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Implications for new physics from $\bar{B}^0 \to \pi^0\pi^0$ and $\bar{B}^0 \to \bar{K}^0 K^0$

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Abstract

We have analyzed the $\bar{B}^0 \to \pi^0\pi^0$ puzzle in three kinds of models beyond the standard model (SM). It is shown that the minimal flavor violation (MFV) models, the minimal supersymmetric standard model (MSSM), and the two Higgs doublet models (2HDM) I and II cannot give an explanation of the $\bar{B}^0 \to \pi^0\pi^0$ puzzle within 1σ experimental bounds and the model III 2HDM can explain the puzzle without a conflict with other experimental measurements. If the constraint on C_{8g} from $b \to sg$ is not imposed, for all kinds of insertions considered there are regions of parameter space, where the scalar quark mass is larger (much larger) than the gluino mass in the case of LR or RL (LL or RR), in which the puzzle can be resolved within 1σ experimental bounds.

The branching ratios of $\bar{B}^0$ decays into two pions have been recently observed [1]:

$$\mathrm{Br}(\bar{B}^0 \to \pi^0\pi^0) = (1.45 \pm 0.29) \times 10^{-6}, \qquad \mathrm{Br}(\bar{B}^0 \to \pi^+\pi^-) = (5.0 \pm 0.4) \times 10^{-6}. \tag{1}$$

The large branching ratio of the B decay into neutral pion final states is unexpected. The decay amplitudes of $\bar{B}^0 \to \pi\pi$ can be generally parameterized as

$$\sqrt{2}A(\bar{B}^0 \to \pi^0\pi^0) = T\left[\left(\frac{P}{T} - \frac{P_{\mathrm{EW}}}{T}\right)e^{i\alpha} - \frac{C}{T}\right], \qquad A(\bar{B}^0 \to \pi^+\pi^-) = -T\left[1 + \frac{P}{T}e^{i\alpha}\right], \tag{2}$$

where T, C, P, and P_{EW} are the tree, color-suppressed tree, penguin, and electro-weak penguin (EWP) amplitudes respectively, and $\alpha = \arg(-\frac{\lambda_u}{\lambda_t})$ is the weak phase, where $\lambda_p = V_{pb}V_{pd}^*$ $(p = u, c, t)$. In SM one has the counting rules: the color-suppressed tree and penguin amplitudes are suppressed by a factor of λ ($\lambda \sim 0.22$ is the Wolfenstein parameter) and the EW penguin is suppressed by a factor of λ^2, with respect to the tree amplitude [2]. So one should expect by the naive counting rules that the branching ratio of the B decay into neutral pion final states is $O(\lambda^2)$ of that for charged pion final states. However, the data (Eq. (1)) indict that the former is $O(\lambda)$ of the later. The observed branching ratio of $\bar{B}^0 \to \pi^0\pi^0 = (1.45 \pm 0.29) \times 10^{-6}$ is much larger than the theoretical prediction, about 0.3×10^{-6}, up to the α_s order in the BBNS approach (QCDF) [3,4] in SM. In Li et al.'s approach (PQCD) the leading order (LO) prediction $\sim 10^{-7}$ [5] is the same order as that of the QCDF prediction. In the recent paper [6] the next leading order (NLO) PQCD calculations have been carried out and the results are that the πK puzzle, the expected relation $A_{CP}(B^\pm \to \pi^0 K^\pm) \approx A_{CP}(B^0 \to \pi^\pm K^\mp)$ disagreed significantly with the data, can be resolved but the predicted branching ratio of $\bar{B}^0 \to \pi^0\pi^0$ is about 0.3×10^{-6} which is still much smaller than the data, i.e., the $\pi^0\pi^0$ puzzle remains. If the large branching ratio persists it could indicate new physics.

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Though $\bar{B}^0 \to \pi^0\pi^0$ is not a pure-penguin process and has the contributions from tree operators, the tree contributions are of the order same as the penguin contributions because of the almost completely cancellation between the two terms in $C_2 + C_1/N_c$ where $C_{1,2}$ are Wilson coefficients of tree operators, so $\bar{B}^0 \to \pi^0\pi^0$ is sensitive to new physics. Therefore, it seems that a lot of new models beyond SM could enhance the branching ratio and consequently resolve the puzzle [7]. However, any new model must simultaneously give an explanation for the branching ratio of $\bar{B}^0 \to \bar{K}^0K^0$ since the two processes are closely related at quark level: the flavor changing neutral current $b \to d$ transition controls $\bar{B}^0 \to \bar{K}^0K^0$ and the same transition gives significant contributions to $\bar{B}^0 \to \pi^0\pi^0$ which are of the order same as the tree contributions in SM. Recently the branching ratio of $\bar{B}^0 \to \bar{K}^0K^0$ has been measured as $(0.96 \pm 0.25) \times 10^{-6}$ [1], which is consistent with the prediction from both the QCDF [4] and PQCD approaches [8]. Therefore, new physics (NP) contributions must satisfy the condition that they make Br of $\bar{B}^0 \to \pi^0\pi^0$ enhanced but keep Br of $\bar{B}^0 \to \bar{K}^0K^0$ basically unchanged, compared with those in SM respectively, which will impose the significant limit on NP models.

