Computers and Mathematics with Applications 59 (2010) 2628–2635

Contents lists available at ScienceDirect

Computers and Mathematics with Applications

journal homepage: www.elsevier.com/locate/camwa

$\boldsymbol{C}$-hypergroupoids obtained by special binary relations

S. Spartalis [a,*], M. Konstantinidou-Serafimidou [b], A. Taouktsoglou [a]

[a] Department of Production Engineering and Management, Faculty of Engineering, Democritus University of Thrace, University Library Building, Kimmeria GR-67 100 Xanthi, Greece

[b] Department of Mathematics, Physics and Computer Sciences, Faculty of Engineering, Aristotle University of Thessaloniki, GR-54 124 Thessaloniki, Greece

ARTICLE INFO

Article history:
Received 2 June 2009
Received in revised form 13 January 2010
Accepted 13 January 2010

Keywords:
Partial hypergroupoids
Hypergroupoids
H_V-semigroups
Binary relation
Hyperoperation relation

ABSTRACT

In this paper we deal with the partial or non-partial $\boldsymbol{C}$-hypergroupoids which are associated with special binary relations defined on H, such as Reflexive, Symmetric, Cyclic and Transitive. Basic properties are investigated and various characterizations are given. The main tool to study the previous special classes of hypergroupoids is the fundamental relation β^* (i.e. the smallest equivalence relation such that the quotient of a hypergroupoid (partial or not) is a groupoid (partial or not))

1. Introduction

Given a partial or non-partial hypergroupoid $(H, *)$, that is, $H \neq \varnothing$ and $*\colon H \times H \to \wp(H)\colon (x, y) \mapsto x * y$ (where $A * B = \cup_{a\in A, b\in B}\, a * b$, $A, B \in \wp(H)$), the *hyperoperation relation* R_* [1] can be defined on H in the following way:

$$\text{(i) } x, y \in H, z \in x * y \neq \varnothing \quad \text{iff } xR_*z, zR_*y, \qquad \text{(ii) } H * H = \varnothing \quad \text{iff } R_* = \varnothing.$$

On the other hand, connections between hyperstructures and binary relations have been analyzed by many researchers, such as Rosenberg [2], Corsini [3–6], Chvalina [7,8], Konstantinidou-Serafimidou [9], Leoreanu [10,11], Serafimidis [12], DeSalvo and LoFaro [13,14], Hort [15], Kehagias [16], Spartalis [17,18,1] and so on. More precisely, given a binary relation R defined on a non-empty set H (i.e. $R \subseteq H \times H$), several hyperoperations can be obtained in many different ways [2,7,8,11,13,15,17]. In the present paper we deal with the ***Corsini's hyperoperation*** [3] defined in the following way:

$$*_R : H \times H \to \wp(H) : (x, y) \mapsto x *_R y = \{z \in H \mid xRz, zRy\}.$$

Since the previous hyperproduct is not always non-empty, the hyperstructure $(H, *_R)$ is a partial hypergroupoid, called "*partial Corsini's hypergroupoid associated with the binary relation R*" (briefly, *partial* $\boldsymbol{C}$*-hypergroupoid*), denoted by H_R. It is clear that a partial $\boldsymbol{C}$-hypergroupoid H_R is a $\boldsymbol{C}$*-hypergroupoid* (i.e. non-partial), if and only if $R \circ R = H \times H$ [3].

Moreover, let $H \neq \varnothing$, $\Re_H = \{R_i \mid R_i \subseteq H \times H\}$, $\tilde{H}$ be the set of all partial or non-partial hypergroupoids defined on H and $\tilde{H}_{\Re} = \{H_{Ri} \mid R_i \in \Re_H\} \subseteq \tilde{H}$. Notice that, if card $H = n$, $n \geq 1$, then, card $\tilde{H}_{\Re} \leq 2^{n^2}$ and card $\tilde{H} = (2^n)^{n^2} = 2^{n^3}$. The following Propositions are valid:

Proposition 1 ([1]). *For each partial or non-partial hypergroupoid, a* $\boldsymbol{C}$*-hypergroupoid (partial or not) exists that includes it.*

* Corresponding author. Tel.: +30 25410 79341; fax: +30 25410 22711.
E-mail addresses: sspart@pme.duth.gr (S. Spartalis), mkonst@gen.auth.gr (M. Konstantinidou-Serafimidou), ataouktsoglou@sch.gr (A. Taouktsoglou).

0898-1221/$ – see front matter © 2010 Elsevier Ltd. All rights reserved.
doi:10.1016/j.camwa.2010.01.031

Proposition 2 ([1]). *Let R_* be the hyperoperation relation of a partial or non-partial hypergroupoid $H_* = (H, *)$, $H \neq \varnothing$, and $\varphi : \Re_H \to \tilde{H} : R_i \to \varphi(R_i) = H_{Ri}$. Then the following hold:*
(i) *If the H_{R_*} is a partial **C**-hypergroupoid, then*

$$R_* = minimum\{R_i \in \Re_H \mid R_i \in \varphi^{-1}(H_{R_*})\}.$$

(ii) *If the H_{R_*} is a **C**-hypergroupoid (non-partial), then*

$$\varphi^{-1}(H_{R_*}) = \{R_*\}.$$

Lemma 1.1. *Let $(H, *_R)$ be a **C**-hypergroupoid (partial or not) defined on $H \neq \varnothing$. Then, for all $A, B, C, D \in \wp(H) - \{\varnothing\}$,*

$$A *_R B \cap C *_R D = A *_R D \cap C *_R B.$$

Proof. Let $A *_R B \cap C *_R D \neq \varnothing$ and $w \in A *_R B \cap C *_R D$. Then, there exist $a \in A, b \in B, c \in C$ and $d \in D$ such that $w \in a *_R b \cap c *_R d$. According to Proposition 2.1 [1],

$$w \in a *_R b \cap c *_R d = a *_R d \cap c *_R b \subseteq A *_R D \cap C *_R B,$$

that is, $A *_R D \cap C *_R B \neq \varnothing$ and $A *_R B \cap C *_R D \subseteq A *_R D \cap C *_R B$. Similarly, if $A *_R D \cap C *_R B \neq \varnothing$, then $A *_R B \cap C *_R D \neq \varnothing$ and $A *_R D \cap C *_R B \subseteq A *_R B \cap C *_R D$, and so $A *_R B \cap C *_R D = A *_R D \cap C *_R B$.

