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# Particle Spacing and Chemical Reaction Effects on Convective Heat Transfer through a Nano-Fluid in Cylindrical Annulus

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## Abstract

Particle size plays vital role in determination of heat transfer through a nano-fluid. But the chemical reaction of copper metal particle with water certainly alters the expected heat transfer rate. So the size of the nano-particle in its flow can be described by Newton's law. All the assumptions lead to a system of coupled non-linear governing partial differential equations. These equations are solved for velocity ( $w$ ) and temperature ( $\theta$ ) using R-K 6<sup>th</sup> order with shooting method subject to the boundary conditions. The profiles of  $w$ ,  $\theta$  and the heat transfer rate ( $Nu$ ) for various parameters are displayed. The inter particle spacing enhances the heat transfer rate due to the Brownian motion of the particles.

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**Keywords:** Nano-fluid; MHD; Heat Transfer; Inter Particle Spacing; Chemical Reaction; R-K 6<sup>th</sup> order

## 1. Introduction

The Effect of thermal buoyancy on vortex shedding behind a square cylinder has been studied by Chatterjee and Mondal [1]. Das S. K. et. al. [2] pronounced the need of the study of convective heat transfer in cylindrical geometries in microelectronics, transportation, nuclear power plants, cooling of microchips in computers, and heat exchangers. Dhiman et. al. [3] reported that the average Nusselt number increases monotonically with an increment

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in Reynolds number and/or Prandtl number in their numerical study across square cylinder. Recently Etminan-Farooji et al. [4] studied the effects of Peclet number and types of nano-fluids on heat transfer from the cylinder rather than fluid flow hydrodynamics around a square cylinder. But the study of inter particle spacing and the chemical reaction between the Cu particle and water in convective heat transfer are missing in most of these studies. We want to study the convective heat transfer through a cylindrical annulus in cu-water nano-fluid using the models proposed by Graham [5] and Jang-Choi [6] due to the presence of size of the particle and inter particle spacing in these models. Mahmoodi [7] studied the mixed convective heat transfer using nano-fluid in rectangular channel with moving bottom wall. Rahnama et al. [8], reported that increasing aspect ratio decreases Nusselt number for all Reynolds numbers numerically. Sarkar et al. [9] studied vortex structure distributions and mixed convective heat transfer around a solid circular cylinder utilizing nano-fluid for unsteady regime. Soltanipour et al.[10] studied the heat transfer enhancement using  $\gamma\text{-Al}_2\text{O}_3\text{-H}_2\text{O}$  nano-fluid in a curved duct with mixed convection inside a rectangular enclosure. . Valipour and ZareGhadi [11] studied the heat transfer of nano-fluid around a solid circular cylinder and reported that the solid volume fraction increases, the magnitude of minimum velocity in the wake region. Their result showed that the Strouhal number increases by increasing solid volume fraction. It also showed that increase in Strouhal numbers leads to reduction in vortex detachment. The core of the vortices grows during subsequent processes of vortex shedding cycle. Also, Yoon et al. [12] investigated the flow past a square cylinder with different incidence angles. It was found that for steady flow, the wave length increases with increasing Reynolds number.

### Nomenclature

$B_0$	Constant applied magnetic field ( $\text{Wb m}^{-2}$ )
$C_p$	Specific heat at constant pressure ( $\text{J kg}^{-1}\text{K}^{-1}$ )
$g$	Gravity acceleration ( $\text{m s}^{-2}$ )
$J$	Current density
$M$	Dimensionless magnetic field parameter
$Q$	Dimensional heat source ( $\text{kJ s}^{-1}$ )
$T$	Local temperature of the nano-fluid (K)
$T_0$	Temperature on inner surface
$T_m$	Temperature on outer surface
$R$	Dimensionless radius
$k_f$	Thermal conductivity
$D_f$	Diffusivity of water
$r$	Radius
$s$	Gap of the Annulus
$k_p$	Diameter of the pore
$Re$	Reynolds number
$u, w$	Velocity components

### Greek symbols

$\alpha$	Thermal diffusivity ( $\text{m}^2 \text{s}^{-1}$ )
$\beta_T$	Thermal expansion coefficient ( $\text{K}^{-1}$ )
$\varepsilon$	Dimensionless small quantity ( $\ll 1$ )
$\Phi$	Solid volume fraction of the nano-particles
$\mu$	Dynamic viscosity ( $\text{Pa s}$ )
$\theta$	Dimensionless temperature
$\sigma$	Electrical conductivity ( $\text{m}^2 \text{s}^{-1}$ )
$\nu$	Kinematic viscosity ( $\text{m}^2 \text{s}^{-1}$ )
$\rho$	Density
nf	Nano-fluid
f	Fluid (water)