In the Letter we search for new models beyond the SM which can account for the data of branching ratios for both the $\bar{B}^0 \to \pi^0\pi^0$ and $\bar{B}^0 \to \bar{K}^0K^0$ processes. To be specific, we concentrate on the well-known three kinds of models: the minimal flavor violation (MFV) models, the two Higgs doublet models (2HDM) and the minimal supersymmetric standard model (MSSM).

The effective Hamiltonian relevant for the two processes in the SM can be expressed as [3]

$$\mathcal{H}_{\rm eff}^{\rm SM} = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p \left(C_1 Q_1^p + C_2 Q_2^p + \sum_{i=3,\ldots,10} C_i Q_i + C_{7\gamma} Q_{7\gamma} + C_{8g} Q_{8g} \right) + \text{h.c.}, \tag{3}$$

where $\lambda_p = V_{pb}V_{pd}^*$, $Q_{1,2}$ and Q_i $(i = 3, \ldots, 10)$ are the tree and penguin operators respectively. Explicit forms for C_1, C_2, C_i, $C_{7\gamma}$, C_{8g} and Q_1^p, Q_2^p, Q_i, $Q_{7\gamma}$, Q_{8g} can be found, e.g., in Ref. [3].

In the QCD factorization approach, the dominant contributions to the decay amplitudes are given by:

$$\begin{aligned} M(\bar{B}^0 \to \pi^0\pi^0) &= \frac{G_F}{\sqrt{2}} f_\pi F^{B\to\pi} m_B^2 \times \frac{1}{\sqrt{2}} \sum_{p=u,c} \left[a_2\lambda_u - \left(a_4^p + r^\pi a_6^p\right)\lambda_p \right], \\ M(\bar{B}^0 \to \bar{K}^0K^0) &= \frac{G_F}{\sqrt{2}} f_K F^{B\to K} m_B^2 \times \sum_{p=u,c} \left[\left(a_4^p + r^K a_6^p\right)\lambda_p \right], \end{aligned} \tag{4}$$

where the definitions of the parameters a_i and the chiral enhancement factors r^π, r^K can be found in Ref. [3]. We take the values of running masses in the $\overline{\rm MS}$ scheme for light quarks such that $r^\pi = r^K \equiv r$ hereafter. The electro-weak penguin and annihilation contributions are neglected in above formula, which leads to the 10% theoretical uncertainty.

First we consider the MFV models. The MFV models beyond the SM discussed in the Letter mean a class of models in which the general structure of flavor changing neutral current (FCNC) processes present in the SM is preserved. In particular, all flavor violating and *CP*-violating transitions are governed by the CKM matrix and the only relevant local operators are the ones that are relevant in the SM [9]. New parameters in the MFV models, e.g., the masses of charginos, squarks, Higgs particles in the MFV scenario of the MSSM, enter into Wilson coefficients of relevant local operators. Therefore, in the MFV models the amplitudes of the two decays are given same as Eq. (4) (with values of a_i generally different from those in SM).

We can model-independently determine $|z| \equiv |\sum_{p=u,c}[(a_4^p + ra_6^p)\lambda_p]|$ from the measured branching ratios of $\bar{B}^0 \to \pi^0\pi^0$ and $\bar{B}^0 \to \bar{K}^0K^0$ and $a_2 = (0.0502 - 0.0689i) \pm (0.0025 + 0.0035i)$ which comes from the known tree contributions. The result is given in Fig. 1 where the 60% theoretical uncertainty (coming mainly from non-perturbative parameters such as form factors, distribution amplitudes and CKM matrix elements) has been taken into account. There is a narrow region which can simultaneously fit the data. However, assuming Wilson coefficients of relevant operators, except the chromo-magnetic dipole operator, change a little compared with SM, which is the case in the MFV models, z in the region corresponds to

$$\left|C_{8g}(m_W)\right| \geqslant 2.6, \tag{5}$$

which cannot be reached in the MFV models [10,11]. That is, the MFV models are excluded within 1σ experimental bounds.

