



Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb



Gauge dynamics in the PNJL model: Color neutrality and Casimir scaling

Hiroaki Abuki^{a,*}, Kenji Fukushima^b

^a Institut für Theoretische Physik, Goethe-Universität, Max-von-Laue-Straße 1, D-60438 Frankfurt am Main, Germany

^b Yukawa Institute for Theoretical Physics, Kyoto University, Oiwake-cho, Kitashirakawa, Sakyo-ku, Kyoto 606-8502, Japan

ARTICLE INFO

Article history:

Received 3 February 2009

Accepted 14 April 2009

Available online 3 May 2009

Editor: J.-P. Blaizot

ABSTRACT

We discuss a gauge-invariant prescription to take the mean-field approximation self-consistently in the PNJL model (Nambu–Jona-Lasinio model with the Polyakov loop). We first address the problem of non-vanishing color density in normal quark matter, which is an artifact arising from gauge-fixed treatment of the Polyakov loop mean-fields. We then confirm that the gauge average incorporated in our prescription resolves this problem and ensures color neutrality. We point out that the proposed method has an advantage in computing the expectation value of any function of the Polyakov loop matrix. We discuss the Casimir scaling as an immediate application of the method.

© 2009 Elsevier B.V. Open access under [CC BY license](#).

1. Introduction

The interplay between the QCD phase transitions of chiral restoration and color deconfinement at finite temperature and/or density has been attracting much interest recently. There are a lot of attempts to describe confinement-deconfinement physics by means of effective models in terms of the Polyakov loop [1–22]. One successful approach that can describe both the chiral and deconfinement transitions (crossovers) is the Polyakov-loop augmented Nambu–Jona-Lasinio (PNJL) model. This model accommodates self-consistent treatment for two approximate order parameters; the Polyakov loop L for deconfinement and the chiral condensate $\langle\bar{\psi}\psi\rangle$ for chiral restoration. Here the former works as an exact order parameter in the quenched limit (i.e. $m_q \rightarrow \infty$), while the latter is exact in the chiral limit (i.e. $m_q \rightarrow 0$). Due to a particular form of coupling between L and $\langle\bar{\psi}\psi\rangle$ the PNJL model has a general tendency to make two crossovers in L and $\langle\bar{\psi}\psi\rangle$ come close to each other [3]. Besides, it has turned out that the bulk thermodynamics resulting from the model shows remarkable agreement with numerical data from the lattice QCD simulation [7].

It is known by now that pathological behavior arises from a simple mean-field ansatz for the Polyakov loop matrix [10,15,16,22]. In Ref. [23] one of the present authors found that a saddle-point approximation on the Polyakov loop matrix leads to unphysical non-zero color density even in the normal phase of quark matter (see also Refs. [24,25]). This is not a principle problem inherent to the PNJL model but rather a practical one associated with the mean-field approximation; we have to assume a certain gauge to make the color density definite, and at the same time, for the

sake of the color density computation it is convenient to take a special gauge in which the Polyakov loop L is diagonal. These two gauge choices are, however, not necessarily compatible. In other words, the color chemical potential matrix and the Polyakov loop matrix are not commutable.

The problem comes from the fact that we need to treat the Polyakov loop mean-field not as a traced quantity $\ell \equiv \frac{1}{N_c} \langle \text{tr} L \rangle$ but as a matrix $L(\varphi_1, \varphi_2) \equiv \text{diag}(e^{i\varphi_1}, e^{i\varphi_2}, e^{-i(\varphi_1+\varphi_2)})$ when we evaluate the color density. Then we face another undesirable situation. That is, in terms of φ_1 and φ_2 , it is hard to realize a difference between the Polyakov loop ℓ and the anti-Polyakov loop $\bar{\ell} \equiv \frac{1}{N_c} \langle \text{tr} L^\dagger \rangle$ which are both real numbers [11,26]. It is claimed in Ref. [18] that the fluctuation around the mean-field induces a difference between ℓ and $\bar{\ell}$. We should note, however, that all these problems do not appear if we treat ℓ and $\bar{\ell}$ as the relevant mean-fields, which works unless we consider color degrees of freedom such as color superconductivity [27]. In fact, in the color-superconducting phase, we cannot express the quasi-quark contribution to the thermodynamic potential solely in terms of ℓ and $\bar{\ell}$, but it inevitably involves the matrix elements of L [10].

The present Letter aims to propose a resolution to circumvent these shortcomings of the simple mean-field approximation. We would emphasize that our prescription not only improves the mean-field approximation but also encompasses correct gauge dynamics from which the neutrality with respect to gauge charge is derived (i.e. the Gauss law).

2. Model and mean-field approximation

We here explain our model, the ingredients of which are the Polyakov-loop matrix model [5,9] and the NJL model [28]. The difference from the PNJL model lies in a mean-field evaluation for the Polyakov loop [1,3,6,29].

* Corresponding author.

E-mail address: h.abuki@rs.kagu.tus.ac.jp (H. Abuki).

