Exciton condensation in a holographic double monolayer semimetal

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A B S T R A C T

The formation of intra-layer and inter-layer exciton condensates in a model of a double monolayer Weyl semi-metal is studied in the strong coupling limit using AdS/CFT duality. We find a rich phase diagram which includes phase transitions between inter-layer and intra-layer condensates as the charge densities and the separation of the layers are varied. The tendency to inter-layer condensation is strongest when the charge densities are balanced so that the weak coupling electron and hole Fermi surfaces would be nested. For systems with multiple species of massless fermions, we find a novel phase transition where the charge balance for nesting occurs spontaneously.

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1. Introduction and results

The possibility that an inter-layer exciton condensate can form in a double monolayer of two-dimensional electron gases has been of interest for a long time [1]. A double monolayer contains two layers, each containing an electron gas, separated by an insulator so that electrons cannot be transferred between the layers. Electrons and holes in the two layers can still interact via the Coulomb interaction. The exciton which would condense is a bound state of an electron in one layer and a hole in the other layer. This idea has recently seen a revival with some theoretical computations for emergent relativistic systems such as graphene or some topological insulators which suggested that a condensate could form at relatively high temperatures, even at room temperature [2]. A room temperature superfluid would have applications in electronic devices where proposals include ultra-fast switches and dispersionless field-effect transistors [3].

An exciton condensate might be more readily achievable in a double monolayer with relativistic electrons due to particle–hole symmetry and the possibility of engineering nested Fermi surfaces of electrons in one layer and the holes in the other layer. This nesting would enhance the effects of the attractive Coulomb interaction between an electron and a hole. Even at very weak coupling, it can be shown to produce an instability to exciton condensation [4]. However, in spite of this optimism, an inter-layer condensate has yet to be observed in a relativistic material, even in experiments using clean graphene sheets with separations down to the

nanometre scale [5]. The difficulty with theoretical computations, where the Coulomb interaction is strong, is the necessity of ad-hoc inclusion of screening, to which the properties of the strongly coupled system have been argued to be sensitive [6].

In this paper we will study a model of a double monolayer of relativistic two-dimensional electron gases. This model has a known AdS/CFT dual which is easy to study and it can be solved exactly in the strong coupling limit. We shall learn that, in this model, the only condensates which form are excitons, bound states of electrons with holes in the same layer (intra-layer) or bound states of electrons in one layer with holes in the other layer (inter-layer). Moreover, even though at very strong coupling, the idea of a Fermi surface loses its meaning, we find that the tendency to form an inter-layer condensate is indeed greatly enhanced by the charge balance which, at weak coupling, would give nested particle and hole Fermi surfaces. We shall see that, in the strong coupling limit, and when the charges are balanced, an inter-layer condensate can form for any separation of the layers. As well as the inter-layer condensate, such a strong interaction will also form an intra-layer condensate. We find that a mixture of the two condensates is favoured for small charge densities and larger layer separations. For sufficiently large charge densities, on the other hand, the only condensate is the inter-layer condensate. These results for charge balanced layers are summarized in Fig. 1. When the charges are not balanced, so that at weak coupling the Fermi surfaces would not be nested, no inter-layer condensate forms, regardless of the layer separation. This dramatic difference is similar to and even sharper than what is seen at weak coupling [4] where condensation occurs in only a narrow window of densities near nesting.