Next we consider models in which there are new operators in addition to those in the SM, e.g., the 2HDM and MSSM. The effective Hamiltonian in the 2HDM and MSSM can be written as [12,13]

$$\mathcal{H}_{\rm eff} = \mathcal{H}_{\rm eff}^{\rm SM} + \mathcal{H}_{\rm eff}^{\rm new}, \tag{6}$$

$$\mathcal{H}_{\rm eff}^{\rm new} = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} V_{pb}V_{pd}^* \left(\sum_{i=11,\ldots,16} [C_i Q_i + C_i' Q_i'] + \sum_{i=3,\ldots,10} C_i' Q_i' + C_{7\gamma}' Q_{7\gamma}' + C_{8g}' Q_{8g}' \right) + \text{h.c.}, \tag{7}$$

where $Q_i^{(\prime)}$, $i = 11, \ldots, 16$, are the neutral Higgs penguin operators and their explicit forms can be found in Refs. [12,13] with the substitution $s \to d$. The primed operators, the counterpart of the unprimed operators, are obtained by replacing the chirality in the corresponding unprimed operators with opposite ones.

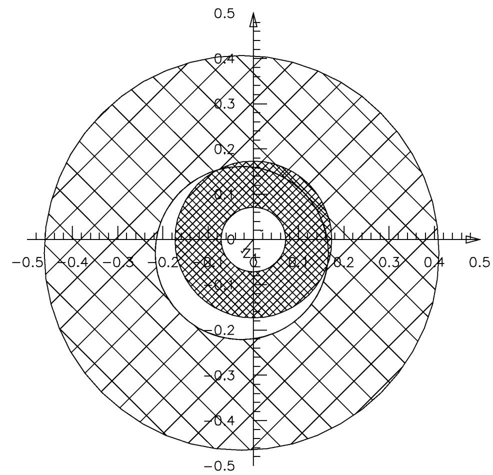

Fig. 1. The constraints on z from the branching ratios of $\bar{B}^0 \to \pi^0\pi^0$ and $\bar{B}^0 \to \bar{K}^0 K^0$. The big lattice denotes the constraint from $\bar{B}^0 \to \pi^0\pi^0$ and the small lattice for the constraint from $\bar{B}^0 \to \bar{K}^0 K^0$.

From the effective Hamiltonian, Eq. (6), it follows that the main contributions to the decay amplitudes from the SM and new physics are given by:

$$M\big(\bar{B}^0 \to \pi^0\pi^0\big) = \frac{G_F}{\sqrt{2}} f_\pi F^{B\to\pi} m_B^2 \times \frac{1}{\sqrt{2}} \sum_{p=u,c} \big[a_2\lambda_u - \big(a_4^p + r a_6^p\big)\lambda_p\big],$$

$$M\big(\bar{B}^0 \to \bar{K}^0 K^0\big) = \frac{G_F}{\sqrt{2}} f_K F^{B\to K} m_B^2 \times \sum_{p=u,c} \left\{\big(a_4^p + r a_6^p\big)\lambda_p + \frac{m_s}{m_b}\big[h_1(C_{11} - C'_{11}) + h_2\big(C_{13}(\mu) - C'_{13}(\mu)\big)\big]\right\}, \tag{8}$$

where we have set $m_d = 0$. Due to the renormalization group equation (RGE) running, Wilson coefficients C_i, $i = 14, 15, 16$, are related to C_{13} and the known constants $h_{1,2}$ represent the running effects. The largest contributions to the hadronic elements of the neutral Higgs penguin operators at the α_s order arise from penguin contractions with b quark in the loop, which are the same for $\bar{B}^0 \to \pi^0\pi^0$ and $\bar{B}^0 \to \bar{K}^0 K^0$ and have been included in a_4 (see, for example, Ref. [14]). Therefore, although they can enhance the branching ratio of $\bar{B}^0 \to \pi^0\pi^0$, they alone cannot resolve the puzzle because the branching ratio of $\bar{B}^0 \to \bar{K}^0 K^0$ will also be enhanced by them, which will not agree with the data.

The new physics contribution, the terms proportional to C_{11} and C_{13}, respectively, which contributes to one mode but not to the another (precisely speaking, the contribution to the another mode is m_d/m_s suppressed) comes from the hadronic matrix elements of Higgs penguin operators at the leading order in α_s. It is the contribution that gives a possibility to make the data be account for without a conflict with all relevant experimental measurements. The key point is if one can have a sizable $C_{13}^{(\prime)}(\mu)$ and/or $C_{11}^{(\prime)}(\mu)$ in the 2HDM and MSSM.

