Available online at www.sciencedirect.com

ScienceDirect

Physics Procedia 58 (2014) 174 – 177

Physics Procedia

26th International Symposium on Superconductivity, ISS 2013

# Numerical investigation on accuracy of defect detection in HTS film by inductive method

Teruou Takayama[a,]*, Soichiro Ikuno[b], Atsushi Kamitani[c]

[a] *Faculty of Engineering, Yamagata University, 4-3-16, Johnan, Yonezawa, Yamagata 992-8510, Japan*
[b] *School of Computer Science, Tokyo University of Technology, 1404-1, Katakura, Hachioji, Tokyo 192-0982, Japan*
[c] *Graduate School of Science and Engineering, Yamagata University, 4-3-16, Johnan, Yonezawa, Yamagata 992-8510, Japan*

---

## Abstract

The inductive method for measuring the critical current density in a high-temperature superconducting (HTS) film has been reproduced numerically. To this end, a numerical code has been developed for analyzing the time evolution of a shielding current density in the HTS film containing a crack. The results of computations show that the accuracy of the inductive method is degraded due to the crack or the film edge. This result means that the inductive method can be applied to the crack detection. However, the crack located near the film edge cannot be detected because the crack is treated the same as the film edge.

Peer-review under responsibility of the ISS 2013 Program Committee

*Keywords:* Critical currents; Defect detection; Numerical analysis; Superconducting films

---

## 1. Introduction

As is well known, a critical current density $j_C$ is one of the most important parameters for engineering applications of high-temperature superconductors (HTSs). For measuring $j_C$ in an HTS film contactlessly, Claassen *et al*. have proposed the inductive method [1]. By applying an ac current to a small coil placed just above an HTS film, they monitored a harmonic voltage induced in the coil. They found that, only when a coil current exceeds a threshold current, the third-harmonic voltage develops suddenly. They conclude that $j_C$ can be evaluated from the

---

* Corresponding author. Tel.: +81-238-26-3789; fax: +81-238-26-3789.
*E-mail address:* takayama@yz.yamagata-u.ac.jp

Peer-review under responsibility of the ISS 2013 Program Committee
doi:10.1016/j.phpro.2014.09.040

threshold current. On the other hand, Mawatari *et al.* elucidated the inductive method on the basis of the critical state model [2]. From their results, they derived a theoretical formula between $j_C$ and threshold current. The inductive method has been successfully employed as the measurement of the $j_C$-distributions and the detection of a crack [3].

The purpose of the present study is to develop a numerical code for analyzing the time evolution of the shielding current density in an HTS film with a crack and to investigate the influence of the film edge on the crack detection.

## 2. Governing equations and numerical method

In Fig. 1, we show a schematic view of an inductive method. A small $M$-turn coil of the outer radius $R$, the inner radius $R_{\mathrm{in}}$, and the height $H$ is placed just above a squared-shaped HTS film of the length $a$ and the thickness $b$. We define a distance $L$ between the coil bottom and the film surface. Also, an ac current $I(t) = I_0 \sin 2\pi ft$ flows in the coil.

The square cross-section of the film is denoted by $\Omega$, and an outer boundary of $\Omega$ is expressed by $C_0$. We assume that a crack is included in $\Omega$ and its shape is given by an inner boundary $C_1$. We adopt the Cartesian coordinate system $\langle O : \boldsymbol{e}_x, \boldsymbol{e}_y, \boldsymbol{e}_z \rangle$, where $z$-axis is the thickness direction, and the origin $O$ is the centroid of the film. In order to determine the coil position, the symmetrical axis of the coil is shown by $(x, y) = (x_A, y_A)$.

A shielding current density $\boldsymbol{j}$ in an HTS is closely related to the electric field $\boldsymbol{E}$. The relation can be written as $\boldsymbol{E} = E(|\boldsymbol{j}|)[\boldsymbol{j}/|\boldsymbol{j}|]$, where a function $E(j)$ is given by the power law: $E(j) = E_C [j / j_C]^N$. Here, $E_C$ is the critical electric field, and $N$ is a constant.