First, let us address the pure gluonic sector. We assume that the pure gluonic dynamics would be described by the nearest neighbor interaction of the traced Polyakov loop as

$$S_g[L] = -N_c^2 e^{-a/T} \sum_{\vec{x}, \hat{n}} l(\vec{x}) l^*(\vec{x} + \hat{n}), \quad (1)$$

with $l \equiv \frac{1}{N_c} \text{tr} L$ and $l^* \equiv \frac{1}{N_c} \text{tr} L^\dagger$. This form of the simplest matrix model [30] is to be postulated from the leading-order contribution in the strong coupling expansion, which specifies the T -dependent interaction strength with a being a model parameter [3].

The action (1) looks like a spin model. We then make use of the Weiss approximation with the neighboring spin sites treated as the mean-fields. Hence, the mean-field action is

$$S_{\text{mf}}[\alpha, \beta] = -N_c \sum_x [\alpha \text{Re} l(x) + i\beta \text{Im} l(x)], \quad (2)$$

where α and β correspond to the Polyakov loop mean-fields. Finite β would be induced by \mathcal{C} -odd terms at finite μ . We denote the Polyakov loop expectation values, hereafter, as $\ell \equiv \langle l \rangle_{\text{mf}}$ and $\bar{\ell} \equiv \langle l^* \rangle_{\text{mf}}$. The expectation value $\langle \dots \rangle_{\text{mf}}$ refers to the average over the Polyakov loop matrix with the mean-field action.

In the Weiss mean-field approximation the free energy is defined by¹

$$\frac{V}{T} f_g(\alpha, \beta) = \langle S_g[L] - S_{\text{mf}}[L] \rangle_{\text{mf}} - \ln \int \mathcal{D}L e^{-S_{\text{mf}}[L]}. \quad (3)$$

It is possible to find a closed analytic expression of $f_g(\alpha, \beta = 0)$ at $\mu = 0$, but we have to rely on numerical calculation to evaluate $f_g(\alpha, \beta)$ for $\mu \neq 0$.

We can fix the parameter a by requiring that the pure gluonic theory has a first-order phase transition of color deconfinement at $T = 270$ MeV when $\mu = 0$. This condition results in

$$a = 542 \text{ MeV}. \quad (4)$$

Next, we shall consider how to add the contribution of dynamical quarks in the mean-field approximation. We simply add dynamical quarks using the quasi-quark approximation with the same Polyakov loop coupling as the PNJL model. In our notation $\frac{V}{T} \Omega_q(\sigma_i, L)$ denotes the quark thermodynamic potential with the chiral condensates, σ_u , σ_d , and σ_s , giving the total mean-field free energy,

$$f_{\text{mf}}(\sigma_i, \alpha, \beta) = f_g(\alpha, \beta) + \langle \Omega_q(\sigma_i; L) \rangle_{\text{mf}}. \quad (5)$$

Although Ω_q is a complex function of L for $\mu \neq 0$, its expectation value as a function of α and β is real. This is because the imaginary part contributing to the thermodynamic potential is odd under \mathcal{C} , i.e. $L \rightarrow L^\dagger$ transformation. That is,

$$\langle \Omega_q(\sigma_i, L) \rangle_{\text{mf}} = \frac{1}{z_{\text{mf}}} \int dL e^{N_c \alpha \text{Re} l} \times [\cos(N_c \beta \text{Im} l) \text{Re} \Omega_q - \sin(N_c \beta \text{Im} l) \text{Im} \Omega_q], \quad (6)$$

where z_{mf} is the normalization given as $z_{\text{mf}} = \int dL e^{N_c \alpha \text{Re} l} \times \cos(N_c \beta \text{Im} l)$, which is manifestly real. As for the Polyakov loop, we readily find

$$\ell = \frac{1}{z_{\text{mf}}} \int dL e^{N_c \alpha \text{Re} l} [\cos(N_c \beta \text{Im} l) \text{Re} l - \sin(N_c \beta \text{Im} l) \text{Im} l], \quad (7)$$

$$\bar{\ell} = \frac{1}{z_{\text{mf}}} \int dL e^{N_c \alpha \text{Re} l} [\cos(N_c \beta \text{Im} l) \text{Re} l + \sin(N_c \beta \text{Im} l) \text{Im} l]. \quad (8)$$

¹ Although the variational principle seems to break down due to the sign problem at $\mu \neq 0$, the saddle-point of this mean-field free energy leads to a good approximation. See Ref. [11] for details.

It is obvious from the above that ℓ and $\bar{\ell}$ are different by the presence of the imaginary (\mathcal{C} -odd) part induced by $\beta \neq 0$ at finite μ [11,31].

We now must specify the concrete form of $\Omega_q(\sigma_i, L)$. To this end, here, we shall augment the NJL model with the Polyakov loop coupling (i.e. the PNJL model). Then $\Omega_q(\sigma_i, L)$ take the following form;

$$\begin{aligned} \Omega_q(\sigma_i, L) &= g_S(\sigma_u^2 + \sigma_d^2 + \sigma_s^2) + 4g_D \sigma_u \sigma_d \sigma_s - 2N_c \sum_i \int \frac{d^3 \mathbf{p}}{(2\pi)^3} \varepsilon_i(\mathbf{p}) \\ &\quad - 2T \sum_i \sum_{\lambda=\pm 1} \int \frac{d^3 \mathbf{p}}{(2\pi)^3} \ln \det(1 + L^\lambda e^{-(\varepsilon_i(\mathbf{p}) - \lambda\mu)/T}), \end{aligned} \quad (9)$$

where the quasi-quark dispersion relations are $\varepsilon_i(\mathbf{p}) = \sqrt{p^2 + M_i^2}$ with the constituent quark masses being $M_u = m_u - 2g_S \sigma_u - 2g_D \sigma_d \sigma_s$, $M_d = m_d - 2g_S \sigma_d - 2g_D \sigma_s \sigma_u$, and $M_s = m_s - 2g_S \sigma_s - 2g_D \sigma_u \sigma_d$. We note that $\lambda = +1$ and -1 in the above are the quasi-quark and quasi-antiquark contributions, respectively.