Let us analyze how large $C_{13}^{(\prime)}$ and/or $C_{11}^{(\prime)}$ are needed to fit the data. Let

$$z = \sum_{p=u,c} \frac{\lambda_p}{\lambda_t}\big(a_4^p + r a_6^p\big), \qquad z_1 = \frac{\lambda_u}{\lambda_t} a_2, \qquad z_2 = \frac{m_s}{m_b}\big[h_1(C_{11} - C'_{11}) + h_2\big(C_{13}(\mu) - C'_{13}(\mu)\big)\big],$$

we have

$$r_{11} \equiv \sqrt{2}\sqrt{\frac{32\pi m_B \,\mathrm{Br}(\pi^0\pi^0)_{\min}}{(G_F f_\pi F^{B\to\pi}\lambda_t)^2 \tau_B}} \leqslant |z - z_1| \leqslant r_{12} \equiv \sqrt{2}\sqrt{\frac{32\pi m_B \,\mathrm{Br}(\pi^0\pi^0)_{\max}}{(G_F f_\pi F^{B\to\pi}\lambda_t)^2 \tau_B}},$$

$$r_{21} \equiv \sqrt{\frac{32\pi m_B \,\mathrm{Br}(\bar{K}^0 K^0)_{\min}}{(G_F f_K F^{B\to K}\lambda_t)^2 \tau_B}} \leqslant |z + z_2| \leqslant r_{22} \equiv \sqrt{\frac{32\pi m_B \,\mathrm{Br}(\bar{K}^0 K^0)_{\max}}{(G_F f_K F^{B\to K}\lambda_t)^2 \tau_B}}. \tag{9}$$

From the data, $1.16 \times 10^{-6} \leqslant \mathrm{Br}(\pi^0\pi^0) \leqslant 1.74 \times 10^{-6}$ and $0.71 \leqslant \mathrm{Br}(\bar{K}^0 K^0) \leqslant 1.21 \times 10^{-6}$ [15], we have $r_{12} > r_{11} > r_{22} > r_{21}$. To satisfy the above two relations, it is necessary to have

$$|z_2 + z_1| \geqslant r_{11} - r_{22}. \tag{10}$$

In the model I and II 2HDMs and MSSM the Wilson coefficients of QCD penguin operators are not changed significantly, compared with those in SM, and the Wilson coefficient of chromo-magnetic operator can have a significant change [16]. Taking the SM values of Wilson coefficients of relevant operators but the chromo-magnetic operator and using RQE running, we can obtain the correlation between $|C_{8g}(m_W) - C'_{8g}(m_W)|$ and $|C_{13}(m_W) - C'_{13}(m_W)|$ from $|z - z_1| \geqslant r_{11}$ and Eq. (10), which is shown in

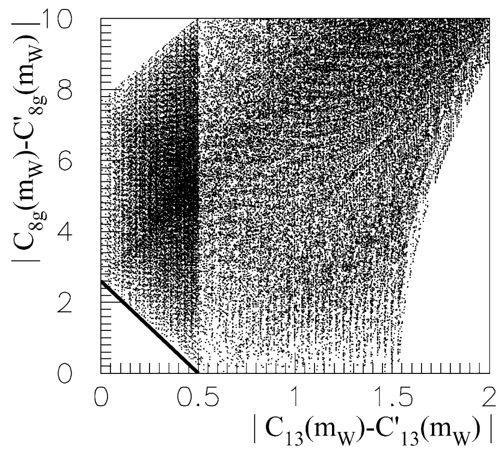

Fig. 2. The correlation between $|C_{8g}(m_W) - C'_{8g}(m_W)|$ and $|C_{13}(m_W) - C'_{13}(m_W)|$.

Fig. 2 where $C_{11} = C_{13}$ has been assumed for simplicity, without losing the generality of discussions.[1] It follows from the figure that $|C_{8g}(m_W) - C'_{8g}(m_W)|_{\min} = 2.6$ when $C_{13}(m_W) - C'_{13}(m_W) = 0$, which reduce to Eq. (5) in the MFV models, as it should be.