In the case that $A *_R B \cap C *_R D = \varnothing$ or $A *_R D \cap C *_R B = \varnothing$ the statement is obvious. □

In this paper we deal with the partial or non-partial **C**-hypergroupoids which are associated with special binary relations defined on H, such as Reflexive, Symmetric, Cyclic and Transitive. Basic properties are investigated and various characterizations are given. The main tool to study the previous special classes of hypergroupoids is the fundamental relation β^* (i.e. the smallest equivalence relation such that the quotient of a hypergroupoid (partial or not) is a groupoid (partial or not)).

2. Partial or non-partial *C*-hypergroupoids defined by reflexive binary relations

Let $\Re_H^{\text{refl}} = \{R \in \Re_H \mid xRx, \text{ for all } x \in H\} \subseteq \Re_H$. Then, $\varDelta_H = \cap_{R \in \Re_H^{\text{refl}}} R$ and for all **C**-hypergroupoids $(H, *_R)$ (partial or not), $x \in x *_R x$, for all $x \in H$.

Proposition 2.1. *Let $(H, *_R)$ be a **C**-hypergroupoid (partial or not) and $R \in \Re_H^{\text{refl}}$. Then, the following hold:*
(i) *For all $a, b \in H$, $aRb \Leftrightarrow \{a, b\} \subseteq a *_R b$.*
(ii) *The class $\overline{R} = \{R_i \in \Re_H \mid H_R = H_{R_i}\}$ is a singleton.*
(iii) *The map $\overline{\varphi} : \Re_H^{\text{refl}} \to \tilde{\mathrm{H}} : R \to \overline{\varphi}(R)$ is an injection.*

Proof. (i) Let aRb. Since aRa, bRb, it follows that $\{a, b\} \subseteq a *_R b$. The converse is obvious.
(ii) Let $H_R = H_{R_i}$, $R_i \in \Re_H$. Since $a \in a *_R a = a *_{Ri} a$, for all $a \in H$, it follows that $R_i \in \Re_H^{\text{refl}}$. Therefore, for each $R_j \in \{R, R_i\}$, from (i) we obtain that

$$aR_jb \Leftrightarrow \{a, b\} \subseteq a *_R b = a *_{Ri} b, \quad \text{and so } R \subseteq R_i, R_i \subseteq R.$$

Therefore, $R = R_i$, that is, $\overline{R} = \{R\}$.
(iii) Let $R, R_i \in \Re_H^{\text{refl}}$, $R \neq R_i$. If $\overline{\varphi}(R) = \overline{\varphi}(R_i)$, that is, $H_R = H_{R_i}$, then, from (ii) it follows that $\overline{R} = \overline{R_i} = \{R\}$. Hence, $R_i = R$, which is a contradiction. Consequently, $H_R \neq H_{R_i}$, i.e. $\overline{\varphi}(R) \neq \overline{\varphi}(R_i)$. □

Proposition 2.2. *Let $(H, *_R)$ be a **C**-hypergroupoid (partial or not) and $R \in \Re_H^{\text{refl}}$. Then, for all $a, b, x \in H$, the following hold:*
(i) *$x \in a *_R b \Leftrightarrow x \in a *_R x \cap x *_R b$.*
(ii) *$b \in a *_R a \Leftrightarrow \{a, b\} \subseteq a *_R b \cap b *_R a$.*

Proof. (i) Let $x \in a *_R b$. Then, from Proposition 2.1 [1] we have that

$$x \in a *_R b \cap x *_R x = a *_R x \cap x *_R b.$$

Conversely, let $x \in a *_R x \cap x *_R b$. Then, aRx, xRb, and so $x \in a *_R b$.
(ii) Let $b \in a *_R a$. Since aRb, bRa, Proposition 2.1(i) of this issue follows that

$$\{a, b\} \subseteq a *_R b \cap b *_R a.$$

Conversely, let $\{a, b\} \subseteq a *_R b \cap b *_R a = a *_R a \cap b *_R b$. Then, $b \in a *_R a$. □

3. Partial or non-partial *C*-hypergroupoids defined by transitive binary relations

Let $\Re_H^{trans} = \{R \in \Re_H \mid xRy, yRz \Rightarrow xRz, \text{ for all } x, y, z \in H\} \subseteq \Re_H$ and $(H, *_R)$, $R \in \Re_H^{trans}$, be a **C**-hypergroupoid (partial or not). Then, $x *_R y \neq \varnothing$, $x, y \in H$, implies that there exists $w \in H$ such that xRw, wRy, and so xRy. Therefore, the following hold:

$$(H, *_R) \text{ is a } \boldsymbol{C}\text{-hypergroupoid iff } R \text{ is total.} \tag{3.1}$$

Proposition 3.1. *Let $(H, *_R)$ be a partial $\boldsymbol{C}$-hypergroupoid and $R \in \mathfrak{R}_H^{trans}$. Then, for all $x, y, z \in H$ the following hold:*
(i) $xRy \Rightarrow x *_R x \cup y *_R y \subseteq x *_R y$.
(ii) $(x *_R y) *_R z \subseteq x *_R z \supseteq x *_R (y *_R z)$.