We take the same parameter set in the NJL part as in Ref. [28]; $\Lambda = 631.4$ MeV, $m_u = m_d = 5.5$ MeV, $m_s = 135.7$ MeV, $g_S \Lambda^2 = 3.67$, and $g_D \Lambda^5 = -9.29$. Then $\sigma_u = \sigma_d$ always holds due to isospin symmetry in the strong interaction.

This model has one more parameter, that is the normalization of $f_g(\alpha, \beta)$. The right-hand side of Eq. (3) is proportional to the number of space points, N , and thus $f_g(\alpha, \beta) \propto T \cdot N/V$ which carries the mass dimension of the energy density. Here, N/V is a model parameter corresponding to b discussed in Ref. [21]. We can fix N/V by the condition that the chiral and deconfinement crossovers take place near $T = 200$ MeV. In this way, we find

$$N/V = 0.02 \Lambda^3. \quad (10)$$

3. Quark and color densities

Once we determine the mean-fields $\{\alpha, \beta, \sigma_i\}$ by solving the gap equations, $\partial f_{\text{mf}}/\partial \alpha = \partial f_{\text{mf}}/\partial \beta = \partial f_{\text{mf}}/\partial \sigma_i = 0$, we can calculate various physical quantities.

The quark number density, i.e. $n_q = -\partial f_{\text{mf}}/\partial \mu$, can be expressed as

$$\begin{aligned} n_q &= \frac{1}{z_{\text{mf}}} \sum_i \int \frac{d^3 \mathbf{p}}{(2\pi)^3} \int dL e^{N_c(\alpha \text{Re} l + i\beta \text{Im} l)} \\ &\quad \times \text{tr} \left[\frac{1}{L^\dagger e^{(\varepsilon_i(\mathbf{p}) - \mu)/T} + 1} - \frac{1}{L e^{(\varepsilon_i(\mathbf{p}) + \mu)/T} + 1} \right]. \end{aligned} \quad (11)$$

Next we consider color densities. Since the phase A_4 of the Polyakov loop matrix, $L = \exp[iA_4/T]$, could be regarded as the color chemical potential, the color density is then given by differentiating the integrand of f_{mf} with respect to $(-iA_4^a)$. This leads to

$$\begin{aligned} n_a &= \frac{1}{z_{\text{mf}}} \sum_i \int \frac{d^3 \mathbf{p}}{(2\pi)^3} \int dL e^{N_c(\alpha \text{Re} l + i\beta \text{Im} l)} \\ &\quad \times \text{tr} \left[\frac{1}{L^\dagger e^{(\varepsilon_i(\mathbf{p}) - \mu)/T} + 1} T_a - \frac{1}{L e^{(\varepsilon_i(\mathbf{p}) + \mu)/T} + 1} T_a \right]. \end{aligned} \quad (12)$$

Here T_a 's are the $SU(N_c)$ algebra in the fundamental representation. In deriving this we have made use of the cyclicity in the trace. The group integration in Eq. (12) is hard to perform in general. Usually we take the Polyakov gauge in which L is diagonal with two angle variables φ_1 and φ_2 , and then we can express dL as $d\varphi_1 d\varphi_2$ accompanied by the $SU(N_c = 3)$ Haar measure

$\mu(\varphi_1, \varphi_2) \equiv [\sin(\varphi_1 - \varphi_2) + \sin(2\varphi_1 + \varphi_2) + \sin(\varphi_1 + 2\varphi_2)]^2 / (3\pi^2)$. This procedure works straightforwardly for Eq. (11) but not for Eq. (12) because of the presence of T_a . The color density is gauge dependent, however, so we should fix the gauge to define this quantity. If we take the Polyakov gauge as usual, then the color density for only the T_3 and T_8 components (belonging to the Cartan subalgebra of $SU(3)$) have non-vanishing integrands. We then can define the red, green, and blue quark densities as $n_r = \frac{1}{3}n_q + \frac{1}{2}n_3 + \frac{1}{2\sqrt{3}}n_8$, $n_g = \frac{1}{3}n_q - \frac{1}{2}n_3 + \frac{1}{2\sqrt{3}}n_8$, and $n_b = \frac{1}{3}n_q - \frac{1}{\sqrt{3}}n_8$.

4. Approximation

Here we introduce an approximation which greatly reduces the computational cost. That is,

$$\langle \ln \det(1 + Le^{-\epsilon_i - \mu}) / T \rangle_{\text{mf}} \rightarrow \ln \langle \det(1 + Le^{-\epsilon_i - \mu}) / T \rangle_{\text{mf}}, \quad (13)$$

and the same for the antiquark part. With this approximation applied to the free energy expression, we can reduce the three-dimensional integral with respect to $\{\varphi_1, \varphi_2, p\}$ to the one-dimensional p -integral with given ℓ and $\bar{\ell}$ which result from the integral over $\{\varphi_1, \varphi_2\}$ independently of p . We have numerically confirmed that this approximation works excellently well.