**2. Mathematical Formulations**

We consider convective flow of Cu-water nano-fluid through a circular cylindrical annulus filled with porous medium in the presence of temperature dependent heat source, with inner and outer walls are maintained at a constant temperature. The flow is fully developed in the axial direction (z). The fluid region has constant physical properties and the flow is a convection flow taking place under thermal buoyancies and uniform axial pressure gradient. The Boussinesq approximation is invoked. Also the flow is unidirectional along the axis of cylindrical annulus. The transverse magnetic field is applied. The geometry of the problem is shown in Schematic Diagram.

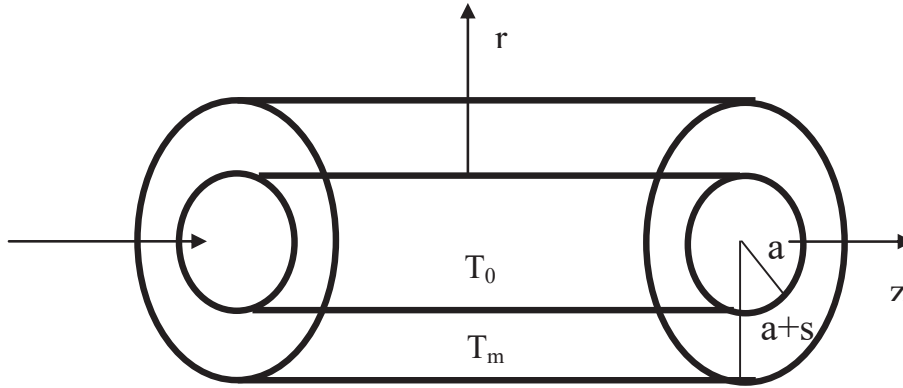


Fig.1 Schematic Diagram

Making use of the above assumptions the governing equations are

$$\frac{\partial}{\partial r} (ru) = 0 \tag{1}$$

$$\mu_{nf} \left( \frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r} \right) - \frac{\mu_{nf}}{k} w - \frac{\sigma B_0^2}{r^2} w + g(\rho\beta_T)_{nf} (T - T_0) = 0 \tag{2}$$

$$w \frac{\partial T}{\partial z} = \frac{\alpha_{nf}}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) - \frac{Q}{(\rho c_p)_{nf}} (T - T_0) \tag{3}$$

The appropriate initial and boundary conditions for the problem are given by

$$T = T_0, w = 0 \text{ on } r = a \text{ and } T = T_m, w = 0 \text{ on } r = a + s \tag{4}$$

Thermo-Physical properties are related as follows:

$$\rho_{nf} = (1 - \phi)\rho_f + \phi\rho_s, \quad (\rho c_p)_{nf} = (1 - \phi)(\rho c_p)_f + \phi(\rho c_p)_s$$

$$\alpha_{nf} = \frac{k_{nf}}{(\rho c_p)_{nf}}, \quad \frac{\mu_{nf}}{\mu_f} = 1 + 2.5\phi + 4.5 \left[ \frac{h}{d_p} \left( 2 + \frac{h}{d_p} \right) \left( 1 + \frac{h}{d_p} \right)^2 \right]^{-1}$$

$$k_{nf} = k_f(1 - \phi) + \beta_1 k_p \phi + c_1 \frac{d_f}{d_p} k_f \text{Re}^2 d_p \text{Pr} \phi; \quad (\rho\beta)_{nf} = (1 - \phi)(\rho\beta)_f + \phi(\rho\beta)_s$$

Due to chemical reaction by the solvent and the metal particle the rate of change of the size of the Particle can be considered as:

$\frac{dd_p}{dt} = -K d_p \Rightarrow d_p = d_0 e^{-K t}$  Where  $d_p$  the diameter of the metal particle,  $d_0$  is the initial size of the particle (assumed as 50 nm) and  $K$  is the reaction rate:

$$\frac{\mu_{nf}}{\mu_f} = 1 + 2.5 \phi + 4.5 \left[ \left( \frac{h e^{Kt}}{50} \right) \left( 2 + \frac{h e^{Kt}}{50} \right) \left( 1 + \frac{h e^{Kt}}{50} \right)^2 \right]^{-1}$$

$$k_{nf} = k_f(1-\phi) + \beta_1 k_p \phi + \frac{c_1}{50} d_f e^{Kt} k_f Re^2 d_p Pr \phi; d_f = 0.384 \text{ nm for water} \quad (5)$$

where  $\beta_1 = 0.01$  is a constant for considering the kapitza resistance per unit area

$$Re d_p = \frac{d_p}{v_f} \frac{\kappa T}{3 \pi \mu_f d_p l_f} = \frac{1.381 \times 10^{-23} T}{v_f 3 \pi \mu_f (0.738)}, \quad Pr = \text{Prandtl number} = \frac{v_f}{\alpha_f}; T = 300k$$

$c_1 = 18 \times 10^6$  is proportionality constant

The dimensionless variables are  $R = \frac{r}{a}, s^* = \frac{s}{a}, Z = \frac{z}{a}, W = \frac{w}{v_f} a, U = \frac{u}{v_f}, \theta = \frac{T-T_0}{T_m-T_0}$  (6)

Using equations (4), (5), (6) the Equations (2) & (3) can be written in the following form:

$$\left( 1 + 2.5 \phi + 4.5 \left[ \left( \frac{h e^{Kt}}{50} \right) \left( 2 + \frac{h e^{Kt}}{50} \right) \left( 1 + \frac{h e^{Kt}}{50} \right)^2 \right]^{-1} \right) \left( \frac{\partial^2 W}{\partial R^2} + \frac{1}{R} \frac{\partial W}{\partial R} \right) - \frac{1}{D} \left( 1 + 2.5 \phi + 4.5 \left[ \left( \frac{h e^{Kt}}{50} \right) \left( 2 + \frac{h e^{Kt}}{50} \right) \left( 1 + \frac{h e^{Kt}}{50} \right)^2 \right]^{-1} \right) W + \frac{1}{R^2} M W + \left( 1 - \phi + \phi \frac{(\rho\beta)_s}{(\rho\beta)_f} \right)^{-1} Gr \theta = 0$$

$$W \frac{\partial \theta}{\partial z} = \frac{1}{Pr} \left[ \left( 1 - \phi + 0.01 \phi \frac{k_p}{k_f} + \frac{k_p}{k_f} \frac{\rho_f^2 c_{pf} e^{Kt}}{50 \mu_f^4} - 28632.9991 \times 10^{-52} \right) \left( 1 - \phi + \phi \frac{(\rho c_p)_s}{(\rho c_p)_f} \right)^{-1} \left[ \frac{\partial^2 \theta}{\partial R^2} + \frac{1}{R} \frac{\partial \theta}{\partial R} \right] - \frac{1}{Pr} Q_H \left[ \left( 1 - \phi + \phi \frac{(\rho c_p)_s}{(\rho c_p)_f} \right)^{-1} \right] \theta \right]$$

Where the corresponding boundary conditions (4) can be written in the dimensionless form as:

$W = 0, \theta = 0$  on  $R = 1$  and  $W = 0, \theta = 1$  on  $R = 1 + s^*$  (We drop \* in the later part)

Here  $Pr$  is the Prandtl number,  $M$  is the magnetic parameter (Hartmann number),  $Q_H$  is the heat Source parameter,  $K$  is the chemical Reaction parameter,  $D^{-1}$  is the Darcy number,  $Gr$  is the Thermal Grashof number, which are defined as:

$$Pr = \frac{v_f}{\alpha_f}, M = \frac{\sigma B_0^2}{\mu_f} a, Q_H = \frac{Q}{k_f} a, K = \frac{k_l}{v_f} a, \frac{1}{D} = \frac{a}{k}, Gr = \frac{g \beta_f a^3 (T_m - T_0)}{v_f^2}$$

The local Nusselt number  $Nu$  in dimension less form:  $Nu = - \frac{k_{nf}}{k_f} \theta' (1 + s)$

### 3. Solution of the Problem and Results

The typical cross section of the cylinder has considered for numerical computations. The governing equations are solved by using R-K 6<sup>th</sup> order with shooting methods with the help of Mathematica package. During the solution the axial temperature and constriction gradients are assumed to be constant. The effect of various parameters viz. solid volume fraction ( $\phi$ ), inter particle spacing ( $h$ ), Magnetic parameter ( $M$ ), gap of the annulus ( $s$ ), heat source parameter ( $Q_H$ ), porous parameter ( $D$ ), thermal Grashof number ( $Gr$ ) and chemical reaction parameter ( $K$ ) on velocity ( $W$ ) and temperature ( $\theta$ ) are exhibited in graphs from Figures 2 to 15. The Prandtl Number ( $Pr$ ) kept constant as 7 (for water).