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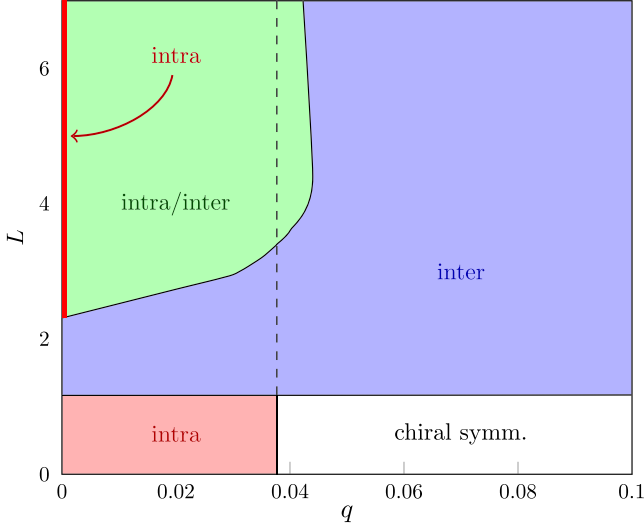
Fig. 1. (Colour online.) Phase diagram of the charge balanced double monolayer (exactly nested Fermi surfaces). The vertical axis is layer separation L in units of the inverse ultraviolet cutoff, R. The horizontal axis is the charge density q in units of R^{-2}. The green region has both inter- and intra-layer condensates. The blue region has only an inter-layer condensate. The red region has only an intra-layer condensate. The white region has no condensates.

However, even in the non-nested case, we can find a novel symmetry breaking mechanism where an inter-layer condensate can form. If each electron gas contains more than one species of relativistic electrons (for example, graphene has four species of massless Dirac electrons and some topological insulators have two species), the electric charge can redistribute itself amongst the species to spontaneously nest one or more pairs of Fermi surfaces, with the unbalanced charge taken up by the other electron species. Then the energy is lowered by formation of a condensate of the nested electrons, the others remaining un-condensed. To our knowledge, this possibility has not been studied before. The result is a new kind of symmetry breaking where Fermi surfaces nest spontaneously and break some of the internal symmetry of the electron gas in each layer. We demonstrate that, for some examples of the charge density, this type of condensate indeed exists as the lowest energy solution.

2. Weak coupling model

The model which we shall consider is a defect quantum field theory consisting of a pair of parallel, infinite, planar $2+1$-dimensional defects in $3+1$-dimensional Minkowski space and separated by a distance L. The defects are each inhabited by N_F species of relativistic massless Dirac fermions. The fermions interact by exchanging massless quanta of maximally supersymmetric Yang–Mills theory which inhabits the surrounding $3+1$-dimensional bulk. In the absence of the defects, the latter would be a conformal field theory. The interactions which it mediates have a $1/r$ fall-off, similar to the Coulomb interaction and, in the large N planar limit which we will consider, like the Coulomb force, the electron–hole interaction is attractive in all channels. The field theory action is

$$S = \int d^4x \frac{1}{g_{YM}^2} \mathrm{Tr}\left[-\frac{1}{2}F_{\mu\nu}F^{\mu\nu} - \sum_{b=1}^{6} D_\mu \Phi^b D^\mu \Phi^b + \dots \right]$$

$$+ \int d^3x \sum_{a=1}^{2}\sum_{i=1}^{N_F} \bar{\psi}_{ai}\left[i\gamma^\mu \partial_\mu + \gamma^\mu A_\mu + \Phi^6 \right]\psi_{ai} \qquad (1)$$

The first term is the action of $\mathcal{N}=4$ supersymmetric Yang–Mills theory where A_μ is the Yang–Mills gauge field and Φ^6 is one of the scalar fields and the second term is the action of the defect fermions. In the second term, the subscript a labels the defects and i the fermion species. The action includes all of the marginal operators which are compatible with the symmetries. It has a global U(1) symmetry which we associate with electric charge.

The defect field theory (1) is already interesting with one layer. It is thought to have a conformally symmetric weak coupling phase for $0 \le \lambda \le \lambda_c$. When $\lambda > \lambda_c$, chiral symmetry is broken by an intra-layer exciton condensate [7]. Near the critical point, the order parameter is thought to scale as $\langle \bar{\psi}_{1i}\psi_{1i}\rangle \sim \Lambda^2 \exp\left(-b/\sqrt{\lambda - \lambda_c}\right)$ where Λ is an ultraviolet (UV) cutoff. In the strong coupling phase, the condensate and therefore the charge gap are finite only when the coupling is tuned to be close to its critical value. The holographic construction examines this theory in the strong coupling limit, where $\lambda \gg \lambda_c$. In that limit, it is cutoff dependent and it can only be defined by introducing a systematic UV cutoff. We will find a string-inspired way to do this, tantamount to defining the model (1) as a limit of the IIB string theory which is finite and resolves the singularities. It will allow us to study the strong coupling limit using the string theory dual of this system.