5. Standard PNJL model treatment

For comparison to the simple mean-field approximation used in literature [10], we shall calculate the same physical quantities using the standard PNJL model,

$$\begin{aligned} \tilde{f}_{\text{mf}}(\sigma_i, L) &= V_{\text{glue}}[l, \bar{l}] + g_S(\sigma_u^2 + \sigma_d^2 + \sigma_s^2) + 4g_D\sigma_u\sigma_d\sigma_s \\ &- 2N_c \sum_i \int \frac{d^3\mathbf{p}}{(2\pi)^3} \epsilon_i(\mathbf{p}) \\ &- 2T \sum_i \sum_{\lambda=\pm 1} \int \frac{d^3\mathbf{p}}{(2\pi)^3} \ln \det(1 + L^\lambda e^{-(\epsilon_i(\mathbf{p}) - \lambda\mu)/T}), \end{aligned} \quad (14)$$

where

$$V_{\text{glue}}[l, \bar{l}] = -bT \{ 54e^{-a/T} \bar{l} + \ln[1 - 6\bar{l}l - 3(\bar{l}l)^2 + 4(l^3 + \bar{l}^3)] \} \quad (15)$$

with $a = 664$ MeV, $b = 0.026\Lambda^3$ so that the transition (crossover) temperatures with and without dynamical quarks are 270 MeV and 200 MeV respectively, as explained previously. We will refer to this model as the *standard* PNJL model hereafter.

The above expression for $\tilde{f}_{\text{mf}}(\sigma_i, L)$ is given in terms of the gauge invariant mean-fields, ℓ and $\bar{\ell}$, so that one can evaluate them without difficulty to find $\bar{\ell} > \ell$ at non-zero μ . The serious problem arises when we are interested in quantities associated with color degrees of freedom. We cannot express the color density, n_a , using ℓ and $\bar{\ell}$ only, as seen from Eq. (12). That is also the case if the color-superconducting phase is considered in the PNJL model.

Practically, in such a situation, one may well assume the mean-fields, φ_1 and φ_2 (or φ_3 and φ_8), to characterize the Polyakov loop matrix as $L = \text{diag}(e^{i\varphi_1}, e^{i\varphi_2}, e^{-i(\varphi_1 + \varphi_2)}) = \exp[i(\varphi_3 T_3 + \varphi_8 T_8)/T]$. This prescription is quite problematic, however, though adopted frequently. The traced Polyakov loops, ℓ and $\bar{\ell}$, become complex with non-zero φ_3 and φ_8 in general. To avoid this artifact, one could assume $\varphi_8 = 0$, but such an assumption is not consistent with $\bar{\ell} \neq \ell$ at finite μ . More seriously an unphysical color density is induced by the mean-fields, which is, of course, an artifact of this prescription. One may want to cancel the color density by introducing color chemical potentials [23,24], but as soon as one

does so, another undesirable problem seems to come out immediately [32].

We define the magnitude of the color density by $n_c = (\sum_{a=1}^8 n_a^2)^{1/2}$ [23]. This quantity is invariant under gauge rotation. Thus, under the assumption $\varphi_8 = 0$, the color density magnitude is $n_c = \frac{2}{\sqrt{3}}|n(\varphi_3) - n(0)|$, where we define

$$\begin{aligned} n(\varphi_3) &= 2 \sum_i \int \frac{d^3\mathbf{p}}{(2\pi)^3} \text{Re} \left[\frac{1}{e^{\epsilon_i(\mathbf{p}) - \mu - i\varphi_3}/T} + 1} \right. \\ &\quad \left. - \frac{1}{e^{\epsilon_i(\mathbf{p}) + \mu + i\varphi_3}/T} + 1} \right]. \end{aligned} \quad (16)$$

We remark that n_c is non-vanishing at finite φ_3 . This is simply because the phase of the Polyakov loop matrix generally has the physical meaning of the color imaginary chemical potential, and so $\varphi_3 \neq 0$ induces $n_c \neq 0$.

6. Order parameters and color densities

We first show the order parameters for chiral restoration and color deconfinement in Fig. 1 as a function of T at $\mu = 0$. The thick curves represent the constituent quark masses and the Polyakov loop obtained from Eq. (5), while the thin curves are the results in the standard PNJL model with Eq. (14). We note that the thick and thin curves are very close, which means that our formulation would not spoil the nice feature established in the standard PNJL model.

Now we shall move on to the finite density case. In Fig. 2 we display the physical quantities as a function of T at $\mu = 300$ MeV. The left figure shows the constituent quark masses and the Polyakov loop in the same way as in Fig. 1. The thin lines are the results from the standard PNJL model again. The right figure shows the quark number density n_q and the color densities $\{n_r, n_g, n_b\}$, where $n_q = n_r + n_g + n_b$ should be fulfilled. We see that the thick curves have significant difference from the thin curves resulting from the standard PNJL model. In the case of our prescription we have $n_r = n_g = n_b$, meaning that $n_c = |n_r + n_g - 2n_b|/\sqrt{3} = 0$, while n_c is non-vanishing in the standard PNJL model as depicted by the thin line with the label n_c . It is also notable that, despite drastic difference in the color densities, the quark number density, n_q , hardly changes; the thin line stays close to the thick line for n_q .