From Fig. 2-9 the profiles of velocity are parabolic in shape indicating that flow is maximum in the mid region of

the annulus for all parameters. The enhancement of solid volume fraction ( $\phi$ ) enhances the flow due to the Brownian motion (Fig. 2). The absence of the nano-particles leads to the almost constant flow of the fluid. The variation of the  $\phi$  up to 5% has been studied and found that flow varies rapidly. The velocity profile for gap of the annulus ( $s$ ) is depicted in Fig.3. As the gap increases, the velocity ( $W$ ) increases significantly. The flow is significantly high when the radius of the outer cylinder is double the radius of the inner cylinder. The natural increase in the flow of the fluid is observed with inter particle spacing (Fig.4). The significant enhancement of velocity ( $W$ ) is also observed for inter particle spacing ( $h$ ) from 2nm to 10nm. The increase in thermal buoyancy increases the velocity from Fig.5. From Fig. 6 the increase in Lorentz force retards the flow; the variation of velocity is very less due to the presence of the metal particles. The variation of velocity with heat source ( $Q_H$ ) is exhibited in Fig. 7. The increase in temperature dependent heat source retards the flow gradually. This variation is due to the presence of the metal particles, which are affected due to the heat source very much. Fig. 8 exhibits the variation of the velocity with chemical reaction parameter ( $K$ ). Velocity is more when the reaction is destructive and it less when the reaction is generative. The velocity is maximum in mid region of annulus and has quick convergence when we move slightly from the mid region to the boundary. The velocity profile for Darcy parameter ( $D^{-1}$ ) is in Fig. 9. The velocity decreases gradually with increase in  $D^{-1}$ .

The temperature profiles are depicted from Fig. 10 to 15. From Fig.10 the increase in solid volume fraction ( $\phi$ ) enhances the temperature. From Fig.11 the rapid reduction of temperature is observed with the reduction of the gap between the inner and outer cylinders ( $s$ ). The variation of temperature is found absolutely uniform throughout the annulus when the radius of the outer cylinder is double the radius of inner cylinder. Fig. 12 exhibits the variation of temperature with inter particle spacing ( $h$ ). The increase in  $h$  (2nm to 10nm) decreases the temperature due to spacing in between the particles. The magnetic parameter ( $M$ ) increases the temperature slightly from Fig. 13. From Fig. 14 the increase in heat source ( $Q_H$ ) reduces the temperature slightly due to the presence of nano-particles, absorbing the heat. Fig. 15 shows the temperature profile for variation of chemical reaction parameter ( $K$ ). Temperature falls from generative to destructive reaction gradually.

Table-1 display the rate of heat transfer for  $\phi$ ,  $h$ ,  $Q_H$  and  $K$  calculated on the outer cylinder. The table signifies the importance of nano fluid from the second and third rows ( $\phi = 2\%$  and  $5\%$  respectively). When compared with first row (non-nano fluid). Interestingly the increase in the spacing among the metal particles slightly enhances the heat transfer. Naturally the heat source enhances the heat transfer. The destructive chemical reaction produces more heat transfer rate than the generative chemical reaction.

Table 1. Nusselt Number

$\phi$	$h = 2, Q_H = 5,$ $K = 0.5$	$h = 5, Q_H = 5,$ $K = 0.5$	$h = 10, Q_H = 5,$ $K = 0.5$	$h = 5, Q_H = 10,$ $K = 0.5$	$h = 2, Q_H = 5,$ $K = - 0.5$
0	2.42008	2.42229	2.42686	3.09479	2.41998
0.02	2.61657	2.61882	2.62341	3.30678	2.61647
0.05	2.91221	2.91449	2.91909	3.62237	2.9121

