]. One might then conclude that higher-dimensional gauge invariance protects those scalars from being sensitive to the UV physics.

Orbifolds [9] play a prominent role in theories with extra dimensions due to their property to create chirality in the massless sector, an indispensable property in any phenomenologically relevant theory. Another interesting feature of orbifolds is their ability to break symmetries, in particular gauge and supersymmetry. While local symmetries remain intact in the bulk by an appropriate choice of parities for the transformation parameters, they are in general broken to smaller subgroups on the boundaries (the fixed points of the orbifold symmetry). As in any quantum field theory, in the effective action we must allow for all operators consistent with the symmetries. Allowed operators not present at tree level will be generated by radiative corrections [10–13].

The orbifold breaking of the bulk gauge symmetry proceeds by projecting out some fields, i.e., only a subset of the 4D gauge bosons A_μ and the 4D scalars A_i $(i = 5, \ldots, D)$ will be non-vanishing at the boundaries. While these A_μ generate the unbroken gauge group $\mathcal{H}$, the A_i transform in some representation of $\mathcal{H}$. It is then necessary to determine how the symmetries restrict possible brane localized operators of those fields, especially possible mass terms for the scalars [12]. Would $\mathcal{H}$ be the only symmetry left on the brane, mass terms for A_i would be perfectly allowed leading to a quadratic sensitivity to the UV cut-off.

In this Letter we demonstrate that the remnant symmetry on the brane is larger than the $\mathcal{H}$ gauge symmetry left over from the bulk. This provides a further restriction on the possible brane terms. We find that brane mass terms for scalars can only occur in $D \geqslant 6$ and only for $U(1)$ factors in $\mathcal{H}$ that were not already present in the bulk gauge group $\mathcal{G}$. These brane mass terms are radiatively generated by bulk fermions.

We will consider a gauge theory (gauge group $\mathcal{G}$) coupled to fermions in $D > 4$ dimensional space–time parametrized by coordinates $x^M = x^\mu, y^i$ where $\mu = 0, 1, 2, 3$ and $i = 5, \ldots, D$. The bulk Lagrangian is

$$\mathcal{L}_D = -\frac{1}{4} F^A_{MN} F^{AMN} + i\overline{\Psi}\gamma^M D_M \Psi, \tag{1}$$

where $F^A_{MN} = \partial_M A^A_N - \partial_N A^A_M - g f^{ABC} A^B_M A^C_N$ with the indices A, B, C running over the adjoint representation of $\mathcal{G}$ and f^{ABC} being the $\mathcal{G}$ structure constants. The local symmetry of (1) is the invariance under the (infinitesimal) gauge transformations

$$\delta_{\mathcal{G}} A^A_M = \frac{1}{g} D^{AB}_M \xi^B = \frac{1}{g} \partial_M \xi^A - f^{ABC} \xi^B A^C_M. \tag{2}$$

We now compactify the $p \equiv D - 4$ extra dimensions on the $T^p/\mathbb{Z}_2$ orbifold with all the radii of the torus equal to R[1] and with the $\mathbb{Z}_2$ action defined as $y^i \to -y^i$.

In the compactified theory the surviving gauge symmetry on the boundaries of the orbifold is a subgroup $\mathcal{H}$ of $\mathcal{G}$, according to the action of $\mathbb{Z}_2$ on the gauge fields

$$A\big(x^\mu, -y^i\big) = \mathcal{P}_{\mathcal{A}} A\big(x^\mu, y^i\big), \qquad \mathcal{P}_{\mathcal{A}} = \Lambda \otimes \mathcal{P}_1. \tag{3}$$

Here $\mathcal{P}_1$ acts on the vector indices and it is the diagonal matrix with eigenvalues $\alpha_\mu = +1, \alpha_i = -1$. Λ acts on the gauge indices and can also be taken diagonal. Its eigenvalues $\eta^A = \pm 1$ then define the breaking pattern. We split the bulk gauge index as $A = a, \hat{a}$ corresponding to the unbroken $(\eta^a = +1)$ and the broken generators $(\eta^{\hat{a}} = -1)$ respectively. The non-zero fields on the brane are the even fields, namely A^a_μ and $A^{\hat{a}}_i$, while $A^{\hat{a}}_\mu$ and A^a_i are odd and thus vanish on the brane. The orbifold consistency constraint on the structure constants comes essentially from the invariance of (1) and it provides the automorphism condition [14]

$$\eta^A \eta^B \eta^C = 1, \quad \text{for } f^{ABC} \neq 0. \tag{4}$$

Finally, in the gauge sector, the Faddeev–Popov ghosts c transform as the μ-components of the gauge fields, and for them the parity action is $\mathcal{P}_c = \Lambda$.