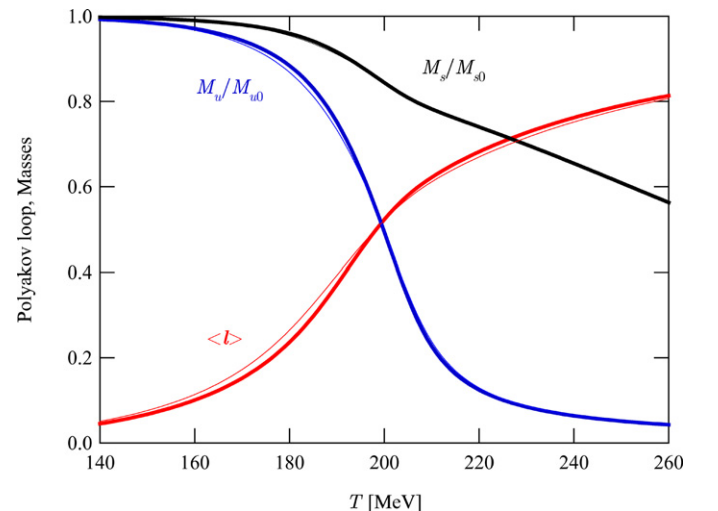


Fig. 1. The chiral and deconfinement crossovers at $\mu = 0$. The constituent quark masses are normalized by the vacuum values, M_{u0} and M_{d0} . The thin lines show the results from the standard PNJL model given in Eq. (14).

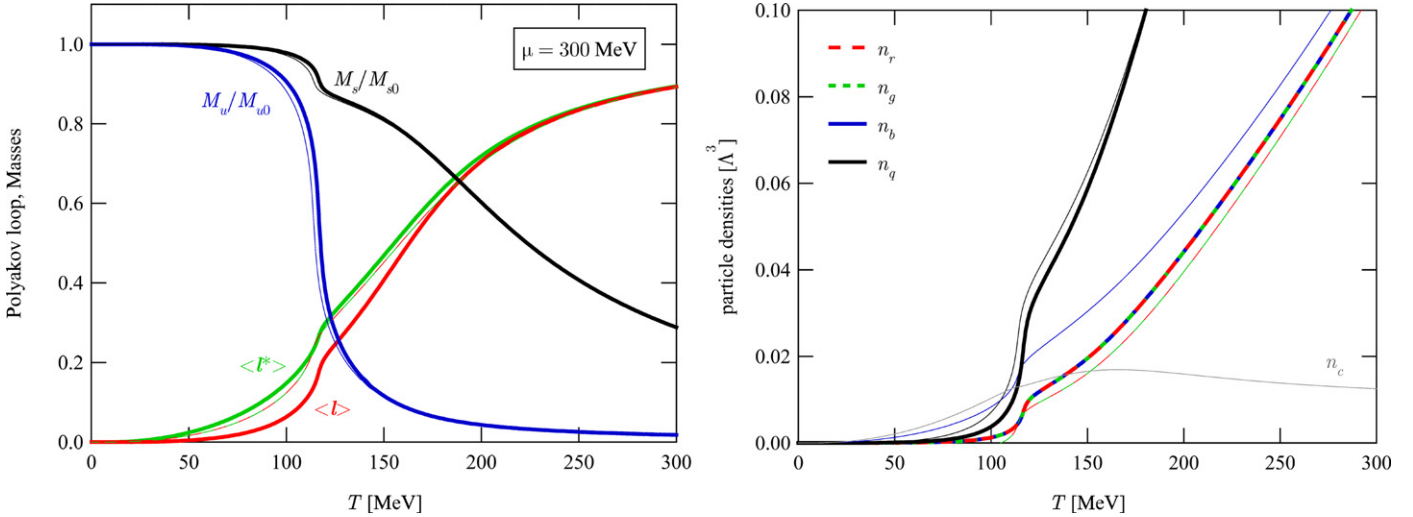


Fig. 2. Left: Constituent quark masses, M_u and M_s , Polyakov loops, ℓ and $\bar{\ell}$, as a function of T at $\mu = 300 \text{ MeV}$. The thin curves are the results from the standard PNJL model. Right: Color densities $\{n_r, n_g, n_b\}$ and the quark number density n_q as a function of T . The net color density $n_c = |n_r + n_g - 2n_b|/\sqrt{3}$ is indicated by the thin line with the label n_c in the case of the standard PNJL model. In our prescription n_c is always zero.

7. Casimir scaling at finite density

As already noted, the mean-field approximation discussed here enables us to compute not only the traced Polyakov loop in the fundamental representation but also the expectation value of arbitrary functions of the Polyakov loop. We shall take a close look at the Polyakov loop in the higher representations as an immediate application.

The Polyakov loop in the higher representations is of special interest with regard to the Casimir scaling hypothesis [33], which may provide a crucial key to understanding non-perturbative aspects of QCD such as confinement [30]. The Casimir scaling hypothesis claims that the color singlet potential between static color sources in the representation r is proportional to the Casimir invariant $C_2(r)$. The statement is rather obvious in the perturbative regime, but it is quite non-trivial at large distances. From the theoretical perspective this hypothesis is verified up to two-loop order in the lattice perturbation theory both in pure gauge theory [34] and in QCD with massless dynamical quarks [35]. Beyond two-loop order the Casimir scaling can be violated, though the violation is tiny [34].