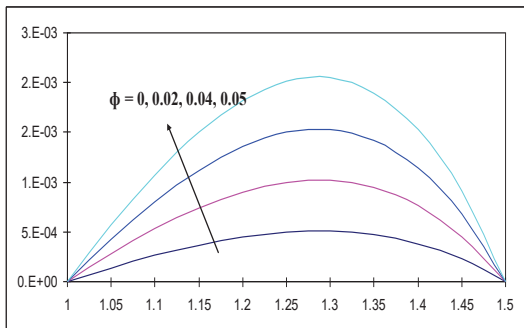


Fig.2 Variation of w with  $\phi$

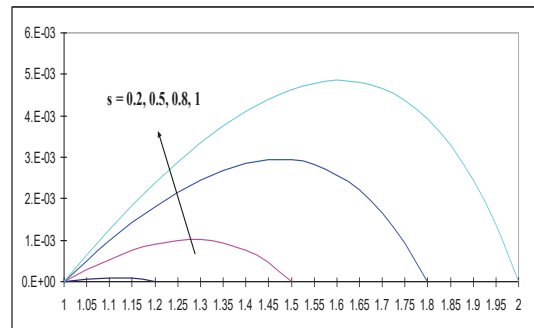


Fig.3 Variation of w with s

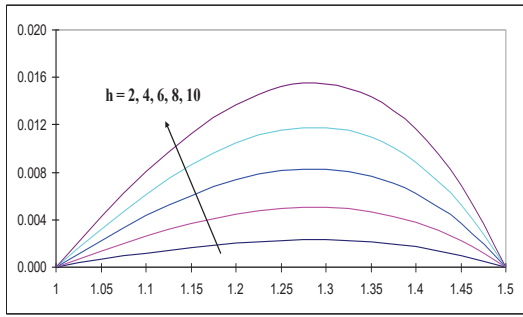


Fig.4 Variation of w with h

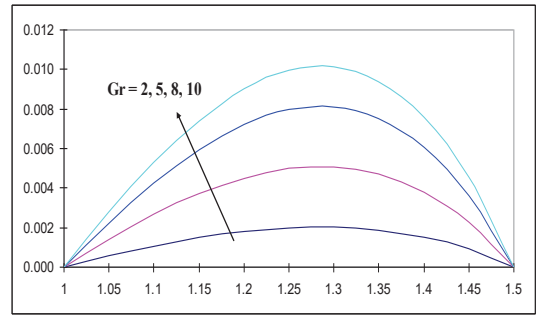


Fig.5 Variation of w with Gr

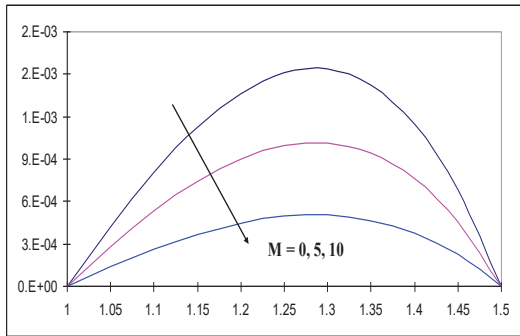


Fig.6 Variation of w with M

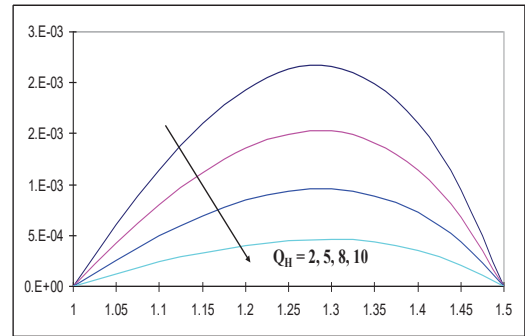


Fig.7 Variation of w with  $Q_H$

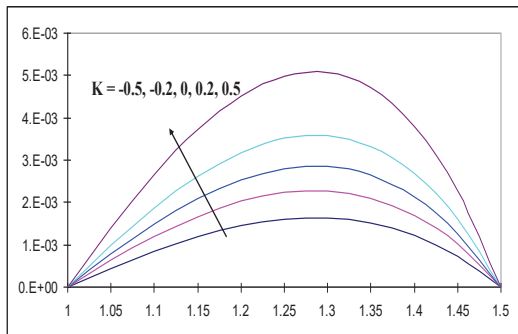


Fig.8 Variation of w with K

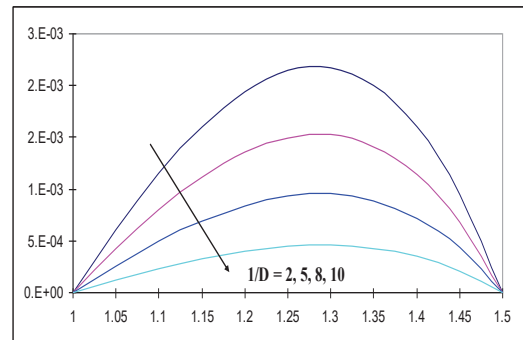


Fig.9 Variation of w with  $D^{-1}$

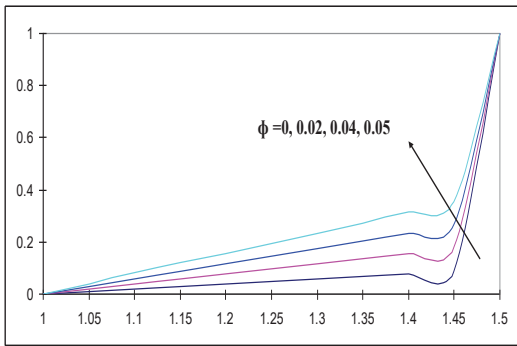


Fig.10 Variation of  $\theta$  with  $\phi$

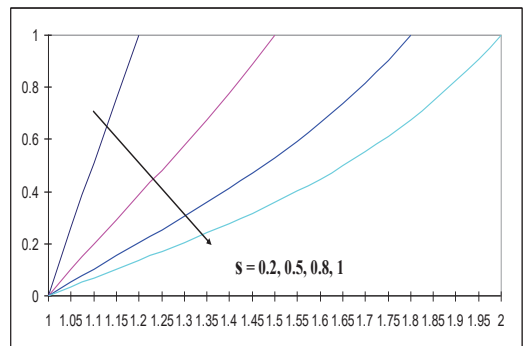