Proof. (i) If $x *_R x = \varnothing = y *_R y$, then the statement is obvious. Let $w \in x *_R x \neq \varnothing$. Then, xRw, wRx and since xRy, it follows that wRy. Therefore, $w \in x *_R y$. Hence, $x *_R x \subseteq x *_R y$. Similarly, $y *_R y \neq \varnothing$ implies that $y *_R y \subseteq x *_R y$. Consequently, $x *_R x \cup y *_R y \subseteq x *_R y$.
(ii) In the case that $(x *_R y) *_R z = \varnothing = x *_R (y *_R z)$, the statement is obvious. Let $w \in (x *_R y) *_R z \neq \varnothing$. Then, there exists $a \in x *_R y$ such that $w \in a *_R z$, which means that xRa, aRy, aRw, wRz. Therefore, xRw, wRz implies that $w \in x *_R z$, and so $(x *_R y) *_R z \subseteq x *_R z$. Similarly, can be proved that $x *_R (y *_R z) \subseteq x *_R z$. □

Proposition 3.2. *Let $(H, *_R)$ be a partial $\boldsymbol{C}$-hypergroupoid and $R \in \mathfrak{R}_H^{trans}$. Then, for all $x, y, z \in H$ the following hold:*

$$\text{(i) } x *_R y \neq \varnothing \neq y *_R z \Rightarrow y \in x *_R z \cap \left(\bigcap_{\substack{w \in x *_R y \\ u \in y *_R z}} w *_R u \right) \quad \textit{and} \quad (x *_R y) *_R (y *_R z) \cup x *_R y \cup y *_R z \subseteq x *_R z.$$

$$\text{(ii) } x *_R (y *_R z) \neq \varnothing \neq (x *_R y) *_R z \Rightarrow x *_R (y *_R z) \cap (x *_R y) *_R z = (x *_R y) *_R (y *_R z) \neq \varnothing.$$

Proof. (i) From the hypothesis we obtain that xRy, yRz, which means that $y \in x *_R z$. Moreover, let $(x *_R y) *_R (y *_R z) = \bigcup_{\substack{w \in x *_R y \\ u \in y *_R z}} w *_R u$. Then, for all $w \in x *_R y$ and for all $u \in y *_R z$ the following hold:

$$xRw, wRy \quad \text{and} \quad yRu, uRz.$$

Since wRy, yRu, it follows that $y \in w *_R u$, and so $y \in \bigcap_{\substack{w \in x *_R y \\ u \in y *_R z}} w *_R u$. Hence,

$$y \in x *_R z \cap \left(\bigcap_{\substack{w \in x *_R y \\ u \in y *_R z}} w *_R u \right).$$

According to the previous, $(x *_R y) *_R (y *_R z) \neq \varnothing$ and let $a \in (x *_R y) *_R (y *_R z)$. Then, $a \in w *_R u$, $w \in x *_R y$, $u \in y *_R z$, which means that wRa, aRu. Therefore, xRw, $wRa \Rightarrow xRa$ and aRu, $uRz \Rightarrow aRz$, and so $a \in x *_R z$, that is, $(x *_R y) *_R (y *_R z) \subseteq x *_R z$.

Moreover, wRy, $yRz \Rightarrow wRz$ and xRy, $yRu \Rightarrow xRu$. Therefore, xRw, $wRz \Rightarrow w \in x *_R z$ and xRu, $uRz \Rightarrow u \in x *_R z$. Consequently,

$$(x *_R y) *_R (y *_R z) \cup x *_R y \cup y *_R z \subseteq x *_R z.$$

(ii) From Lemma 1.1 of this issue and for $A = \{x\}$, $B = y *_R z$, $C = x *_R y$, $D = \{z\}$ it follows that

$$x *_R (y *_R z) \cap (x *_R y) *_R z = x *_R z \cap (x *_R y) *_R (y *_R z).$$

Moreover, from the hypothesis it is obtained that $x *_R y \neq \varnothing \neq y *_R z$. Then, according to (i),

$$x *_R (y *_R z) \cap (x *_R y) *_R z = (x *_R y) *_R (y *_R z) \neq \varnothing. \quad \square$$

Corollary 3.3. *Let $(H, *_R)$ be a partial $\boldsymbol{C}$-hypergroupoid, $R \in \mathfrak{R}_H^{trans}$ and $x, y, z \in H$ such that $x \in x *_R y$, $z \in y *_R z$. Then,*

$$(x *_R y) *_R z = x *_R (y *_R z) \neq \varnothing.$$

Proof. From the hypothesis it is obtained that $x *_R z \subseteq (x *_R y) *_R z \cap x *_R (y *_R z)$. According to Proposition 3.1(ii) of this issue, we have the following

$$x *_R z \subseteq (x *_R y) *_R z \subseteq x *_R z \quad \text{and} \quad x *_R z \subseteq x *_R (y *_R z) \subseteq x *_R z.$$

Therefore, $(x *_R y) *_R z = x *_R z = x *_R (y *_R z)$. Moreover, since $x *_R y \neq \varnothing \neq y *_R z$, Proposition 3.2(ii) of this issue follows that $y \in x *_R z$, and so

$$(x *_R y) *_R z = x *_R (y *_R z) \neq \varnothing. \quad \square$$

Proposition 3.4. *Let $(H, *_R)$ be a partial $\boldsymbol{C}$-hypergroupoid, $R \in \mathfrak{R}_H^{trans}$ and $x \in H$ such that $x *_R x \neq \varnothing$. Then, the following hold:*
(i) *For all $w \in x *_R x$,*
$$\{x, w\} \subseteq x *_R x = x *_R w = w *_R x = w *_R w = (x *_R x) *_R x = x *_R (x *_R x).$$
(ii) *For all $w, u \in x *_R x$, $a, b \in H$, $w *_R a = u *_R a$ and $b *_R w = b *_R u$.*
(iii) *For all $a \in H$, $x *_R x \cap a *_R a = \varnothing \Rightarrow x *_R a = \varnothing$ or $\alpha *_R x = \varnothing$.*