The scaling hypothesis is also tested numerically in the lattice simulation. In the SU(3) pure gauge theory at $T = 0$ the hypothesis has been verified up to the string breaking distance [33]. The Casimir scaling hypothesis also brings a strong constraint on the Polyakov loop expectation value; the traced Polyakov loop in any representation r should satisfy the following scaling relation irrespective of the renormalization of the Polyakov loop [30];

$$\ell_r^{1/d_r} \approx \ell_3. \quad (17)$$

Here $\ell_3 = \ell$ is the Polyakov loop in the fundamental representation, d_r is the ratio of the quadratic Casimir invariant; $d_r \equiv C_2(r)/C_2(3) = \frac{3}{4}C_2(r)$. The relation (17) between the Polyakov loops in different representations actually provides a useful test for the hypothesis, and in Ref. [30] this test has been extensively performed with use of lattice QCD data both in pure gauge theory and in $N_f = 2$ QCD. It has been found that the scaling violation is visible only in the very vicinity of the first-order phase transition in pure gauge theory, while the deviation from the scaling is more evident in $N_f = 2$ QCD particularly below the crossover temperature. The scaling (17) is almost exact at high temperature where ℓ is substantially large.

It has been reported recently that the Casimir scaling of the Polyakov loop in the fundamental (3) and adjoint (8) representations is well realized in the PNJL model at $\mu = 0$ [36]. We here present the first systematic model study on the Casimir scaling at non-zero chemical potential. We compute the Polyakov loop in various representation from **3** to **27** as listed in Table 1. We can construct the Polyakov loop matrix in higher representations by the direct products of L and L^\dagger in the fundamental representation using the Clebsch–Gordan coefficients. In the left of Fig. 3 we show the various Polyakov loops as a function of T at $\mu = 300 \text{ MeV}$ as well as at $\mu = 0$. We see that the scaling is good at high T in both cases. The scaling regime is reached faster in the $\mu = 0$ case; the violation of the Casimir scaling is small at the crossover around $T = 200 \text{ MeV}$, while in the case of $\mu = 300 \text{ MeV}$ the deviations in ℓ_r^{1/d_r} are significant at the crossover temperature $T = 120 \text{ MeV}$. The presence of finite density tends to enhance the scaling violation. At the same time we should be careful about the interpretation; in Ref. [30] it has been shown that this kind of matrix-based model fails in reproducing the exact Casimir scaling $\ell_r^{1/d_r} = \ell_3$ observed on the lattice in the pure gauge theory. One comment which we should mention here is that the scaling violation at higher representations may be attributed to the fact that we limit ourselves to the simplest version of the matrix model in Eq. (1). It could be possible that the inclusion of the Polyakov loops in higher representations may diminish artificial violation of the Casimir scaling. For example we could consider, $l_3(\vec{x})l_6(\vec{x} + \hat{n})$, $l_6(\vec{x})l_6^*(\vec{x} + \hat{n})$, etc., in the model action, which are allowed by Z_3 center symmetry in the pure gluonic sector. Moreover, as for the Polyakov loops in the triality zero representations such as **8**, **10**, and **27**, one may well add their arbitrary functions in the model action. It would be an interesting future problem to take account of the Polyakov loop in higher representations into the matrix model in such a way that the model preserves charge conjugation symmetry [31].

In the right of Fig. 3 we show the Polyakov loop ℓ_r as a function of μ for $T = 100 \text{ MeV}$ and for $T = 150 \text{ MeV}$. At $T = 100 \text{ MeV}$ the first-order chiral transition occurs at $\mu = 315 \text{ MeV}$, while chiral restoration is smooth crossover with increasing μ when $T = 150 \text{ MeV}$. Again we notice the significant scaling violation, though the violation is exaggerated on the logarithmic plot. We observe in Fig. 3 that some ℓ_r^{1/d_r} 's cross each other as μ increases. We see that the change in **6** is milder than those in **3** and **15**. This is rea-

Table 1

Group theoretical factors in various representations; r is the representation labeled by its dimension $D(r)$, (p, q) is the corresponding weight factor, $t = p - q$ modulo 3 is the triality, $z \equiv e^{i2\pi/3}$ is an element of Z_3 , $C_2(r)$ is the quadratic Casimir invariant, and d_r defines the ratio $d_r \equiv C_2(r)/C_2(3)$. The dimension is given by $D(r) = (p + 1) \times (q + 1)(p + q + 2)/2$. In the triality zero representation (i.e. $z^t = 1$), the Polyakov loop is insensitive to center symmetry, and thus it does not serve as an order parameter of deconfinement.