There are restrictions on the fermion representations as well. In even dimensions the bulk fermion representation has to be chosen anomaly free. Furthermore, for any number of extra dimensions, the resulting four-dimensional massless fermion spectrum must also be anomaly free. In addition, there are orbifold consistency conditions analogous to (4). The $\mathbb{Z}_2$ action on the fermions is

$$\Psi\big(x^\mu, -y^i\big) = \mathcal{P}_\Psi \Psi\big(x^\mu, y^i\big), \qquad \mathcal{P}_\Psi = \lambda \otimes \mathcal{P}_{\frac{1}{2}}, \tag{5}$$

[1] From now on we will work in units where $R \equiv 1$. Restoring the R dependence as well as introducing different radii R^i for different dimensions should be straightforward.

where λ is a matrix acting on the representation indices. The constraint comes from the requirement that the coupling $iA_M^A\bar{\Psi}\gamma^M T^A\Psi$ is $\mathbb{Z}_2$ invariant. One obtains [12] for any number of dimensions[2]

$$[\lambda, T^a] = 0, \qquad \{\lambda, T^{\hat{a}}\} = 0. \tag{6}$$

$\mathcal{P}_{\frac{1}{2}}$ is the orbifold action on the spinor indices and will be given explicitly later on.

The non-vanishing fields on the branes are of the general form

$$\prod_{i=5}^{D} \partial_i^{n_i}\Phi\bigg|_{\text{brane}} \equiv \partial^n\Phi, \tag{7}$$

where $n \equiv \sum_i n_i$ is even (odd) for even (odd) fields. Similarly, the gauge parameters ξ^a are even fields and $\xi^{\hat{a}}$ are odd. They couple to the branes according to (7).

The effective four-dimensional Lagrangian can be written as

$$\mathcal{L}_4^{\text{eff}} = \int d^p y \left[\mathcal{L}_D + \mathcal{L}_4^{\text{brane}} \prod_i \{\delta(y^i) + \delta(y^i - \pi)\}\right] \tag{8}$$

where $\mathcal{L}_D$ is given by (1) and $\mathcal{L}_4^{\text{brane}}$ should be the most general Lagrangian consistent with the symmetries. The latter can be nothing but the original bulk symmetry (2) modded out by the orbifold action and subsequently evaluated at the location of the brane. Let us call the transformation resulting from this operation δ_ξ. Applying this rule to (2) acting on the massless even fields, one obtains the transformations

$$\delta_\xi(A_\mu^a) = \frac{1}{g}\partial_\mu\xi^a - f^{abc}\xi^b A_\mu^c, \tag{9}$$

$$\delta_\xi(A_i^{\hat{a}}) = \frac{1}{g}\partial_i\xi^{\hat{a}} - f^{\hat{a}b\hat{c}}\xi^b A_i^{\hat{c}}. \tag{10}$$

In the above equations and in what follows, all fields should be interpreted as coupled to the brane in (8) according to (7).

The brane symmetry is however much larger than the transformations (9) and (10). In fact, there is an infinite number of non-zero independent fields on the brane, i.e., $\partial^{2k}\{A_\mu^a, A_i^{\hat{a}}\}$ and $\partial^{2k+1}\{A_\mu^{\hat{a}}, A_i^a\}$, and an infinite number of corresponding transformation parameters $\{\partial^{2k}\xi^a\}$ and $\{\partial^{2k+1}\xi^{\hat{a}}\}$ induced by the bulk. Using (2), one can derive the transformation of any non-zero brane field. We show explicitly only the first two at the next level:

$$\delta_\xi(\partial_j A_i^a) = \frac{1}{g}\partial_j(\partial_i\xi^a) - f^{a\hat{b}\hat{c}}(\partial_j\xi^{\hat{b}})A_i^{\hat{c}} - f^{abc}\xi^b(\partial_j A_i^c), \tag{11}$$

$$\delta_\xi(\partial_i A_\mu^{\hat{a}}) = \frac{1}{g}\partial_\mu(\partial_i\xi^{\hat{a}}) - f^{\hat{a}\hat{b}c}(\partial_i\xi^{\hat{b}})A_\mu^c - f^{\hat{a}b\hat{c}}\xi^b(\partial_i A_\mu^{\hat{c}}). \tag{12}$$

It is convenient to separate the above transformations into two different classes:

$$\delta_\xi = \delta_{\mathcal{H}} + \delta_{\mathcal{K}}$$
$$\text{with } \delta_{\mathcal{H}} = \{\xi^a\}, \quad \delta_{\mathcal{K}} = \{\partial^{2k}\xi^a, \partial^{2k+1}\xi^{\hat{a}}\}. \tag{13}$$

This is a natural separation because $\delta_{\mathcal{H}}$ is the surviving gauge transformation on the brane reflecting its $\mathcal{H}$ gauge invariance. One can see immediately by inspection of Eqs. (9)–(13) that A_μ^a are the gauge bosons of $\mathcal{H}$ while all other fields transform homogeneously in either the adjoint of $\mathcal{H}$, $(T^a)_{bc} = if^{abc}$, or in the representation spanned by $(T^a)_{\hat{b}\hat{c}} = if^{a\hat{b}\hat{c}}$.[3] The rest of the transformations is a set of local (but not gauge) transformations which we named $\delta_{\mathcal{K}}$.