It is well known that the experimental upper bound of branching ratio for $B_s \to \mu^+\mu^-$ constrains severely parameters in the MSSM and model I and II 2HDMs [17]. Similarly, we show that the corresponding bound for $B_d \to \mu^+\mu^-$ implies that the Wilson coefficients of new operators in the MSSM and model I and II 2HDMs cannot be large. The branching ratio $B_d \to \mu^+\mu^-$ in the 2HDM and MSSM is given as

$$\mathrm{Br}\big(B_d \to \mu^+\mu^-\big) = \frac{G_F^2 \alpha_{\mathrm{em}}^2}{64\pi^3} m_{B_d}^3 \tau_{B_d} f_{B_d}^2 |\lambda_t|^2 \sqrt{1 - 4\hat{m}^2} \Big[\big(1 - 4\hat{m}^2\big)\big|C_{Q_1}(m_b) - C'_{Q_1}(m_b)\big|^2 + \big|C_{Q_2}(m_b) - C'_{Q_2}(m_b) + 2\hat{m}\big(C_{10}(m_b) - C'_{10}(m_b)\big)\big|^2\Big], \tag{11}$$

where $\hat{m} = m_\mu / m_{B_d}$. In the moderate and large $\tan\beta$ cases the term proportional to $(C_{10} - C'_{10})$ in Eq. (11) can be neglected. The new CDF and D0 combined experimental upper bound of $\mathrm{Br}(B_d \to \mu^+\mu^-)$ is 3.2×10^{-8} [18] at 90% confidence level. We have the constraint

$$\sqrt{\big|C_{Q_1}(m_W) - C'_{Q_1}(m_W)\big|^2 + \big|C_{Q_2}(m_W) - C'_{Q_2}(m_W)\big|^2} \lesssim 2.2, \tag{12}$$

where $C_{Q_{1,2}}^{(\prime)}$ are the Wilson coefficients of the operators $Q_{1,2}^{(\prime)}$ which are Higgs penguin induced in leptonic and semileptonic B decays and their definition can be found in Refs. [19,20]. By substituting the quark-Higgs vertex for the lepton-Higgs vertex, it is straightforward to obtain Wilson coefficients relevant to hadronic B decays in the MSSM and model I and II 2HDMs. To translate $C_{Q_{1,2}}$ into $C_{Q_{11,13}}$, we have $C_{Q_{11,13}}^{(\prime)}(m_W) \sim 0.037$. Then it follows from Fig. 2 that $|C_{8g}(m_W) - C'_{8g}(m_W)|$ must be larger than 2.4 in order to resolve the puzzle.

The Wilson coefficients $C_{8g}^{(\prime)}$ in the $b \to d$ transition are constrained by $\mathrm{Br}(B \to X_d g)$. Because there is no data for Br of the $B \to X_d g$ decay and the difference between the $B \to X_d g$ and $B \to X_s g$ decays in the SM comes from CKM matrix elements, we assume the constraint on $C_{8g}^{(\prime)}$ same as that from $b \to sg$. In the presence of new physics a model-independent analysis gives that $|C_{8g}(m_W) - C'_{8g}(m_W)| < 2.01$ when $\mathrm{Br}(b \to sg) < 9\%$ [21]. That is, $|C_{8g}(m_W) - C'_{8g}(m_W)|$ cannot satisfy the condition, larger than 2.4, because of the $b \to sg$ constraint. Therefore, we come to the conclusion that the puzzle of $\bar{B}^0 \to \pi^0\pi^0$ cannot get resolved within 1σ experimental bounds in the MSSM and model I and II 2HDMs.

If one does not impose the $b \to sg$ constraint it is possible to resolve the puzzle in the MSSM because the Wilson coefficient $C_{8g}^{(*)}$ can reach values larger than 2.6 in some regions of parameter space. We have carried out detailed numerical calculations in the MSSM, imposing the constraints from the $\bar{B}_d^0$–$\bar{B}_d^0$ system, the mass difference $\Delta M_d = (0.509 \pm 0.004)\ \mathrm{ps}^{-1}$, mixing induced CP-violation phase angle β measured in charmonium B decays, $\sin 2\beta = 0.687 \pm 0.032$ [1], and $\bar{B}^0 \to X_d\gamma$, in addition to the constraint from $B_d \to \mu^+\mu^-$, however, without imposing the $b \to sg$ constraint. $\delta_{13}^{LL,RR}$ and $\delta_{13}^{LR,RL}$ are constrained to be order of 10^{-1} and 10^{-2} respectively with moderate sparticle masses (say, 500 GeV) [22,23]. In particular, $\mathrm{Br}(\bar{B}^0 \to X_d\gamma) \leqslant 1 \times 10^{-5}$ extracted from exclusive $B \to \rho(\omega)\gamma$, as advocated in Ref. [23], gives a more stringent constraint. $\mathrm{Br}(\bar{B}^0 \to X_d\gamma)$ directly constrains

[1] In the figure $|C_{8g}(m_W) - C'_{8g}(m_W)|$ has no upper bound because we do not impose $|z - z_1| \leqslant r_{12}$. In all models beyond SM known so far, $|C_{8g}(m_W) - C'_{8g}(m_W)|$ never reach very large value (say, 5) when all relevant experimental constraints are imposed. We do not need to know the upper bound for the analysis in the Letter.