Under the above assumptions, a shielding current density $\boldsymbol{j}$ can be written as $\boldsymbol{j} = (2/b)\nabla S \times \boldsymbol{e}_z$, and the time evolution of the scalar function $S(\boldsymbol{x}, t)$ is governed by the following integro-differential equation [4]:

$$\mu_0 \partial_t (\hat{W}S) + \partial_t \langle \boldsymbol{B} \cdot \boldsymbol{e}_z \rangle + (\nabla \times \boldsymbol{E}) \cdot \boldsymbol{e}_z = 0. \qquad (1)$$

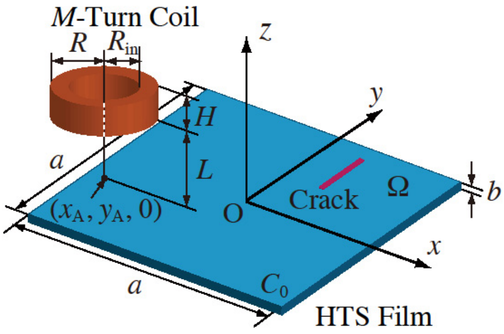


Fig. 1. A schematic view of an inductive method.

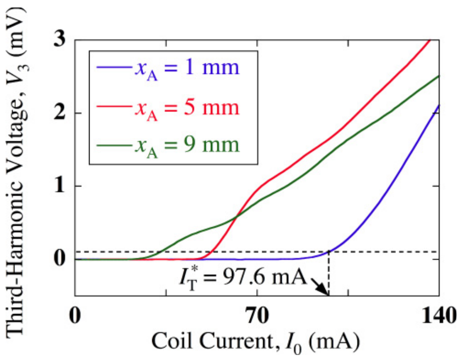
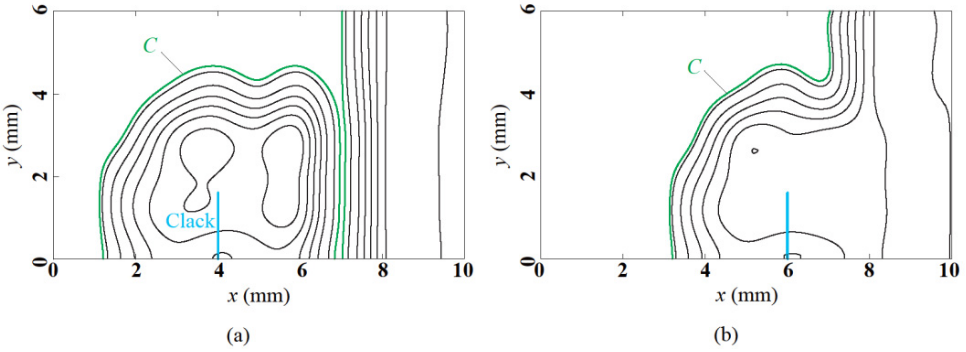


Fig. 2. $I_0$-$V_3$ curves for the case with $x_c$ = 4 mm and $R$ = 2.5 mm.

Here, a bracket $\langle \ \rangle$ denotes an average operator over the thickness of the film, and $\boldsymbol{E}$ is an electromagnetic field. $\hat{W}S$ is defined by $\hat{W}S \equiv \iint_\Omega Q(|x - x'|) S(x', t) d^2x' + (2/b) S(x, t)$, where both $\boldsymbol{x}$ and $\boldsymbol{x}'$ are position vectors in the $xy$-plane. The explicit form of $Q(r)$ is described in [4]. The initial condition to (1) is given bv $S$ = 0 at $t$ = 0, and boundary conditions are assumed as follows: $S = 0$ on $C_0$, $\partial S / \partial s = 0$ on $C_1$ $h[\boldsymbol{E}] \equiv \bigcirc$ Here, $s$ is an arclength along $C_1$.

We assume the thin-layer approximation [4] and we adopt virtual voltage method [5] proposed by Kamitani *et al.* Under a numerical method, a numerical code is developed for analyzing the time evolution of a shielding current density $\boldsymbol{j}$ in an HTS film containing a crack.