Once the symmetries under which the brane action should be invariant are known, one can start constructing the allowed terms by these symmetries. A useful guiding principle in this task is the gauge symmetry $\mathcal{H}$. We know that it is a necessary condition that the building blocks should be $\mathcal{H}$-covariant combinations of the fields since this (and only this) can ensure that the square of these covariant objects are $\delta_{\mathcal{H}}$-invariant. Given a set of $\mathcal{H}$-covariant objects, invariance under $\delta_{\mathcal{K}}$ is a sufficient condition for their square to be invariant under both $\delta_{\mathcal{H}}$ and $\delta_{\mathcal{K}}$ and therefore to be an allowed terms in the effective action. The reason for which we required $\mathcal{K}$-invariance is because there is no notion of $\mathcal{K}$-covariance, since $\mathcal{K}$ is not a gauge symmetry. Thus, even though at this point we have not proved that $\mathcal{K}$-invariance is not only a sufficient but also a necessary condition, we will enforce it.

[2] Note that conditions (6) determine λ up to a sign.

[3] As a simple example consider the breaking $SU(3) \to SU(2) \otimes U(1)$. The adjoint of $SU(3)$, $f^{ABC} = \mathbf{8}$ then splits into the $SU(2)$ representations $f^{abc} = \mathbf{3} \oplus \mathbf{1}$ ($\mathcal{H}$ is not simple and hence its adjoint is reducible) and $f^{a\hat{b}\hat{c}} = \mathbf{2} \oplus \mathbf{2}$.

A simple and very important example is the field $A_i^{\hat{a}}$. By looking at (10) one can see that this field is indeed $\delta_{\mathcal{H}}$-covariant but not $\delta_{\mathcal{K}}$-invariant. A naive interpretation would then be that an explicit brane mass term as $(A_i^{\hat{a}} M_{\hat{a}\hat{b}} A_j^{\hat{b}})$ is forbidden in the four-dimensional effective action. However, as we will see below, under particular circumstances such a term can be part of a $\delta_{\mathcal{H}}$- and $\delta_{\mathcal{K}}$-invariant term in the Lagrangian in which case such a term can be generated radiatively.

The terms which are at the same time $\mathcal{H}$-covariant and $\mathcal{K}$-invariant are easily found from the transformation properties:

$$\delta_{\mathcal{H}} F_{\mu\nu}^a = -f^{abc}\xi^b F_{\mu\nu}^c, \qquad \delta_{\mathcal{K}} F_{\mu\nu}^a = 0 \tag{14}$$

$$\delta_{\mathcal{H}} F_{i\mu}^{\hat{a}} = -f^{\hat{a}b\hat{c}}\xi^b F_{i\mu}^{\hat{c}}, \qquad \delta_{\mathcal{K}} F_{i\mu}^{\hat{a}} = 0 \tag{15}$$

$$\delta_{\mathcal{H}} F_{ij}^a = -f^{abc}\xi^b F_{ij}^c, \qquad \delta_{\mathcal{K}} F_{ij}^a = 0. \tag{16}$$

Note the different structure of $F_{\mu\nu}^a \equiv \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - g f^{abc} A_\mu^b A_\nu^c$ and $F_{ij}^a \equiv \partial_i A_j^a - \partial_j A_i^a - g f^{a\hat{b}\hat{c}} A_i^{\hat{b}} A_j^{\hat{c}}$ in the non-linear terms. Further terms could be constructed from covariant derivatives of these operators. At the renormalizable level the following terms can appear in the Lagrangian:

$$\begin{aligned}\mathcal{L}_4^{\text{brane}} = &-\frac{1}{4}\mathcal{Z}_{ab} F_{\mu\nu}^a F^{b\mu\nu} \\ &- \frac{1}{4}\mathcal{Z}_{\hat{a}\hat{c}}^{ij} F_{i\mu}^{\hat{a}} F_j^{\hat{c}\mu} - \frac{1}{4}\mathcal{Z}_{ab}^{ijkl} F_{ij}^a F_{kl}^b + \mathcal{Z}_\alpha^{ij} F_{ij}^\alpha \\ &+ \mathcal{Z}_\alpha^{klij} D_k^{\alpha A} D_l^{AB} F_{ij}^B,\end{aligned} \tag{17}$$

where the $\mathcal{Z}$ tensors in extra-dimensional indices must be proportional to either the torus metric g^{ij} or to possible invariant tensors under the symmetry group of the torus. We differentiate in the last two terms of (17) possible $U(1)$ factors of $\mathcal{H}$ from the remaining semi-simple part and denote these $U(1)$ generators by T^α. In fact Eq. (16) implies that the field strength of a $U(1)$ gauge field is invariant by itself allowing for the term[4]

$$F_{ij}^\alpha = 2\partial_{[i} A_{j]}^\alpha - g f^{\alpha\hat{b}\hat{c}} A_i^{\hat{b}} A_j^{\hat{c}} \tag{18}$$

that can give rise to a quadratic renormalization. In a similar way, the term $D_k^{\alpha A} D_l^{AB} F_{ij}^B$ is invariant allowing for the last term in (17). It is dimension four and gives rise to a logarithmic renormalization, as we will see.