r	(p, q)	z^t	$C_2(r)$	d_r	direct product expression of $V_r \equiv D(r)l_r$
3	(1, 0)	z	4/3	1	$V_3 = \text{tr } L$
$\bar{3}$	(0, 1)	z^*	4/3	1	$V_{\bar{3}} = \text{tr } L^\dagger$
6	(2, 0)	z^*	10/3	2.5	$V_6 = (V_3^2 - V_{\bar{3}})$
8	(1, 1)	1	3	2.25	$V_8 = (V_3 ^2 - 1)$
10	(3, 0)	1	6	4.5	$V_{10} = (V_3 V_6 - V_8)$
15 _a	(2, 1)	z	16/3	4	$V_{15_a} = (V_3 V_6 - V_3)$
15 _s	(4, 0)	z	28/3	7	$V_{15_s} = (V_3 V_{10} - V_{15_a})$
24	(3, 1)	z^*	25/3	6.25	$V_{24} = (V_3 V_{10} - V_6)$
27	(2, 2)	1	8	6	$V_{27} = (V_6 ^2 - V_8 - 1)$

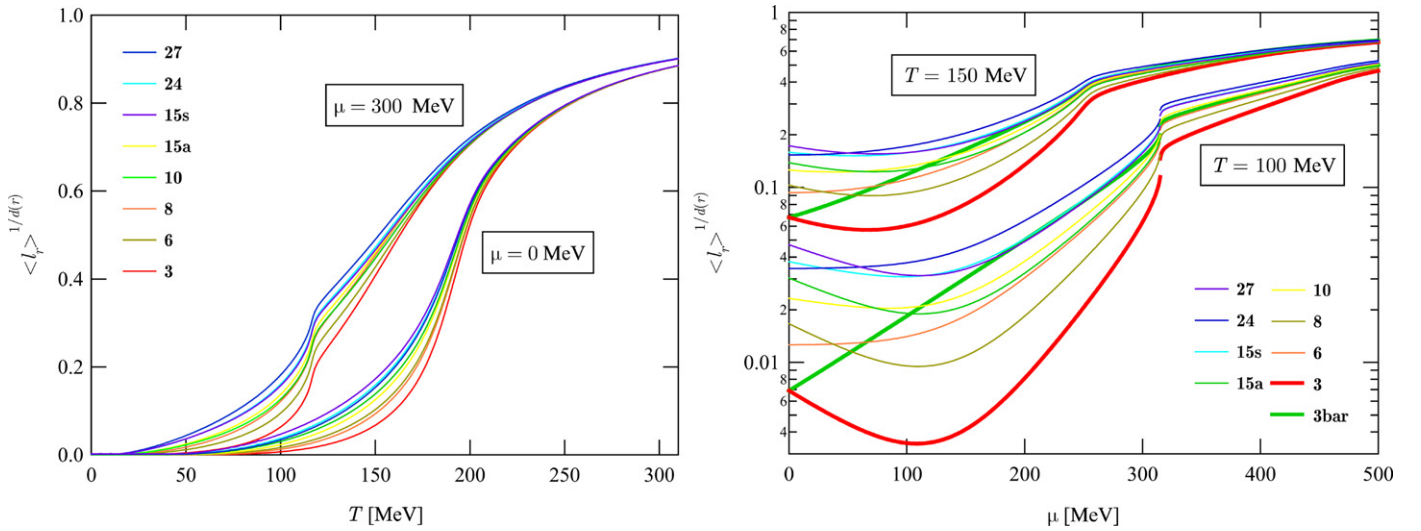


Fig. 3. Left: Scaled Polyakov loop ℓ_r^{1/d_r} as a function of T for $\mu = 0$ and for $\mu = 300$ MeV in various representations with $d_r = C_2(r)/C_2(3)$ being the ratio of the quadratic Casimir invariant. Right: Scaled Polyakov loop as a function of μ for $T = 100$ MeV and for $T = 150$ MeV. It should be noted that the vertical axis is logarithmic.

sonable, for the excitation with the triality z^* (like **6**) should be easier than that with z (like **3** and **15**) in a medium carrying the triality z at $\mu > 0$.

8. Conclusion

We showed that the pathological problems associated with a simple mean-field approximation in the PNJL model can be resolved by the use of the Weiss mean-field approximation. We explicitly demonstrated that the color density is vanishing in the normal phase of quark matter. This vanishing color density is guaranteed by the integration with respect to the Polyakov loop, which can translate into the Gauss law resulting from the A_4 -integration. We also confirmed that $\bar{\ell} > \ell$ at finite μ is naturally realized.

Our mean-field prescription allows us to compute the expectation value of any function of the Polyakov loop matrix, L and L^\dagger , easily. As a demonstration, we computed the Polyakov loops in various representations from **3** to **27**. We observed that the Casimir scaling is violated at finite μ more than at zero density, which is quite natural. More interestingly, the Polyakov loop with the triality z turned out to have decreasing behavior as a function of μ as long as μ is small. This means that the quark excitation bearing the same triality charge with the background medium is less favored. It would be an interesting future work to incorporate couplings between the Polyakov loops in higher representations into the matrix model and investigate the scaling violation in wider model space.

An interesting extension of the present work would be the QCD phase structure with the diquark condensation taken into account. It would be of particular interest how the diquark condensates and the colored Polyakov loop matrix are entangled in a color-superconducting medium. The work along this line certainly deserves future investigations.