Proof. (i) Let $w \in x *_R x$. Then, xRw, wRx and according to Proposition 5.2(i) of this issue,

$$x *_R x \cup w *_R w \subseteq x *_R w \cap w *_R x.$$

On the other hand, Lemma 1.1 of this issue implies that

$$x *_R w \cap w *_R x = x *_R x \cap w *_R w.$$

Therefore, $x *_R x = w *_R w = x *_R w \cap w *_R x$. Moreover, let $a \in x *_R w$, $b \in w *_R x$, that is, xRa, aRw, wRb, bRx. Hence, $aRw, wRx \Rightarrow aRx$ and $xRw, wRb \Rightarrow xRb$, and so $\{a, b\} \subseteq x *_R x$. Therefore, $x *_R w \subseteq x *_R x \supseteq w *_R x$, which means that

$$x *_R x = x *_R w = w *_R x = w *_R w.$$

Furthermore, since xRx and wRw, we obtain that $x \in x *_R x$, $w \in w *_R w$, and so

$$\{x, w\} \subseteq x *_R x = x *_R w = w *_R x = w *_R w.$$

In addition, since $x \in x *_R x$, Corollary 3.3 of this issue implies that $(x *_R x) *_R x = x *_R (x *_R x)$. Moreover, $x *_R x \subseteq (x *_R x) *_R x$. Let $a \in (x *_R x) *_R x$. Then, there exists $w \in x *_R x$ such that $a \in w *_R x$ and since $w *_R x = x *_R x$, it follows that $a \in x *_R x$. Therefore, $(x *_R x) *_R x \subseteq x *_R x$, and so $(x *_R x) *_R x = x *_R x$.
(ii) According to (i), $w, u \in x *_R x$ implies that wRu, uRw. Let $e \in w *_R a$. Then, wRe, eRa and since uRw, we obtain that uRe. Therefore, $e \in u *_R a$, that is, $w *_R a \subseteq u *_R a$. Conversely, let $e \in u *_R a$. Then, uRe, eRa and since wRu, we obtain that wRe. Therefore, $e \in w *_R a$, that is, $u *_R a \subseteq w *_R a$, and so $w *_R a = u *_R a$. Similarly, $b *_R w = b *_R u$.
(iii) Let $x *_R y \neq \varnothing \neq y *_R x$. Then, Proposition 5.2(i) of this issue implies that $a \in x *_R x$, and according to (i), $x *_R x = a *_R a$, which is a contradiction. Therefore, $x *_R a = \varnothing$ or $a *_R x = \varnothing$. $\square$

Proposition 3.5. *Let $H \neq \varnothing$, $R \in \Re_H^{\text{refl}} \cap \Re_H^{trans}$ and $(H, *_R)$ be the associated $\boldsymbol{C}$-hypergroupoid. The following hold:*

(i) *If H is a non-partial $\boldsymbol{C}$-hypergroupoid, then it is total.*
(ii) *If H is a partial $\boldsymbol{C}$-hypergroupoid, then the fundamental equivalence relation β^* is the transitive closure of the relation $\bar{R} \in \Re_H$ defined for all $x, y \in H$ as follows*

$$x\bar{R}y \Leftrightarrow xRy \quad or \quad yRx.$$

Moreover, $(H/\beta^*, \cdot)$ is a partial groupoid where

$$\beta^*(x) \cdot \beta^*(y) = \begin{cases} \beta^*(x), & \text{if } \beta^*(x) = \beta^*(y), x, y \in H \\ \varnothing, & \text{elsewhere} \end{cases}$$

Proof. (i) Let $(H, *_R)$ be a $\boldsymbol{C}$-hypergroupoid (non-partial), i.e. $x *_R y \neq \varnothing$ for all $x, y \in H$. Then, since R is a transitive relation, (3.1) of this issue implies that R is total, and so H is total.
(ii) Let $(H, *_R)$ be a partial $\boldsymbol{C}$-hypergroupoid and $\overline{R}^*$ be the transitive closure of the relation $\bar{R}$. Then, for all $x, y \in H$,

$$x\beta^*y \text{ implies that } \exists z_1, z_2, \dots, z_{n+1} \in H, z_1 = x, z_{n+1} = y \quad \text{and} \quad \exists u_{ij} \in H \text{ and } \exists I_i, i \in \{1, 2, \dots, n\}$$

finite sets of indices such that

$$\{z_i, z_{i+1}\} \subseteq *_R \prod_{j \in I_i} u_{ij}, \quad i \in \{1, 2, \dots, n\}.$$

Since the expression $*_R \prod_{j \in I_i} u_{ij}$ is a finite "product" with respect to the hyperoperation "$*_R$", Proposition 3.1(ii) of this issue implies for all $i \in \{1, 2, \dots, n\}$ the following:

$$*_R \prod_{j \in I_i} u_{ij} = u_{i1} *_R u_{i2} *_R \cdots *_R u_{ij_i} \subseteq u_{1i} *_R u_{ij_i}.$$

In addition, we set for all $i \in \{1, 2, \dots, n\}$,

$$u_{i1} = a_{2i-1} \quad \text{and} \quad u_{ij_i} = a_{2i}.$$

Therefore,

$$\{z_i, z_{i+1}\} \subseteq a_{2i-1} *_R a_{2i}, \quad i \in \{1, 2, \dots, n\}.$$

Hence,

$$z_i \, R \, a_{2i}, \quad z_{i+1} R \, a_{2i,} \quad i \in \{1, 2, \dots, n\},$$

and so

$$z_i \, \bar{R} \, a_{2i}, \quad a_{2i} \bar{R} \, z_{i+1,} \quad i \in \{1, 2, \dots, n\}.$$

Consequently, $x = z_1 \overline{R}^* z_{n+1} = y$, which means that $\beta^* \subseteq \overline{R}^*$.