Throughout the present study, the geometrical and physical parameters are fixed as follows: $a$ = 20 mm, $b$ = 600 nm, $E_C$ = 1 mV/m, $j_C$ = 1 MA/cm$^2$, $N$ = 20, $H$ = 2 mm, $L$ = 0.5 mm, $M$ = 400 and $f$ = 1 kHz. The ratio between the outer radius $R$ and inner radius $R_{\mathrm{in}}$ of the coil is given by 2. Also, a crack shape is a line segment with the center points $(x, y) = (x_c, 0)$ in the $xy$-plane. In the following, the crack size $L_c$ is given by $L_c$ = 3.2 mm.



(a)

(b)

Fig. 3. Contour lines of the relative error $\varepsilon$ for (a) $x_c = 4$ mm and (b) $x_c = 6$ mm. Here, the outer radius of the coil $R = 2.5$ mm. In addition, a green thick line $C$ indicates the contour line of $\varepsilon = 7$ %.

## 3. Crack detection by inductive method

### 3.1. Mawatari's theory

According to Mawatari's theory [2], a critical current density $j_C$ can be easily calculated from

$$j_C = 2F(r_{\max})I_T / b. \quad (2)$$

Here, $F(r_{\max})$ is the maximum value of a primary coil-factor function $F(x)$ [2] determined from the configuration of the coil and the HTS. In addition, $I_T$ is a lower limit of the coil current $I_0$ when a third-harmonic voltage $V_3$ begins to develop in the coil. An important point is that (2) is also applicable only to an HTS film without any cracks.

For determining $j_C$, it is necessary to evaluate a threshold current $I_T$ when a third-harmonic voltage $V_3$ begins to develop in the coil. Since it is difficult to determine $I_T$ accurately, we use the conventional voltage criterion [2] $V_3 = 0.1\ \text{mV} \Leftrightarrow I_0 = I_T^*$. Hence, we get an estimated value $I_T^*$ of the threshold current $I_T$.

The third-harmonic voltage $V_3$ is calculated as a function $I_0$ and is depicted in Fig. 2. We see from this figure that $V_3$ gradually develops above the certain value of $I_0$. By applying the voltage criterion to the $I_0$-$V_3$ curves, we get $I_T^*$ = 97.6, 52.5, and 33.1 mA for $x_A$ = 1 mm, 5 mm, and 9 mm, respectively. On the other hand, the analytic value $I_T^A (\equiv j_C b / [2F(r_{\max})])$ of $I_T$ can be obtained from (2), and its value is $I_T^A = 93.1$ mA. The results of the computations show that, if the orthographic projection of the coil overlaps with the crack or the film edge, the accuracy of the inductive method is hardly degraded.

### 3.2. Crack detection

Let us first investigate the influence of the film edge on the crack detection. To this end, we calculate a spatial distribution of the threshold current $I_T^*$ in $\Omega$. Furthermore, as a measure of the accuracy, we define a relative error $\varepsilon \equiv \left| I_T^{\max} - I_T^* \right| / I_T^{\max}$. Here, $I_T^{\max}$ is denoted by

$$I_T^{\max} = \max_{(x_A, y_A) \in \Omega} I_T^*(x_A, y_A).$$

In the present study, we divide the region $\Omega$ into a grid to make an observation point. The grid shape is a square, and its size is 1 mm.

In Figs. 3 (a) and (b), we show the contour lines of the relative error $\varepsilon$ for the case with $x_c = 4$ mm and $x_c = 6$ mm, respectively. In these figures, a tolerance error $\varepsilon = 7$ % is depicted by a thick line $C$. We see from Fig. 3 (a) that, for $x_c$ = 4 mm, two types of the line $C$ can be obtained. One is a line containing the crack. The other is a line

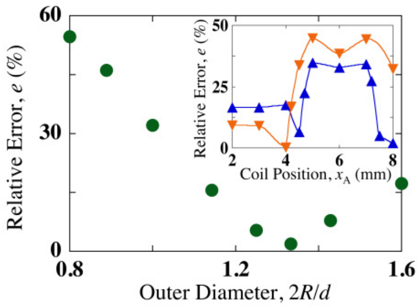


Fig. 4. Dependence of the relative error $e$ on the outer diameter $2R/d$ for the case with $y_A = 8$ mm, $y_A = 0$ mm and $x_c = 6$ mm. The inset shows the dependence of the relative error $e$ on the coil position $x_A$ for the case with $y_A = 0$ mm and $x_c = 6$ mm. Here, symbols, ▲ and ▼, indicate $R = 1.5$ mm and $R = 2$ mm.