When there are two monolayers, the field theory (1) can also have an inter-layer exciton condensate with order parameter $\langle \bar{\psi}_{1i}\psi_{2i}\rangle$. The results of reference [4] suggest that, with balanced charge densities and nested Fermi surfaces, the inter-layer condensate occurs even for very weak coupling. Not much is known as to how it would behave at strong coupling. It is the strong coupling limit of this model which we will now solve using its string theory dual.

3. Strong coupling model

The string theory dual of the defect field theory is the D3–probe-D7 brane system of IIB string theory [8]. A monolayer is a single stack of N_F D7 coincident branes. A double monolayer has two parallel stacks, one of N_F D7 branes and another of N_F anti-D7 branes separated by a distance L. In both cases, the D7 brane stacks overlap $N \gg N_F$ coincident D3 branes. With the appropriate orientation, the lowest energy states of open strings which connect the D3 to the D7 branes are massless two-component relativistic fermions that propagate on $2+1$ dimensions and are the defect fields in (1). In the large N and strong coupling limits, the D3 branes are replaced by the $AdS_5 \times S^5$ background and solving the theory reduces to extremizing the classical Born–Infeld action $S \sim N_F T_{D7} \int d^8\sigma \sqrt{-\det(\gamma + 2\pi\alpha' F)}$ for the D7 brane embedded with world-volume gauge field strength F and metric γ_{ab} in $AdS_5 \times S^5$. However, there is an immediate problem with this setup. Any D7 brane geometry which approaches the appropriate D7 brane boundary conditions at the boundary of AdS^5 is unstable. This is a reflection of the fact that the strong coupling limit of the quantum field theory on a single D7 brane is not conformally symmetric. We shall use a suggestion by Davis et al. [9] who regulated the D7 brane by embedding it in the extremal black D3 brane geometry, with metric

$$\frac{ds^2}{R^2} = \frac{r^2\left(-dt^2 + dx^2 + dy^2 + dz^2\right)}{\sqrt{1 + R^4 r^4}}$$

$$+ \sqrt{1 + R^4 r^4}\left(\frac{dr^2}{r^2} + d\psi^2 + \sin^2\psi \sum_{i=1}^{5}(d\theta^i)^2 \right) \qquad (2)$$

where $\sum_{i=1}^{5}(\theta^i)^2 = 1$ and $R^4 = \lambda\alpha'^2$. The asymptotic, large r limit of this metric is 10-dimensional Minkowski space. It has a horizon at $r = 0$. In the near horizon limit, which produces the IIB string on

$AdS_5 \times S^5$, $rR \ll 1$, it approaches the Poincaré patch of $AdS_5 \times S^5$. Since R contains the string scale α', $1/R$ can be regarded as a (UV) cutoff.