Conversely, let $x, y \in H$, $x\overline{R}^{*}y$. Then, there exist $b_1, b_2, \ldots, b_{n+1} \in H$, $b_1 = x$, $b_{n+1} = y$ such that

$$b_i\,\bar{R}b_{i+1},\quad i \in \{1,2,\ldots,n\} \Leftrightarrow b_iR\,b_{i+1}\quad \text{or}\quad b_{i+1}R\,b_i,\quad i \in \{1,2,\ldots,n\}.$$

Propositions 2.1(i) and 3.1(i) of this issue follow that

$$\{b_i, b_{i+1}\} \subseteq b_i *_R b_{i+1}\quad \text{or}\quad \{b_i, b_{i+1}\} \subseteq b_{i+1} *_R b_i,\quad i \in \{1,2,\ldots,n\}.$$

Therefore, $x = b_1\beta^* b_{n+1} = y$, and so $\overline{R}^{*} \subseteq \beta^*$. Consequently, $\beta^* = \overline{R}^{*}$.

Let now, in the quotient set H/β^*, $\beta^*(x)\cdot\beta^*(y) = \{\beta^*(z) \mid z \in \beta^*(x) *_R \beta^*(y)\}$ be the usual multiplication of the classes.

Let x, y be two arbitrary elements of H such that $\beta^*(x) \neq \beta^*(y)$. Then, obviously $\beta^*(x) \cap \beta^*(y) = \varnothing$. So, if there exists $z \in H$ such that $\beta^*(z) \in \beta^*(x)\cdot\beta^*(y)$, then $z \in \beta^*(x) *_R \beta^*(y)$. Thus, there exist $a \in \beta^*(x)$, $b \in \beta^*(y)$ such that $z \in a *_R b$. So aRz, zRb, and so $z \in \beta^*(x) \cap \beta^*(y) = \varnothing$, which is a contradiction. Hence, $\beta^*(x)\cdot\beta^*(y) = \varnothing$.

On the other hand, $\beta^*(x)\cdot\beta^*(x) = \{\beta^*(z) \mid z \in \beta^*(x) *_R \beta^*(x)\}$. But for all $z \in H$ such that $\beta^*(z) \in \beta^*(x)\cdot\beta^*(x)$ holds $z \in \beta^*(x) *_R \beta^*(x)$. Then, $z \in \beta^*(x)$. That is, $\beta^*(z) = \beta^*(x)$, and so $\beta^*(x)\cdot\beta^*(x) \subseteq \{\beta^*(x)\}$. Obviously, $\beta^*(x) \subseteq \beta^*(x) *_R \beta^*(x)$. So $\beta^*(x)\cdot\beta^*(x) = \{\beta^*(x)\}$. Hence, we can denote $\beta^*(x)\cdot\beta^*(x) = \beta^*(x)$. □

Example 3.6. Let $H = \{1,2,3,4,5,6,7\}$ and $R = \{(x,x) \mid x \in H\} \cup \{(1,6),(2,6),(4,5),(5,4),(7,4),(7,5)\}$.

Then, $(H, *_R)$ is the partial ***C***-hypergroupoid

$*_R$	1	2	3	4	5	6	7
1	$\{1\}$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\{1,6\}$	$\varnothing$
2	$\varnothing$	$\{2\}$	$\varnothing$	$\varnothing$	$\varnothing$	$\{2,6\}$	$\varnothing$
3	$\varnothing$	$\varnothing$	$\{3\}$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$
4	$\varnothing$	$\varnothing$	$\varnothing$	$\{4,5\}$	$\{4,5\}$	$\varnothing$	$\varnothing$
5	$\varnothing$	$\varnothing$	$\varnothing$	$\{4,5\}$	$\{4,5\}$	$\varnothing$	$\varnothing$
6	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\{6\}$	$\varnothing$
7	$\varnothing$	$\varnothing$	$\varnothing$	$\{4,5,7\}$	$\{4,5,7\}$	$\varnothing$	$\{7\}$

and $H/\beta^* = \{\beta^*(1), \beta^*(3), \beta^*(4)\}$, where $\beta^*(1) = \{1,2,6\}$, $\beta^*(3) = \{3\}$, $\beta^*(4) = \{4,5,7\}$ such that

$\cdot$	$\beta^*(1)$	$\beta^*(3)$	$\beta^*(4)$
$\beta^*(1)$	$\beta^*(1)$	$\varnothing$	$\varnothing$
$\beta^*(3)$	$\varnothing$	$\beta^*(3)$	$\varnothing$
$\beta^*(4)$	$\varnothing$	$\varnothing$	$\beta^*(4)$

Remark 3.7. In the case that $R \in \mathfrak{R}_H^{\mathrm{refl}} \cap \mathfrak{R}_H^{trans}$, the equivalence class $\beta^*(x)$ of an arbitrary element $x \in H$ can also be constructed in the following way: If $A_1(x) = x *_R x$ and $A_{i+1}(x) = \cup_{(a *_R b)\cap A_i \neq \varnothing}\, a *_R b$ $(i = 1,2,\ldots)$, then obviously $A_1 \subseteq A_2 \subseteq A_3 \subseteq \cdots$ and there exists a positive integer $k = k(x)$ such that $A_k = A_{k+1} = \cdots$. Then, $A_k = \beta^*(x)$.