One might think that the term $\mathrm{tr}(\lambda_R T_R^a) F_{ij}^a$, where λ_R satisfies Eqs. (6) and the index R denotes some arbitrary irreducible representation, would give a further invariant linear in F_{ij}.[5] However, for T_R^a belonging to a simple factor of $\mathcal{H}$, λ_R must act as the identity in this subspace by Eqs. (6) and Schur's lemma, so the trace vanishes. Only $U(1)$ factors will thus contribute to the trace and we do not get any new invariant. We conclude that the terms F_{ij}^α are the most general linear terms.

We will be concerned mainly with the appearance of scalar mass terms in $\mathcal{L}_4^{\text{brane}}$. For a general unbroken gauge group $\mathcal{H}$ the most general renormalizable Lagrangian allowed by the symmetries of the theory contains the terms in (17). The first term in (17) corresponds to kinetic terms for the four-dimensional gauge bosons, the second one corresponds to kinetic terms for the even scalars (plus some interactions), while the third term contains brane mass terms for the odd scalars. One consequence of the appearance of brane mass terms in this particular way is that their renormalization is expected to be governed by the (wave function) renormalization of F^2, which does not contain quadratic divergences. They are expected to pick up only logarithmically divergent renormalization effects. Brane mass terms for even scalars can appear in $\mathcal{L}_4^{\text{brane}}$ in the case where there are $U(1)$ group factors in $\mathcal{H}$ corresponding to unbroken generators T^α. Under this circumstance we have seen that the operator (18) is allowed by all symmetries on the brane and we expect that both a tadpole for the derivative of odd fields, $\partial_i A_j^\alpha$, and a mass term for the even fields, $f^{\alpha\hat{b}\hat{c}} A_i^{\hat{b}} A_j^{\hat{c}}$, will be generated on the brane by bulk radiative corrections. Moreover, since these operators have dimension two, we expect that their respective renormalizations will lead to quadratic divergences, making the theory ultraviolet sensitive.

We would like to confirm by explicit calculation that the allowed terms are indeed generated radiatively on the brane. In particular, mass terms for brane scalars (extra-dimensional components of gauge

[4] Notice that unbroken $U(1)$ factors in $\mathcal{G}$ do not give rise in (18) to bilinear terms in even fields.

[5] We thank C. Csáki for pointing this out to us and thus making us aware of the possibility of having terms linear in F_{ij} on the brane.

Table 1
Discrete Lorentz symmetries broken/conserved by the orbifold. The corresponding reflected coordinates are indicated

	$\vec{x}, y^5, y^6$ (6D parity)	$\vec{x}$ (4D parity)	y^5	y^6
$\mathcal{P}_{\frac{1}{2}}$	conserved	conserved	broken	broken
$\mathcal{P}'_{\frac{1}{2}}$	broken	broken	conserved	conserved

bosons) are contained in the third term of (17) for the odd scalars A_i^a, and in (18) for the even scalars $A_i^{\hat{a}}$ when there are $U(1)$ group factors in $\mathcal{H}$. In all cases they arise from effective operators proportional to F_{ij}. An important special case is $D = 5$, i.e., a five-dimensional gauge theory compactified on $S^1/\mathbb{Z}_2$. In this case the term F_{ij} does not exist and therefore we do not expect any type of brane mass terms to appear in $\mathcal{L}_4^{\text{brane}}$. This result has been confirmed by explicit one loop calculation in Ref. [12]. However for $D > 5$ F_{ij} does exist and we expect, from the previous symmetry arguments, the corresponding mass terms to be generated on the brane by radiative corrections. The rest of this Letter will be devoted to an explicit calculation of these mass terms in a $D = 6$ model compactified on the orbifold $T^2/\mathbb{Z}_2$.

In $D = 6$ the Clifford algebra is spanned by eight-dimensional matrices $\Gamma_M = \Gamma_\mu, \Gamma_i$ satisfying $\{\Gamma_M, \Gamma_N\} = 2g_{MN}$. For an appropriate choice of the representation of the Γ^M, Dirac spinors in six dimensions are of the form $\Psi = (\Psi_1, \Psi_2)^{\mathrm{T}}$ where $\Psi_{1,2}$ are Dirac spinors in the four-dimensional sense. We can define the six-dimensional Weyl projector leading to the corresponding six-dimensional chirality such that $\Gamma_7\Psi_\pm = \pm\Psi_\pm$ where $\Psi_\pm$ are six-dimensional chiral spinors. The chiral fermions $\Psi_\pm$ contain the degrees of freedom of a four-dimensional Dirac spinor. A theory with six-dimensional chiral fermions is not free from six-dimensional anomalies, generated by box diagrams, and anomaly freedom should be enforced by appropriately restricting the fermion content.[6] After orbifolding on $T^2/\mathbb{Z}_2$, half of the degrees of freedom of the chiral fermions $\Psi_\pm$ is odd and the zero mode sector becomes chiral from the four-dimensional point of view; four-dimensional anomaly freedom for zero modes is a further requirement of any consistent theory.