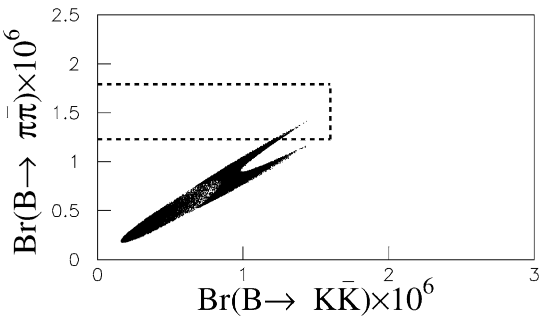

Fig. 3. The correlation between $\bar{B}^0 \to \pi^0\pi^0$ and $\bar{B}^0 \to \bar{K}K$ in MSSM with the LL insertion.

$|C_{7\gamma}(m_b)|^2 + |C'_{7\gamma}(m_b)|^2$ at the leading order. Due to the strong enhancement factor $m_{\tilde{g}}/m_b$ associated with single $\delta_{13}^{LR(RL)}$ insertion term in $C_{7\gamma}^{(\prime)}(m_b)$, $\delta_{13}^{LR(RL)}$ ($\sim 10^{-2}$) are more severely constrained than $\delta_{13}^{LL(RR)}$. However, if the left–right mixing of scalar bottom quark δ_{33}^{LR} is large (~ 0.5), $\delta_{13}^{LL(RR)}$ is constrained to be order of 10^{-2} since the double insertion term $\delta_{13}^{LL(RR)}\delta_{33}^{LR(LR*)}$ is also enhanced by $m_{\tilde{g}}/m_b$. In the case of LL (RR) insertion the precisely measured mass difference ΔM_d imposes a more severe constraint on $\delta_{13}^{LL(RR)}$.

In numerical analysis we fix $\tan\beta = 10$, vary $m_{\tilde{g}}$ and $m_{\tilde{q}}$ in the region between 300 GeV and 2 TeV, and the NHB masses in the ranges of 91 GeV $\leqslant m_h \leqslant$ 135 GeV, 91 GeV $\leqslant m_H \leqslant$ 200 GeV with $m_h < m_H$ and 200 GeV $\leqslant m_A \leqslant$ 240 GeV for the fixed mixing angle $\pi/2$ of the CP even NHBs, and scan δ_{13}^{dAB} in the range $|\delta_{13}^{dAB}| \leqslant 0.1$ for $A = B$ and 0.05 for $A \neq B$ ($A = L, R$). The numerical result for the correlation between branching ratios of $\bar{B}^0 \to \pi^0\pi^0$ and $\bar{B}^0 \to \bar{K}^0K^0$ is shown in Fig. 3 for the case of LL insertion. Due to the combined constraints mentioned above, in most of parameter space $C_{8g}(m_W)$ is not large enough to accommodate the data in 1σ region. However, there are small regions of parameter space with $x \gg 1$ (say $x \sim 40$, x is the square of the ratio between scalar quark and gluino masses) where $C_{8g}(m_W)$ is large enough to resolve the puzzle. In the case of both LL and RR insertion, the result is similar. In the cases of LR, both LR and RL insertions, we also have similar results. For $x > 1$ we have the regions with large enough $C_{8g}(m_W)$ and the regions are larger than those in the cases of LL, both LL and RR insertions. In short, numerical results of Br show that the MSSM can explain the puzzle within 1σ experimental bounds under all relevant experimental constraints except that from Br($b \to sg$).

Finally we consider the model III 2HDM [24]. In the model III 2HDM there are tree-level flavor changing neutral currents (FCNC). After diagonalizing the mass matrix of quark fields, the flavor changing (FC) part of the Yukawa Lagrangian is [24]

$$\mathcal{L}_{Y,\mathrm{FC}}^{(\mathrm{III})} = \xi_{ij}^U \bar{Q}_{i,L}\tilde{H}_2 U_{j,R} + \xi_{ij}^D \bar{Q}_{i,L} H_2 D_{j,R} + \mathrm{h.c.} \tag{13}$$