4. Partial or non-partial *C*-hypergroupoids defined by cyclic binary relations

Let $\mathfrak{R}_H^{cyc} = \{R \in \mathfrak{R}_H \mid xRy, yRz \Rightarrow zRx, \text{ for all } x, y, z \in H\} \subseteq \mathfrak{R}_H$ and $(H, *_R)$, $R \in \mathfrak{R}_H^{cyc}$, be a ***C***-hypergroupoid (partial or not). Then, $x *_R y \neq \varnothing$, $x, y \in H$ implies that there exists $w \in H$ such that xRw, wRy, and so yRx. Therefore, the following holds:

$$(H, *_R) \text{ is a } \boldsymbol{C}\text{-hypergroupoid}\quad \text{iff } R \text{ is total} \tag{4.1}$$

Proposition 4.1. *Let $(H, *_R)$ be a partial **C**-hypergroupoid and $R \in \mathfrak{R}_H^{cyc}$. Then, for all $x, y, z \in H$ the following hold:*

(i) $x \in y *_R z \Rightarrow y \in z *_R x$ *and* $z \in x *_R y$

(ii) $x *_R y \neq \varnothing \neq y *_R z \Rightarrow y \in \left(\bigcap_{\substack{w \in x *_R y\\ u \in y *_R z}} w *_R u\right) \cap z *_R x$

Proof. (i) Let $x \in y *_R z$. Then, yRx, xRz implies that zRy. Moreover, $zRy, yRx \Rightarrow y \in z *_R x$ and $xRz, zRy \Rightarrow z \in x *_R y$.

(ii) Since $x *_R y \neq \varnothing \neq y *_R z$, then, for all $w \in x *_R y$ and for all $u \in y *_R z$ (i) implies that $y \in w *_R x \cap z *_R u$. Moreover, according to Lemma 1.1 of this issue, $y \in w *_R u \cap z *_R x$ and therefore, $y \in \left(\bigcap_{\substack{w \in x *_R y\\ u \in y *_R z}} w *_R u\right) \cap z *_R x$. □

Proposition 4.2. *Let $(H, *_R)$ be a partial **C**-hypergroupoid and $R \in \mathfrak{R}_H^{cyc}$. Then, for all $x, y, z \in H$ the following hold:*

(i) $x *_R x \neq \varnothing \Rightarrow$ *for all* $w, u \in x *_R x$,

$$\{x, w, u\} \subseteq x *_R x = w *_R u$$

(ii) $x *_R x \neq \varnothing \Rightarrow (x *_R x) *_R x = x *_R (x *_R x) = x *_R x$

(iii) xRy *and* $(x *_R x \neq \varnothing$ *or* $y *_R y \neq \varnothing) \Rightarrow y \in x *_R x$

(iv) $(x *_R y) *_R z \neq \varnothing \neq x *_R (y *_R z) \Rightarrow (x *_R y) *_R z = x *_R x = x *_R (y *_R z)$.

Proof. (i) Let $w \in x *_R x \neq \varnothing$. According to Proposition 4.1(i) of this issue, we obtain that

$$x \in x *_R w \cap w *_R x = x *_R x \cap w *_R w.$$

In the same way, since $x \in w *_R w$, we obtain that

$$w \in w *_R x \cap x *_R w = w *_R w \cap x *_R x.$$

And so $\{x, w\} \subseteq x *_R x$ and $x *_R x \subseteq x *_R w$, $x *_R x \subseteq w *_R w$ (I).

Conversely, let $v \in x *_R w$. Then, $x \in w *_R v$, but also $x \in x *_R x$. So $x \in w *_R v \cap x *_R x = w *_R x \cap x *_R v$. Thus, $x \in x *_R v$ implies that $v \in x *_R x$, that is, $x *_R w \subseteq x *_R x$. Hence, $x *_R x = x *_R w$ (II).

Since $x \in w *_R w$, we similarly obtain that $w *_R w = w *_R x$ and $w *_R w \subseteq x *_R x$. Considering also (I) and (II), we finally obtain that

$$\text{for all } w \in x *_R x \neq \varnothing \quad \text{holds } \{x, w\} \subseteq x *_R w = x *_R x = w *_R w = w *_R x \text{ (III)}.$$

Let also $u \in x *_R x \neq \varnothing$. Then according to (III), we have $\{x, u\} \subseteq x *_R x = w *_R w$. But, since $u \in w *_R w$, (III) implies again that $w *_R u = w *_R w = u *_R u = u *_R w$. So, for all $w, u \in x *_R x$ holds $\{x, w, u\} \subseteq x *_R x = w *_R u$.

(ii) Since $x *_R x \neq \varnothing$, we have $x \in x *_R x$, and so $x *_R x \subseteq (x *_R x) *_R x$ and $x *_R x \subseteq x *_R (x *_R x)$. Conversely, let $\alpha \in (x *_R x) *_R x$ (resp. $\alpha \in x *_R (x *_R x)$), then there exists $w \in x *_R x$ such that $\alpha \in w *_R x$ (resp. $\alpha \in x *_R w$). Since $w *_R x = x *_R w = x *_R x$, we have that $\alpha \in x *_R x$, and so $(x *_R x) *_R x \subseteq x *_R x$ (resp. $x *_R (x *_R x) \subseteq x *_R x$). Hence, $(x *_R x) *_R x = x *_R x = x *_R (x *_R x)$.

(iii) Let now xRy and $x *_R x \neq \varnothing$. From (i) we have $x \in x *_R x$, so xRx. But then, $x \in x *_R y$ and according to Proposition 4.1(i) of this issue, also $y \in x *_R x$. Then, (III) implies that

$$\{x, y\} \subseteq x *_R y = x *_R x = y *_R y = y *_R x.$$

In the case xRy and $y *_R y \neq \varnothing$, the proof is similar.