Fig.11 Variation of  $\theta$  with  $s$

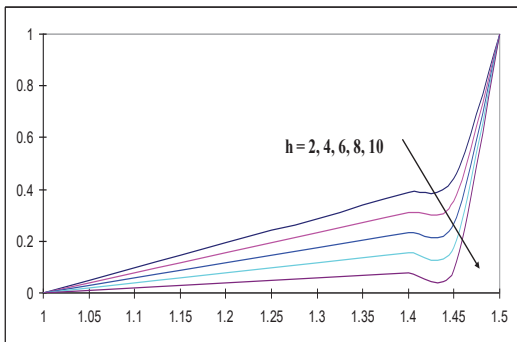


Fig.12 Variation of  $\theta$  with  $h$

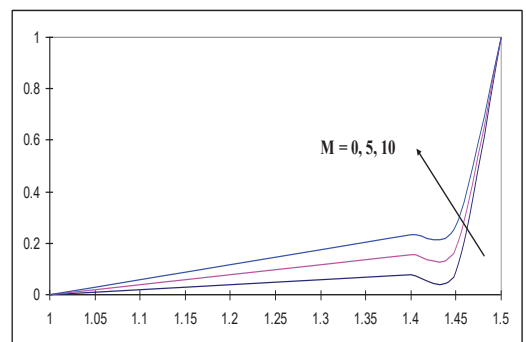


Fig.13 Variation of  $\theta$  with  $M$

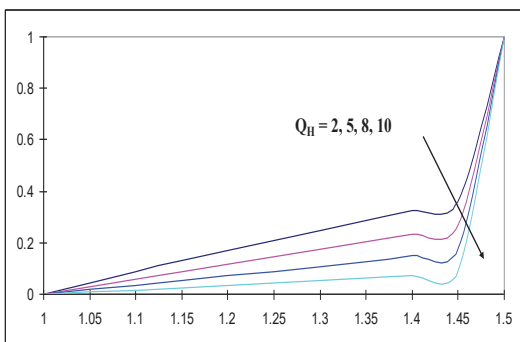


Fig.14 Variation of  $\theta$  with  $Q_H$

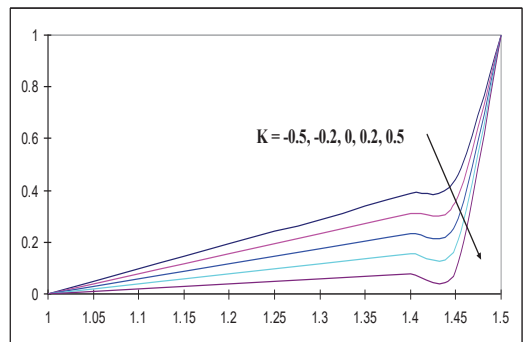


Fig.15 Variation of  $\theta$  with  $K$

#### 4. Conclusions

- The particle spacing is directly proportional to flow, inversely proportional to temperature.
- The chemical reaction is directly proportional to flow, inversely proportional to temperature.
- The destructive chemical reaction enhances the thermal boundary.

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