The D7 and anti-D7 world-volumes are almost entirely determined by symmetry. They have $2 + 1$-dimensional Poincaré invariance and wrap (t, x, y). The model (1) has an $SO(5)$ R-symmetry. The D7's must therefore wrap $(\theta^1, \ldots, \theta^5)$ to form an S^4. For the remaining world-volume coordinate, we use the radius r in (2). The dynamical variables are then $\psi(r)$ and the positions $z_1(r)$ and $z_2(r)$ of D7 and anti-D7, which by symmetry can only be functions of r. $\psi = \frac{\pi}{2}$ is a point of higher symmetry, corresponding to parity in the defect field theory with massless fermions. $\psi(r) = \frac{\pi}{2} + \frac{c}{r^2} + \ldots$ is required to approach $\frac{\pi}{2}$ at $r \to \infty$ and, if it becomes r-dependent at all ($c \neq 0$), parity is broken by an intra-layer condensate. Parity can be restored if pairs of branes have condensates of opposite signs. This would break flavour symmetry when N_F is even, $U(N_F) \to U(N_F/2) \times U(N_F/2)$. Whether this sort of flavour symmetry breaking or parity and time reversal breaking takes place is an interesting dynamical question which will be studied elsewhere. Finally, it will turn out that, either $z_{1,2}(r)$ are constants, or the D7 and anti-D7 meet and smoothly join together at a minimum radius, r_0. Asymptotically, $z_{1,2}(r) = \pm L/2 \mp R r_0^4/r^4 + \ldots$.

We have performed numerical computations to determine the lowest energy embeddings of the D7 (and anti-D7) branes as a function of the charge density (q) and the brane–anti-brane separation L. In the following we outline the results of these computations. The formalism for studying the embeddings of the probe D branes is already well-known in the literature and we refer the reader there for details. Examples for double monolayers can be found in references [10–16].

When we suspend a single D7 brane in the black D3 brane metric (2), we find that the lowest energy solution truncates before it reaches the horizon. This is called a "*Minkowski embedding*". The function $\psi(r)$ moves from $\psi = \frac{\pi}{2}$ at $r \sim \infty$ to $\psi = 0$ or $\psi = \pi$ at the r where the brane pinches off. The S^4 which the world-volume wraps shrinks to a point there and this collapsing cycle is what makes the truncation smooth. This brane geometry is interpreted as a charge-gapped state. The lowest energy charged excitation is a fundamental string which would be suspended between the D7 brane and the horizon. In this case, that string has a minimum length and therefore a mass gap.

We can introduce a charge density q on the single monolayer. When the D7 brane carries a charge density, its world volume must necessarily reach the horizon. This is called a "*black hole embedding*". Charge in the quantum field theory corresponds to D7 brane world-volume electric field $E_r \sim q$. This hedgehog-like electric field points outward from the centre of the brane. The radial lines of flux of the electric field can only end if there are sources. Such sources would be fundamental strings, suspended between the D7 brane and the horizon. However, the strings have a larger tension than the D7 brane and they pull the D7 brane to the horizon resulting in a gapless state. This is confirmed by numerical solutions of the embedding equation of a single brane and, indeed, we find that the S^4 which is wrapped by the world volume shrinks to a point as it enters the horizon. This state no longer has a charge gap. Even in the absence of a charge gap, we find that, for small charge densities, there is still an intra-layer exciton condensate. Our numerical studies show that it persists up to a quantum phase transition at a critical density $q_{\text{crit.}} \approx 0.0377/R^2$. At densities greater than the critical one, $\psi = \frac{\pi}{2}$, is a constant.

Now, consider the double monolayer with D7 and anti-D7 branes. A D7–anti-D7 pair of branes would tend to annihilate. We prevent this annihilation by requiring that they be separated by a distance L as they approach the boundary at $r \to \infty$. When their world volume enters the bulk, they can still come together and

Table 1
Types of possible solutions for the balanced charge $(q, -q)$ case, where (Mink, BH) stand for (Minkowski, black-hole) embeddings.

	$z_2 - z_1 = L$, const.	$z_2(r) - z_1(r) \to 0$ at r_0
$c = 0$	**Type 1** un-joined, $\psi = \frac{\pi}{2}$ BH, no condensate	**Type 2** joined, $\psi = \frac{\pi}{2}$ inter
$c \neq 0$	**Type 3** un-joined, $\psi(r)$ r-dependent Mink ($q = 0$) intra BH ($q \neq 0$) intra	**Type 4** joined, $\psi(r)$ r-dependent intra + inter only when $q \neq 0$

annihilate – their world volumes fusing together at a minimal radius r_0. This competes with the tendency of a monolayer brane to pinch off at some radius. Indeed, when the charge density is zero, we see both behaviours. When the stacks of branes are near enough, that is, $L < L_c \simeq 2.31R$ is small enough, they join. This state has an inter-layer condensate. When they are farther apart, they remain un-joined. Instead, they pinch off to form Minkowski embedding, corresponding to a state with intra-layer condensates.