Acknowledgements

H.A. thanks D. Blaschke, T. Brauner, M. Buballa, D. Rischke, M. Ruggieri and F. Sandin for discussions. The present work was supported in part by the Alexander von Humboldt Foundation. Numerical calculations were performed using the facilities of the Frankfurt Center for Scientific Computing. K.F. is grateful to D. Rischke for the hospitality at the Institut für Theoretische Physik of Johann Wolfgang Goethe-Universität Frankfurt am Main, where this work was initiated. K.F. is supported by Japanese MEXT grant No. 20740134 and also supported in part by Yukawa International Program for Quark Hadron Sciences.

References

- [1] A. Gocksch, M. Ogilvie, Phys. Rev. D 31 (1985) 877.
- [2] E.M. Ilgenfritz, J. Kripfganz, Z. Phys. C 29 (1985) 79.
- [3] K. Fukushima, Phys. Lett. B 591 (2004) 277.
- [4] P.N. Meisinger, M.C. Ogilvie, T.R. Miller, Phys. Lett. B 585 (2004) 149.
- [5] A. Dumitru, Y. Hatta, J. Lenaghan, K. Orginos, R.D. Pisarski, Phys. Rev. D 70 (2004) 034511.

- [6] E. Megias, E. Ruiz Arriola, L.L. Salcedo, Phys. Rev. D 74 (2006) 065005.
- [7] C. Ratti, M.A. Thaler, W. Weise, Phys. Rev. D 73 (2006) 014019.
- [8] S.K. Ghosh, T.K. Mukherjee, M.G. Mustafa, R. Ray, Phys. Rev. D 73 (2006) 114007, arXiv:hep-ph/0603050.
- [9] R.D. Pisarski, Phys. Rev. D 74 (2006) 121703.
- [10] S. Roessner, C. Ratti, W. Weise, Phys. Rev. D 75 (2007) 034007.
- [11] K. Fukushima, Y. Hidaka, Phys. Rev. D 75 (2007) 036002.
- [12] W.J. Fu, Z. Zhang, Y.x. Liu, Phys. Rev. D 77 (2008) 014006.
- [13] M. Ciminale, R. Gatto, N.D. Ippolito, G. Nardulli, M. Ruggieri, Phys. Rev. D 77 (2008) 054023.
- [14] B.J. Schaefer, J.M. Pawłowski, J. Wambach, Phys. Rev. D 76 (2007) 074023.
- [15] D. Blaschke, M. Buballa, A.E. Radzhabov, M.K. Volkov, Yad. Fiz. 71 (2008) 2012, Phys. At. Nucl. 71 (2008) 1981.
- [16] M. Ciminale, G. Nardulli, M. Ruggieri, R. Gatto, Phys. Lett. B 657 (2007) 64.
- [17] K. Kashiwa, H. Kouno, M. Matsuzaki, M. Yahiro, Phys. Lett. B 662 (2008) 26.
- [18] S. Roessner, T. Hell, C. Ratti, W. Weise, Nucl. Phys. A 814 (2008) 118.
- [19] H. Abuki, M. Ciminale, R. Gatto, N.D. Ippolito, G. Nardulli, M. Ruggieri, Phys. Rev. D 78 (2008) 014002.
- [20] H. Abuki, R. Anglani, R. Gatto, G. Nardulli, M. Ruggieri, arXiv:0805.1509 [hep-ph].
- [21] K. Fukushima, Phys. Rev. D 77 (2008) 114028;
K. Fukushima, Phys. Rev. D 78 (2008) 039902, Erratum.
- [22] T. Hell, S. Roessner, M. Cristoforetti, W. Weise, arXiv:0810.1099 [hep-ph].
- [23] H. Abuki, M. Ciminale, R. Gatto, G. Nardulli, M. Ruggieri, Phys. Rev. D 77 (2008) 074018;
H. Abuki, arXiv:0805.3076 [hep-ph].
- [24] D. Gomez Dumm, D.B. Blaschke, A.G. Grunfeld, N.N. Scoccola, Phys. Rev. D 78 (2008) 114021.
- [25] H. Abuki, M. Ciminale, R. Gatto, M. Ruggieri, arXiv:0811.1512 [hep-ph].
- [26] C.R. Allton, et al., Phys. Rev. D 71 (2005) 054508.
- [27] For reviews on color superconductivity, see K. Rajagopal, F. Wilczek, arXiv:hep-ph/0011333;
M.G. Alford, A. Schmitt, K. Rajagopal, T. Schafer, Rev. Mod. Phys. 80 (2008) 1455.
- [28] T. Hatsuda, T. Kunihiro, Phys. Rep. 247 (1994) 221, arXiv:hep-ph/9401310.
- [29] J.B. Kogut, M. Snow, M. Stone, Nucl. Phys. B 200 (1982) 211.
- [30] S. Gupta, K. Huebner, O. Kaczmarek, Nucl. Phys. A 785 (2007) 278;
S. Gupta, K. Huebner, O. Kaczmarek, Phys. Rev. D 77 (2008) 034503.
- [31] A. Dumitru, R.D. Pisarski, D. Zschiesche, Phys. Rev. D 72 (2005) 065008.
- [32] D. Blaschke, F. Sandin, private communication.
- [33] G.S. Bali, Phys. Rev. D 62 (2000) 114503.
- [34] Y. Schroder, Phys. Lett. B 447 (1999) 321.
- [35] G.S. Bali, P. Boyle, arXiv:hep-lat/0210033.
- [36] H.M. Tsai, B. Muller, arXiv:0811.2216 [hep-ph].