(iv) $(x *_R y) *_R z \neq \varnothing \neq x *_R (y *_R z)$ implies that there exist $w, k, v \in H$ such that $w \in x *_R y$, $k \in w *_R z$, $v \in y *_R z$. Then, $w \in x *_R y \cap z *_R k = x *_R k \cap z *_R y$, and so $y \in w *_R z$.

Furthermore, $z \in k *_R w \cap v *_R y = k *_R y \cap v *_R w$, and so $y \in z *_R k$. Thus, $y \in w *_R z \cap z *_R k = w *_R k \cap z *_R z$, but since $y \in z *_R z$, we obtain through (III) that

$$z *_R z = z *_R y = y *_R z = y *_R y.$$

Moreover, since $x *_R y \neq \varnothing \neq y *_R z$, we have that $y \in z *_R x$, and so $x \in y *_R z = z *_R z = y *_R y$. Hence, $z *_R z = z *_R x = x *_R z = x *_R x = y *_R x = x *_R y$. Then, $(x *_R y) *_R z = (x *_R x) *_R x = x *_R x$ (according to (ii)) and also $x *_R (y *_R z) = x *_R (x *_R x) = x *_R x$. Consequently, $(x *_R y) *_R z = x *_R (y *_R z)$. □

Proposition 4.3. *Let $H \neq \varnothing$, $R \in \mathfrak{R}_H^{cyc}$ and H_R be the associated partial* **C***-hypergroupoid. Then, for all $x, y \in H$ the following hold:*

(i) $x *_R x \cap y *_R y = \varnothing$ *or* $x *_R x = y *_R y$
(ii) $x *_R x \neq \varnothing \Rightarrow$ *for all* $y \in H$, $x *_R y = \varnothing$ *or* $x *_R y = x *_R x = y *_R y = y *_R x$

Proof. (i) Let $x *_R x \cap y *_R y \neq \varnothing$. Then, there exists $w \in H$ such that $w \in x *_R x \cap y *_R y$. So, according to Proposition 4.2(i) of this issue, $x *_R x = w *_R w = y *_R y$.

(ii) Let $x *_R x \neq \varnothing$ and y be an arbitrary element of H. If $x *_R y \neq \varnothing$, there exists $w \in H$ such that $w \in x *_R y$. Then, since $R \in \mathfrak{R}_H^{cyc}$, yRx and according to Proposition 4.2(iii) of this issue, we obtain that $x \in y *_R y$. So, Proposition 4.2(i) of this issue implies that $x *_R y = x *_R x = y *_R y = y *_R x$. □

Proposition 4.4. *Let $H \neq \varnothing$, $R \in \mathfrak{R}_H$ and H_R be the associated partial* **C***-hypergroupoid. Let R_{*_R} be the hyperoperation relation of H_R. Then, the following hold:*

(i) $R \in \mathfrak{R}_H^{cyc} \Leftrightarrow R_{*_R} \in \mathfrak{R}_H^{cyc}$
(ii) *If* $R \in \mathfrak{R}_H^{cyc}$, *then* $R = R_{*_R} \Leftrightarrow y *_R x \neq \varnothing$, *for all* $(x, y) \in R$.

Proof. (i) Let $R \in \mathfrak{R}_H^{cyc}$. For all $(x, y), (y, z) \in R_{*_R}$ holds that $(x, y), (y, z) \in R$, since $R_{*_R} \subseteq R$. Then, $(z, x) \in R$. But $(y, z), (z, x) \in R$ implies that $z \in y *_R x$. So $yR_{*_R}z$, $zR_{*_R}x$. So $(z, x) \in R_{*_R}$. That is, $R_{*_R} \in \mathfrak{R}_H^{cyc}$. Conversely, let $R_{*_R} \in \mathfrak{R}_H^{cyc}$. For all $(x, y), (y, z) \in R$ holds that $y \in x *_R z$. So $(x, y), (y, z) \in R_{*_R}$. Since $R_{*_R} \in \mathfrak{R}_H^{cyc}$, $(z, x) \in R_{*_R} \subseteq R$. So $(z, x) \in R$. This means that $R \in \mathfrak{R}_H^{cyc}$.

(ii) $R = R_{*_R} \Leftrightarrow$ for all $(x, y) \in R$ there exists $z_1 \in H$ such that $(y, z_1) \in R$, or there exists $z_2 \in H$ such that $(z_2, x) \in R \Leftrightarrow$ for all $(x, y) \in R$ there exists $z_1 \in H$ such that $y \in x *_R z_1$, or there exists $z_2 \in H$ such that $x \in z_2 *_R y$. Since $R \in \mathfrak{R}_H^{cyc}$, according to Proposition 4.1(i) of this issue, we have $R = R_{*_R} \Leftrightarrow$ for all $(x, y) \in R$ there exists $z_1 \in H$ such that $z_1 \in y *_R x$, or there exists $z_2 \in H$ such that $z_2 \in y *_R x \Leftrightarrow y *_R x \neq \varnothing$ for all $(x, y) \in R$. □

Proposition 4.5. *Let $H \neq \varnothing$, $R \in \mathfrak{R}_H^{\text{refl}} \cap \mathfrak{R}_H^{cyc}$ and H_R be the associated partial* **C***-hypergroupoid. Then, for all $x, y, z \in H$ the following hold:*

$$(x *_R y) *_R z \neq \varnothing \quad or \quad x *_R (y *_R z) \neq \varnothing \Rightarrow (x *_R y) *_R z = x *_R x = x *_R (y *_R z).$$

Proof. Let $(x *_R y) *_R z \neq \varnothing$. Then, there exist $a, b \in H$ such that $a \in x *_R y$ and $b \in a *_R z$. So xRa, aRy. Since $R \in \mathfrak{R}_H^{cyc}$, we have yRx and since $R \in \mathfrak{R}_H^{\mathrm{refl}}$, we have xRx. So xRy. This means $y \in x *_R x$, and according to Proposition 4.2(i) of this issue, we have $x *_R y = x *_R x$. But then, $a \in x *_R x$, so $x *_R x = a *_R a$. Similarly, $b \in a *_R z$ implies that $b \in a *_R z = a *_R a = x *_R x$. So $b \in x *_R x$. That is, $(x *_R y) *_R z \subseteq x *_R x$ (I). Notice that, since $b \in x *_R x = a *_R z$, we have xRb and bRz. So zRx. But also xRx. So xRz.