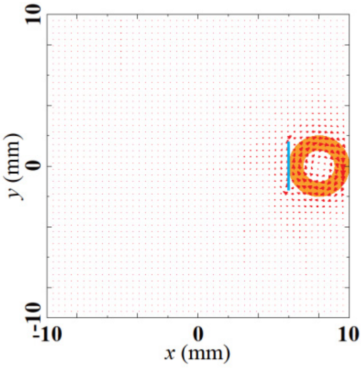


Fig. 5. Spatial distribution of the shield current density $\boldsymbol{j}$ for the case with $I_0 = I_T$ at time $ft = 1.2$. Here, $x_A = 8$ mm, $y_A = 0$ mm, $R = 2$ mm and $x_c = 6$ mm. Moreover, the thick line and the circle region indicate the crack and the orthogonal projection of the coil.

substantially parallel to the $y$-axis direction. Therefore, cracks can be detected for this case. On the other hand, the crack is not included in the line $C$ due to the film edge in Fig. 3 (b).

Next, we investigate the influence of the accuracy on the outer radius $R$ of the coil. For this purpose, we define another relative error $e \equiv \left| I_T^A - I_T^* \right| / I_T^A$. In the inset of Fig. 4, we show the dependence of the relative error $e$ on the coil position $x_A$. We see from this figure that, for $R = 2$ mm, the relative error $e$ increases with $x_A$ for $x_A > 4$ mm. In particular, the value of $e$ becomes large for the case with $x_A = 8$ mm (i.e. the outer diameter $2R$ comes in contact with the film edge). As a result, the accuracy is degraded when the outer diameter comes in contact with the film edge. For the case with $R = 1.5$ mm, the value of $e$ is about $e = 17$ % or less for $x_A < 4.5$ mm and $x_A > 7.5$ mm.

Finally, we investigate the accuracy for the case where the coil is located between the crack and the film edge. In the following, the distance $d$ between and the crack and edge is defined by $d \equiv a/2 - x_c$. The relative error $e$ is calculated as a function of the outer diameter $2R/d$ and is depicted in Fig. 4. From this figure, the value of relative error is $e = 30$ % or more for $2R/d \leq 1$. Particularly, the accuracy is degraded if the outer diameter $2R$ comes in contact with the film edge (i.e. $2R/d = 1$). This is mainly because the spatial distribution of shielding current density $\boldsymbol{j}$ is asymmetric about the symmetry axis of the coil due to the film edge (see Fig. 5). From this figure, it is found that a slightly larger $\boldsymbol{j}$ flows in the edge when compared to the crack.

## 4. Conclusion

Conclusions obtained in the present study are summarized as follows: the accuracy is degraded if the orthogonal projection of the coil overlaps the crack. A similar trend is also shown when the outer diameter comes in contact with the film edge. The crack located near the film edge cannot be detected because the crack is treated the same as the film edge. In addition, the results of the computations show that there exists the optimum outer diameter.

## References

[1] Claassen JH, Reeves ME, Soulen Jr RJ. A contactless method for measurement of the critical current density and critical temperature of superconductiong films. Rev. Sci. Instrum 1991:62;996-1004.

[2] Mawatari Y, Yamasaki H, Nakagawa Y. Critical current density and third-harmonic voltage in superconducting films. Appl. Phys. Lett 2002:81;2424-6.

[3] Kim SB. The defect detection in HTS films on third-harmonic voltage method using various inductive coils. Physica C 2007:463,702-6.

[4] Kamitani A, Ohshima S. Magnetic shielding analysis of axisymmetric HTS plates in mixed state.IEICE Trans. Electron. 1999: 82-C;766-73.

[5] Kamitani A, Takayama T. Numerical simulation of shielding current density in high-temperature superconducting film: influence of film edge on permanent magnet method IEEE Trans Magn. 2012:48;727-30.

[6] Kamitani A, Takayama T, Ikuno S. Shielding current analysis by current-vector-potential method: application to HTS film with multiply-connected structure Physica C. 2013:494;168-72.