In order to obtain naturally small FCNC one assumes the Cheng–Sher parameterization [25]

$$\xi_{ij}^D = \lambda_{ij}\frac{\sqrt{m_i m_j}}{v}. \tag{14}$$

Phenomenological constraints on parameters of the models have been extensively discussed [26]. For $b \to ds\bar{s}$, the couplings $\lambda_{bd,db,ss}$ are involved. λ_{bd} can reach 0.4 without a conflict with the measured mass difference ΔM_{B_d} if the mass of pseudo-scalar Higgs boson M_A is large (say, $\sim$1 TeV) [27] and it is also allowed by the recent data for $\bar{B}^0 \to \rho(\omega)\gamma$. The constraint on λ_{ss} from the analysis on $\bar{B}_s^0$–$\bar{B}_s^0$ shows that λ_{ss} can reach $O(100)$ [28] which means that the coupling of Higgs to s quark is $O(10^{-2})$. It should be emphasized that the constraint from $B_d \to \mu^+\mu^-$ is irrelevant in the model III 2HDM because the decay involves $\lambda_{\mu\mu}$ besides λ_{bd} and $\lambda_{\mu\mu}$ has no relation to λ_{ss}, which is different from the MSSM and 2HDMs I and II.

In numerical calculations, we use $m_h = 120$ GeV, $m_d = 6$ MeV, $\lambda_{bd} = 0.3$, $\lambda_{ss} = 150$ and consequently obtain $C_{13}(m_W) = 0.41$. Corresponding to this value, $|C_{8g}(m_W)| \geqslant 0.6$, which can be satisfied under all relevant constraints. The numerical result for the values of parameters given above shows that

$$\mathrm{Br}\big(\bar{B}^0 \to \pi^0\pi^0\big) = 1.3 \times 10^{-6}, \qquad \mathrm{Br}\big(\bar{B}^0 \to \bar{K}^0K^0\big) = 0.9 \times 10^{-6}.$$

That is, the data of branching ratios of $\bar{B}^0 \to \pi^0\pi^0$ and $\bar{B}^0 \to \bar{K}^0K^0$ are accounted for, as expected. At the same time, we have checked that the NP contribution to Br($\bar{B}^0 \to \pi^+\pi^-$) is very small and negligible.

In conclusion, we have analyzed the $\bar{B}^0 \to \pi^0\pi^0$ puzzle in three kinds of models beyond the SM. In the analysis 1σ experimental bounds and 60% theoretical uncertainty which mainly comes from the input of non-perturbative parameters have been taken into account. It is shown that the minimal flavor violation models, the minimal supersymmetric standard model, and the two Higgs doublet models I and II cannot give an explanation of the $\bar{B}^0 \to \pi^0\pi^0$ puzzle within 1σ experimental bounds when all relevant experimental constraints are imposed and the model III 2HDM can explain the puzzle without a conflict with other experimental measurements. Therefore, if the data of Br for $\bar{B}^0 \to \pi^0\pi^0$ and $\bar{B}^0 \to \bar{K}^0K^0$ persist in the future, the MFV models, MSSM and model I, II 2HDMs will be excluded within 1σ experimental bounds and the model III 2HDM will be survived to resolve the puzzle.

As it is obvious, the analysis depends on the estimate of theoretical uncertainty. Our estimate comes from the uncertainties of input parameters (form factors, distribution amplitudes, CKM matrix elements, etc.) as well as the error estimate of neglecting electroweak penguin and annihilation contributions. If the uncertainty were 70%, $|C_{8g}(m_W) - C'_{8g}(m_W)|_{\min}$, which should be reached in a model in order to account for the data, would be 2.2 in the MFV models and 2.0 in the MSSM respectively so that the MFV models could not give an explanation of the data and the MSSM could. We have also analyzed the puzzle in the case without imposing the $b \to sg$ constraint in the MSSM. For all kinds of insertions there are regions of parameter space where the puzzle can be resolved within 1σ experimental bounds. We expect that similar effects appear in decays with PV final states, $B \to \pi^0\rho^0$ and $B \to \bar{K}^0 K^{0*}$.