Compactification of a six-dimensional theory to four dimensions is done by means of usual techniques. The Kaluza–Klein (KK) number is now a two-dimensional vector denoted by $\vec{m}$. Generalizing the techniques of [10,12] to six dimensions is easy, the only missing ingredient being the orbifold action on the fermions $\mathcal{P}_{\frac{1}{2}}$ in (5). Requiring that

$$\mathcal{P}_{\frac{1}{2}}^2 = 1, \qquad \mathcal{P}_{\frac{1}{2}}\Sigma_{MN}\mathcal{P}_{\frac{1}{2}} = (\mathcal{P}_1)_M^R(\mathcal{P}_1)_N^S\Sigma_{RS}, \tag{19}$$

where $\Sigma_{MN} = \frac{i}{4}[\Gamma_M, \Gamma_N]$, we can identify two possible solutions:

$$\mathcal{P}_{\frac{1}{2}} = i\Gamma_5\Gamma_6, \qquad \mathcal{P}'_{\frac{1}{2}} = i\Gamma_5\Gamma_6\Gamma_7. \tag{20}$$

Both projections differ in their action on possible discrete space-reflection symmetries, which might be broken by the orbifold or not. The situation is summarized in Table 1.

The main difference is that starting from a 6D Dirac spinor, using $\mathcal{P}'_{\frac{1}{2}}$ the massless spectrum contains two 4D Weyl spinors of the same chirality while with $\mathcal{P}_{\frac{1}{2}}$ it contains a 4D Dirac spinor. This is consistent with the fact that $\mathcal{P}'_{\frac{1}{2}}$ breaks 4D parity while $\mathcal{P}_{\frac{1}{2}}$ conserves it. We would like to stress that the distinction is completely irrelevant in case the discrete symmetries are broken in the first place, as is the case when dealing with 6D Weyl fermions. We can obtain the same massless field content with $\mathcal{P}_{\frac{1}{2}}$ and $\mathcal{P}'_{\frac{1}{2}}$ since we are now allowed to choose different λ for different chiralities.[7] Without loss of generality we will choose $\mathcal{P}_{\frac{1}{2}}$ for 6D Weyl fermions.

[6] Of course a theory with six-dimensional Dirac fermions does not have six-dimensional anomalies.

[7] Indeed, $\lambda_i \otimes \mathcal{P}_{\frac{1}{2}}$ produces the same zero mode spectrum as $\lambda'_i \otimes \mathcal{P}'_{\frac{1}{2}}$ with $\lambda'_i = \varepsilon_i\lambda_i$, ε_i being the 6D chirality of the fermions species ψ_i.

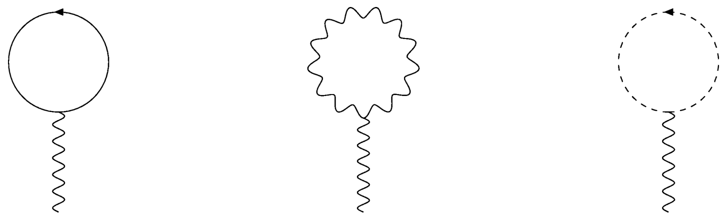

Fig. 1. One loop tadpole diagrams.

The propagator of the $\vec{m}$-mode of an arbitrary field Φ (a gauge boson A_M, a ghost field c or a fermion Ψ) in the six-dimensional space compactified on the orbifold $T^2/\mathbb{Z}_2$ can be written as

$$\langle \Phi^{\vec{m}'} \overline{\Phi}^{\vec{m}} \rangle = \frac{1}{2}(\delta_{\vec{m}'-\vec{m}} + \mathcal{P}_\Phi \delta_{\vec{m}'+\vec{m}}) G^{(\Phi)}(p_\mu, p_i), \tag{21}$$

where $G^{(\Phi)}(p_\mu, p_i)$ is the propagator of the corresponding field in flat six-dimensional space and $\mathcal{P}_\Phi$ the parity as defined in (3) and (5).