When we introduce balanced charges q and $-q$ on the D7 and anti-D7, respectively, there are four modes of behaviour which are summarized in Table 1. Each of these behaviours occurs in the phase diagram in Fig. 1. Type 1 solutions are maximally symmetric with $\psi = \frac{\pi}{2}$ and $z_{1,2} = \pm L/2$. They occur in the white region of Fig. 1. They have no exciton condensates at all. Type 3 solutions occur in the red region. They have $\psi(r)$ a nontrivial function, but $z_{1,2} = \pm L/2$. The branes do not join. They are Minkowski embeddings when $q = 0$ and black hole embeddings when $q \neq 0$. Type 3 has an intra-layer exciton condensate only. There is a quantum phase transition between type 1 and type 3 solutions at $q_c = 0.0377$. Both type 1 and type 3 solutions occur only for very small layer separations, or order the UV cutoff. Type 2 solutions occupy the blue region. They have $\psi = \frac{\pi}{2}$, constant, $z_{1,2}(r)$ are nontrivial functions. The D7 and anti-D7 branes join at a radius, $r_0 \neq 0$. The intra-layer condensate vanishes and there is a non-zero inter-layer condensate. In type 4 solutions, both $\psi(r)$ and $z_{1,2}(r)$ have nontrivial profiles. The D7 and anti-D7 branes join and $\psi(r)$ also varies with radius. This phase has both and inter- and intra-layer condensate. This solution exists only when q is nonzero and, then, only for small values of q. For $r_0 \simeq 0$ we have $q < 0.0377$, when r_0 grows, the allowed values of q decrease.

4. Un-balanced charges

Consider a double monolayer with un-balanced charges, $Q > 0$ on the D7 and $-\bar{Q} < 0$ on the anti-D7 brane. The same argument as to why a single charged D7 brane must have a Minkowski embedding applies and, on the face of it, it is impossible for the branes to join before they reach the horizon. There is, however, another possibility which arises when there are more than one species of fermions on each brane, that is, $N_F > 1$. In that case, one or more of the fermion species can nest spontaneously, with the deficit of charge residing in the other species. This would break internal symmetry. For example, if $Q > \bar{Q} > 0$, k branes take up charge $\bar{Q}$ and the remaining $N_F - k$ take up the remainder $Q - \bar{Q}$, this would break $U(N_F) \times U(N_F) \to U(N_F - k) \times U(k) \times U(N_F - k) \times U(k)$. Then the branes with matched charges ($\bar{Q}$) would join, further breaking the symmetry $U(N_F - k) \times U(k) \times U(N_F - k) \times U(k) \to U(N_F - k) \times U(k) \times U(N_F - k)$. Then, $N_F - k$ charged D7 branes and $N_F - k$ uncharged anti-D7 branes either break parity or some of the remaining $U(N_F - k) \times U(N_F - k)$ symmetry. The uncharged branes must take up a Minkowski embedding. We have computed the energies of some of these symmetry breaking states for the case where $N_F = 2$. We find a range of charge densities

where spontaneous nesting is energetically preferred. The implications of this idea for double monolayer physics is clear. The fermion and hole densities of individual monolayers would not necessarily have to be fine tuned in order to nest the Fermi surfaces. It could happen spontaneously.

The intra-layer and inter-layer condensates discussed here have not been seen in graphene to date (with a possible exception [17]), presumably because the coupling is not strong enough. Our results show that the inter-layer condensation is extremely sensitive to Fermi surface nesting, even in the strong coupling limit. It would be interesting to better explore spontaneous nesting, since creating favourable conditions for it could be a way forward with graphene.

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