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References

[1] The Heavy Flavor Averaging Group, http://www.slac.stanford.edu/xorg/hfag/.
[2] M. Gronau, O.F. Hernández, D. London, J.L. Rosner, Phys. Rev. D 52 (1995) 6356.
[3] M. Beneke, G. Buchalla, M. Neubert, C.T. Sachrajda, Nucl. Phys. B 606 (2001) 245.
[4] M. Beneke, M. Neubert, Nucl. Phys. B 675 (2003) 333, hep-ph/0308039.
[5] C.D. Lü, K. Ukai, M.Z. Yang, Phys. Rev. D 63 (2001) 074009;
Y.Y. Keum, A.I. Sanda, Phys. Rev. D 67 (2003) 054009.
[6] H.-n. Li, S. Mishima, A.I. Sanda, hep-ph/0508041.
[7] Y.-D. Yang, R.-M. Wang, G.R. Lu, Phys. Rev. D 73 (2006) 015003, hep-ph/0509273;
S. Baek, F.J. Botella, D. London, J.P. Silva, Phys. Rev. D 72 (2005) 114007.
[8] C.H. Chen, H.N. Li, Phys. Rev. D 63 (2001) 014003.
[9] A.J. Buras, P. Gambino, M. Gorbahn, S. Jager, L. Silvestrini, Phys. Lett. B 500 (2001) 161, hep-ph/0007085;
A.J. Buras, Acta Phys. Pol. B 34 (2003) 5615, hep-ph/0310208.
[10] A. Ali, E. Lunghi, C. Greub, G. Hiller, Phys. Rev. D 66 (2002) 034002.
[11] G. Degrassi, P. Gambino, P. Slavich, hep-ph/0601135.
[12] J.-F. Cheng, C.-S. Huang, Phys. Lett. B 554 (2003) 155.
[13] J.-F. Cheng, C.-S. Huang, X.-H. Wu, Phys. Lett. B 585 (2004) 287, hep-ph/0306086.
[14] J.-F. Cheng, C.-S. Huang, X.-H. Wu, Nucl. Phys. B 701 (2004) 54.
[15] C.W. Bauer, I.Z. Rothstein, I.W. Stewart, hep-ph/0510241.
[16] See, e.g., C.-S. Huang, P. Ko, X.-H. Wu, Y.-D. Yang, hep-ph/0511129, and references therein.
[17] C.-S. Huang, Q.-S. Yan, Phys. Lett. B 442 (1998) 209;
C.-S. Huang, W. Liao, Q.-S. Yan, Phys. Rev. D 59 (1999) 011701;
S.R. Choudhury, N. Gaur, Phys. Lett. B 451 (1999) 86;
K.S. Babu, C. Kolda, Phys. Rev. Lett. 84 (2000) 228;
C.-S. Huang, et al., Phys. Rev. D 63 (2001) 114021;
C.-S. Huang, et al., Phys. Rev. D 64 (2001) 059902, Erratum;
P.H. Chankowski, L. Slawianowska, Phys. Rev. D 63 (2001) 054012;
C. Bobeth, et al., Phys. Rev. D 64 (2001) 074014, hep-ph/0204225;
For a recent review, see, e.g., C. Kolda, in: Proceedings of the 12th International Conference on Supersymmetry and Unification of Fundamental Interactions, Tsukuba, Japan, June 2004, hep-ph/0409205.
[18] R. Bernhard, et al., CDF Collaboration, D0 Collaboration, hep-ex/0508058.
[19] Y.-B. Dai, C.-S. Huang, H.-W. Huang, Phys. Lett. B 390 (1997) 257.
[20] C.-S. Huang, X.-H. Wu, Nucl. Phys. B 657 (2003) 304.
[21] C. Greub, P. Liniger, Phys. Lett. B 494 (2000) 237;
C. Greub, P. Liniger, Phys. Rev. D 63 (2001) 054025;
G. Hiller, F. Krüger, Phys. Rev. D 69 (2004) 074020.
[22] D. Becirevic, et al., Nucl. Phys. B 634 (2002) 105, hep-ph/0112303.
[23] P. Ko, J.-h. Park, G. Kramer, Eur. Phys. J. C 25 (2002) 615, hep-ph/0206297.
[24] W.S. Hou, Phys. Lett. B 296 (1992) 179;
A. Antaramian, L.J. Hall, A. Rasin, Phys. Rev. Lett. 69 (1992) 1871;
L.J. Hall, S. Weinberg, Phys. Rev. D 48 (1993) R979;
M.J. Savage, Phys. Lett. B 266 (1991) 135;
L. Wolfenstein, Y.L. Wu, Phys. Rev. Lett. 73 (1994) 2809.
[25] T.P. Cheng, M. Sher, Phys. Rev. D 35 (1987) 3484;
T.P. Cheng, M. Sher, Phys. Rev. D 44 (1991) 1461.
[26] D.B. Chao, K. Cheung, W.-Y. Keung, Phys. Rev. D 59 (1999) 115006;
E.O. Iltan, G. Turan, I. Turan, J. Phys. G 28 (2002) 307;
Z. Xiao, L. Guo, Phys. Rev. D 69 (2004) 014002;
Y.-F. Zhou, J. Phys. G 30 (2004) 783;
R.A. Diaz, R. Martinez, C.E. Sandoval, Eur. Phys. J. C 41 (2005) 305.
[27] D. Atwood, L. Reina, A. Soni, Phys. Rev. D 54 (1996) 3296.
[28] C.-S. Huang, T.J. Li, Int. J. Mod. Phys. A 20 (2005) 161, and references therein.