The diagrams appearing in Fig. 1 contribute to the renormalization of the first term in Eq. (18), the dimension two operator $\partial_i A_j^a$, as well as to the renormalization of the dimension four operator $\partial_k \partial_l \partial_i A_j^\alpha$ contained in the last term of Eq. (17). In the first diagram of Fig. 1 six-dimensional fermions circulate. The contribution of a chiral fermion $\Psi_\pm$ turns out to be

$$i g \operatorname{tr}(\lambda_R T_R^B) \epsilon_{ij} m^j \int \frac{d^4 q}{(2\pi)^4} \frac{1}{q^2 - \vec{m}^2/4}, \quad m_5, m_6 \text{ even}, \tag{22}$$

where the external leg corresponds to the 4D scalar A_i^B (we have defined $\epsilon_{56} = -\epsilon_{65} = +1$). It leads to the terms[8]

$$\left(\mathcal{Z}_\alpha^{ij} F_{ij}^\alpha + \mathcal{Z}_\alpha^{klij} D_k^{\alpha A} D_l^{AB} F_{ij}^B\right)\left[\delta(y_5) + \delta(y_5 - \pi)\right] \times \left[\delta(y_6) + \delta(y_6 - \pi)\right], \tag{23}$$

where α runs over the different $U(1)$ factors of $\mathcal{H}$ and $\mathcal{Z}_\alpha^{ij}$ and $\mathcal{Z}_\alpha^{klij}$ are given by

$$\mathcal{Z}_\alpha^{ij} = \epsilon^{ij} \frac{g}{32\pi^2} \zeta^\alpha \Lambda^2, \qquad \zeta^\alpha = \operatorname{tr}(\lambda_R T_R^\alpha), \tag{24}$$

$$\mathcal{Z}_\alpha^{klij} = \delta^{kl} \epsilon^{ij} \frac{g}{64\pi^2} \zeta^\alpha \log \frac{\Lambda}{\mu}, \tag{25}$$

where Λ is the ultraviolet and μ the infrared cut-off.

A further comment concerns the gauge contribution to the tadpole. At one loop it is given by the second (contribution from gauge fields A_M) and third (contribution from ghosts c) diagrams in Fig. 1. Each one is proportional to the corresponding trace

$$\operatorname{tr}(\lambda_{\mathrm{Adj}} T_{\mathrm{Adj}}^\alpha) = \eta^A \delta^{AB} f^{\alpha AB} = 0 \tag{26}$$

and thus vanish by the asymmetry of the structure constants. Note that this is a generic feature of real representations.

We have also computed the one loop contribution to the terms $(F_{ij}^a)^2$ in (17) and we found the logarithmic divergence we anticipated:

$$-\frac{1}{4} F_{ij}^a \mathcal{Z}_{ab}^{ijkl} F_{kl}^b \left[\delta(y_5) + \delta(y_5 - \pi)\right] \times \left[\delta(y_6) + \delta(y_6 - \pi)\right], \tag{27}$$

where

$$\mathcal{Z}_{ab}^{ijkl} = \frac{1}{2}\left(\delta^{ik}\delta^{jl} - \delta^{jk}\delta^{il}\right) \frac{g^2}{2\pi^2} \times \left(C_2(\mathcal{H}_a) - \frac{1}{2} C_2(\mathcal{G})\right) \delta_{ab} \log \frac{\Lambda}{\mu}. \tag{28}$$

Here, $C_2(\mathcal{H}_a)$ is by definition the Casimir of the group factor in $\mathcal{H}$ to which the generator T^a belongs (we define it to be zero for $U(1)$ factors). One expects a corresponding logarithmic contribution from the fermion sector.

Since we have seen that, in all cases, the one loop contribution to the renormalization of (18) from six-dimensional gauge bosons and fermions is proportional to $\operatorname{tr}(\lambda T^\alpha)$, where λ represents the parity action on the corresponding representation, two main issues can be addressed. The first issue concerns the vanishing of the tadpole from the gauge sector (or in general from real representations) at higher orders in perturbation theory. In order to answer this question we have

[8] We have confirmed explicitly that the terms $f^{\alpha\hat{a}\hat{b}} A_i^{\hat{a}} A_j^{\hat{b}}$ in (18) receive the same renormalization $\mathcal{Z}_\alpha^{ij}$ at one loop as the tadpole.

computed the two loop contribution to the tadpole. We have verified (see Appendix A) that the contribution of real representations to the tadpole vanish at higher loop order and we can expect that it vanishes at all orders in perturbation theory although we do not have an explicit proof beyond two loops.

The second issue concerns the possible relation between the tadpole and the generation of the four-dimensional anomalies on the brane by (chiral) fermions in the bulk [15,16]. In fact we have seen that given a collection of six-dimensional chiral fermions Ψ_ε, where $\varepsilon = \pm$, the generated tadpole for a given $U(1)$ factor is

$$\zeta^\alpha = \sum_{\Psi_\varepsilon} \mathrm{tr}\big(\lambda_{\Psi_\varepsilon} T^\alpha_{\Psi_\varepsilon}\big), \tag{29}$$

where $\varepsilon = \pm$ is the six-dimensional chirality of the field Ψ_ε. On the other hand, the four-dimensional anomaly on the brane is generated by bulk triangular loop diagrams where chiral fermions Ψ_ε circulate in the loop, while gauge bosons and/or gravitons are external legs. In particular the mixed $U(1)$-gravitational anomaly on the brane is easily seen to be proportional to

$$\mathcal{A}^\alpha = \sum_{\Psi_\varepsilon} \varepsilon\, \mathrm{tr}\big(\lambda_{\Psi_\varepsilon} T^\alpha_{\Psi_\varepsilon}\big). \tag{30}$$

Notice that different chiralities contribute with the same sign to the tadpole $\mathcal{Z}^\alpha$ while they contribute with different signs to the anomaly $\mathcal{A}^\alpha$. Let us also note that with the choice $\mathcal{P}'_{\frac{1}{2}}$ the ε would move from Eq. (30) to Eq. (29). Keeping the physics constant requires however to make the change $\lambda \to -\lambda$ for the negative chirality fermions (see footnote 7), which is consistent with Eqs. (29) and (30).

By looking at (29) and (30) one may conclude that imposing $\zeta^\alpha = \mathcal{A}^\alpha = 0$ results in the tadpole cancellation to be equivalent to the $U(1)$-gravitational anomaly cancellation in the positive and negative chirality sectors separately. However, in models originating from string theories anomalies can be cancelled by a generalized Green–Schwarz mechanism. In those cases the cancellation of the anomaly $\mathcal{A}^\alpha$ in Eq. (30) is no longer a necessary condition and therefore the tadpole cancellation as given by Eq. (29) remains as the only constraint in the model. We will illustrate the above ideas with the six-dimensional model of Ref. [8] compactified on $T^2/\mathbb{Z}_2$ with gauge group $\mathcal{G} = SU(3)_c \times SU(3)_w \times U(1)_{\mathcal{Q}_3} \times U(1)_{\mathcal{Q}_2}$ broken by the orbifold boundary conditions to $\mathcal{H} = SU(3)_c \times SU(2)_w \times U(1)_{\mathcal{Q}_1} \times U(1)_{\mathcal{Q}_3} \times U(1)_{\mathcal{Q}_2}$. Fermions are in representations $L_f = (\mathbf{1},\mathbf{3})^+_{(0,1)}$, $U_f = (\mathbf{3},\mathbf{1})^+_{(1,0)}$, $Q_f = (\mathbf{3},\mathbf{3})^{\varepsilon_f}_{(1,1)}$ where $f = 1,2,3$, $\varepsilon_{1,2} = -$, $\varepsilon_3 = +$, and the notation $(\mathbf{r_3},\mathbf{r_2})^\varepsilon_{(q_3,q_2)}$ represents a six-dimensional Weyl fermion with chirality ε in the representation $\mathbf{r_3}$ and $\mathbf{r_2}$ of $SU(3)_c$ and $SU(3)_w$, respectively, and $U(1)$ charges q_3 and q_2 under the generators $\mathcal{Q}_3$ and $\mathcal{Q}_2$. Orbifold compactification breaks $SU(3)_w \to SU(2)_w \times U(1)_{\mathcal{Q}_1}$, where $\mathcal{Q}_1 = \mathrm{diag}(1,1,-2)$ and for $SU(3)_w$ triplets the matrix satisfying (6) is $\lambda = \mathrm{diag}(1,1,-1)$. The Standard Model hypercharge is related to $\mathcal{Q}_i$ by $Y = \mathcal{Q}_1/6 - 2\mathcal{Q}_2/3 + 2\mathcal{Q}_3/3$ and the fields are decomposed under $SU(2)_w \times U(1)_{\mathcal{Q}_1}$ as

$$L_L = \begin{pmatrix} \ell_L \\ \tilde{e}_L \end{pmatrix}, \qquad L_R = \begin{pmatrix} \tilde{\ell}_R \\ e_R \end{pmatrix},$$
$$Q_L = \begin{pmatrix} q_L \\ \tilde{d}_L \end{pmatrix}, \qquad Q_R = \begin{pmatrix} \tilde{q}_R \\ d_R \end{pmatrix},$$
$$U_L = \tilde{u}_L, \qquad U_R = u_R, \tag{31}$$

where untilded (tilded) fields are (mirrors of) Standard Model fields.

Given (31) the parity properties of fields is given by: $\lambda_L = \lambda$, $\lambda_{Q_3} = \lambda$, $\lambda_{Q_{1,2}} = -\lambda$, $\lambda_U = -1$. Thus their contribution to ζ^1 cancels while $\mathcal{A}^1 = 12(1+N_c)$ since, as stressed in Ref. [8], $\mathcal{Q}_1$ is anomalous.

An alternative way to the Green–Schwarz mechanism, that can be used to cancel the (bulk-induced) brane anomalies in Eq. (30), is by means of chiral fermions localized on the brane. Localized fermions do not possess tree level couplings with A_i^A, or their derivatives, and thus they provide no one-loop contribution to the tadpole. Moreover their unique two loop contribution, given by the third diagram of Fig. 2 where the dashed (ghost) line is replaced by a localized chiral fermion, vanishes as can be easily checked.

We want to conclude this Letter by stressing the fact that the conditions for tadpole cancellation on the brane do not coincide with those required from bulk-induced anomaly cancellation. As such the tadpole is not expected to be (as the anomaly) a purely one loop effect and in a general theory with fermions we expect a tadpole generation at least at the two loop level. However for theories with low (TeV) cut-off scale the latter will provide a mild (tiny) dependence on the

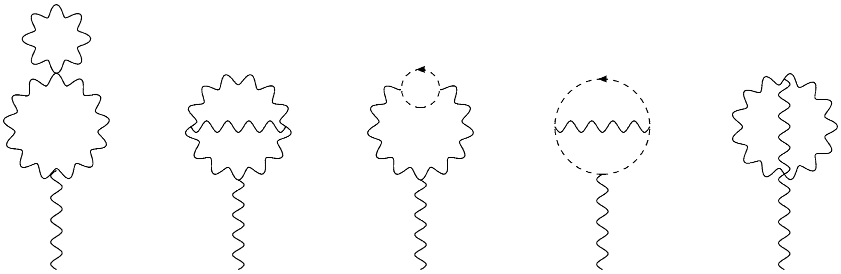

Fig. 2. Two loop tadpole diagrams from the gauge sector.

cut-off that should not disturb the stability of the low energy effective theory.

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Appendix A. Two loop contribution to the tadpole

The diagrams contributing to the tadpole at two loops are given in Fig. 2. Note that fermionic diagrams are obtained by just replacing in the diagrams of (2) ghost propagators by fermion propagators. To judge whether or not there are contributions to the tadpole, it is sufficient to examine the gauge index structure. Recall that the gauge and ghost propagators have the general structure (see Eq. (21))

$$\delta_+\delta_{AB} + \delta_-\Lambda_{AB} = (\delta_+ + \delta_-\eta_A)\delta_{AB} \tag{A.1}$$

while for fermions in the representation R we have

$$\delta_+\delta_{ij} + \delta_-(\lambda_R)_{ij}. \tag{A.2}$$

Here δ_+ symbolizes extra-dimensional momentum conservation and δ_- extra-dimensional momentum flip. The first four diagrams are reducible in the sense that they just correspond to wave function renormalization insertions of the gauge or ghost propagators. Using the contraction identities[9]

$$f^{BDE} f^{CDE} = C_2(\mathcal{G})\delta^{BC}, \tag{A.3}$$

$$f^{BDE} f^{CDE}\eta^D = \left(C_2(\mathcal{H}_B) - \frac{1}{2}C_2(\mathcal{G})\right) \times (\eta_B + 1)\delta^{BC}, \tag{A.4}$$

$$f^{BDE} f^{CDE}\eta^D\eta^E = C_2(\mathcal{G})\eta^B\delta^{BC}, \tag{A.5}$$

one can verify immediately that all these insertions are matrices $\mathcal{Z}_{BC}$ which are symmetric in BC. These are then to be contracted with f^{ABC}, giving zero. In a similar way it can be seen that the last diagram does not contribute either.

Finally, fermions contribute at two loops unless they transform in a real representation, $T_R^{\mathrm{T}} = -T_R$. The corresponding contraction identities read:

$$\mathrm{tr}\big(T_R^B T_R^C\big) = C_R\delta^{BC}, \tag{A.6}$$

$$\mathrm{tr}\big(T_R^B\lambda_R T_R^C\lambda_R\big) = C_R\eta^B\delta^{BC}, \tag{A.7}$$

and in addition $\mathrm{tr}(T_R^B T_R^C\lambda_R) = \mathrm{tr}(T_R^C T_R^B\lambda_R)$ if R is real. We conclude that in this case the third diagram (with the ghost replaced by a fermion) is zero, while for general R there will be a contribution from the antisymmetric part of $\mathrm{tr}(T_R^B T_R^C\lambda_R)$.[10] Finally, the fourth diagram can be seen to give terms proportional to the four tensors (the sum over B is understood)

$$\mathrm{tr}\big(T^aT^BT^B\big),\quad \eta_B\,\mathrm{tr}\big(T^aT^BT^B\big),$$
$$\mathrm{tr}\big(T^a\lambda T^BT^B\big),\quad \eta_B\,\mathrm{tr}\big(T^a\lambda T^BT^B\big). \tag{A.8}$$

The Casimirs T^BT^B and $\eta_BT^BT^B$ are symmetric matrices which commute with T^a. Together with

[9] The tensor in Eq. (A.4) vanishes for $B = \hat{b}$. It has already been encountered in Eq. (28).

[10] It is easy to verify that this corresponds to a brane term.

Eq. (6) this implies that all four traces vanish for R real.

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