Now, let $c \in x *_R x$. Then, cRx and xRz. So zRc. Also zRz. So cRz. But then, xRc and cRz. So $c \in (x *_R x) *_R z$. That is, $x *_R x \subseteq (x *_R x) *_R z = (x *_R y) *_R z$ (II). (I) and (II) imply that $x *_R x = (x *_R y) *_R z$.

Similarly it can be proved that $x *_R x = x *_R (y *_R z)$. □

Proposition 4.6. *Let $H \neq \varnothing$, $R \in \mathfrak{R}_H^{\mathrm{refl}} \cap \mathfrak{R}_H^{cyc}$ and $(H, *_R)$ be the associated **C**-hypergroupoid. Then, the following hold:*

(i) *If H is a non-partial **C**-hypergroupoid, then it is total.*
(ii) *If H is a partial **C**-hypergroupoid, then the fundamental equivalence relation β^* is exactly the relation R. Moreover,*
  (a) *$\beta^*(x) = x *_R x$, for all $x \in H$,*
  (b) *$(H/\beta^*, \cdot)$ is a partial groupoid, where*

$$\beta^*(x) \cdot \beta^*(y) = \begin{cases} \beta^*(x), & \text{if } \beta^*(x) = \beta^*(y), x, y \in H \\ \varnothing, & \text{elsewhere} \end{cases}$$

  (c) *H is a complete partial hypergroupoid.*

Proof. (i) Let $(H, *_R)$ be a non-partial **C**-hypergroupoid, i.e. $x *_R y \neq \varnothing$ for all $x, y \in H$. Then, since R is a cyclic relation, (4.1) of this issue implies that R is total, and so H is total.
(ii) Let $(H, *_R)$ be a partial **C**-hypergroupoid. Since Proposition 4.5 of this issue implies that for all $i \in \{1, 2, \ldots, n\}$

$$*_R \prod_{j \in I_i} u_{ij} = u_{i1} *_R u_{i2} *_R \cdots *_R u_{ij_i} = u_{i1} *_R u_{i1},$$

then, according to Proposition 4.3(i) of this issue, for all $x, y \in H$, $x\beta^*y$ leads finally to $\{x, y\} \subseteq u_{11} *_R u_{11} = x *_R x$. Consequently, xRy, that is, $\beta^* \subseteq R$. Conversely, for all $x, y \in H$, xRy implies, according to Proposition 4.2(iii) of this issue, that $\{x, y\} \subseteq x *_R x$. So $x\beta^*y$. That is, $R \subseteq \beta^*$. So $\beta^* = R$.

For the equivalence class of an arbitrary element $x \in H$ holds $R(x) = \{y \in H \mid xRy\} = x *_R x$, according to Proposition 4.2(iii) of this issue. Let now, in the quotient set H/R, $R(x) \cdot R(y) = \{R(z) \mid z \in R(x) *_R R(y)\}$ be the usual multiplication of the classes. Let x, y be two arbitrary elements of H such that $x\not{R}y$, that is, $R(x) \neq R(y)$, according to Proposition 4.3(i) of this issue. Then, $R(x) *_R R(y) = (x *_R x) *_R (y *_R y) = \varnothing$. So $R(x) \cdot R(y) = \varnothing$. In the case that xRy, according to Proposition 4.2(iii) and 4.2(i) of this issue, we have $R(x) = R(y)$ and according to Proposition 4.2(ii) of this issue, $R(x) *_R R(x) = (x *_R x) *_R (x *_R x) = x *_R x = R(x)$. So $R(x) \cdot R(x) = \{R(x)\}$. Hence, we can denote $R(x) \cdot R(x) = R(x)$.

Furthermore, $\beta^*(x *_R y) = R(x *_R y) = \cup_{z \in x *_R y} R(z) = \begin{cases} R(x *_R x) = x *_R y & \text{if } xRy \\ \varnothing = x *_R y & \text{if } x\not{R}y \end{cases}$. Consequently, $(H, *_R)$ is a complete partial hypergroupoid. □

Proposition 4.7. *Let $(H, *)$ be a partial hypergroupoid, in which the following hold*

(i) *$x \in x * x$, for all $x \in H$*
(ii) *$x * x \cap y * y = \varnothing$ or $x * x = y * y$, for all $x, y \in H$*
and (iii) *$x * y = \varnothing$ or $x * y = x * x = y * y = y * x$, for all $x, y \in H$.*

*Then, there exists a binary relation $R \in \mathfrak{R}_H^{\mathrm{refl}} \cap \mathfrak{R}_H^{cyc}$ such that $(H, *)$ identify with the partial **C**-hypergroupoid $(H, *_R)$. R is exactly the hyperoperation relation R_* of $(H, *)$.*

Proof. Let R_* be the hyperoperation relation of $(H, *)$. According to condition (i), R_* is reflexive. Furthermore, for all $x, y \in H$, $xR_*y \Leftrightarrow$ (there exists $w_1 \in H$ such that $x \in w_1 * y$) or (there exists $w_2 \in H$ such that $y \in x * w_2$). According to condition (iii), it holds $xR_*y \Leftrightarrow x \in y * y$ or $y \in x * x$. This means $x * x \cap y * y \neq \varnothing$. So, according to condition (ii), we have that $xR_*y \Leftrightarrow x * x = y * y$. Then, R_* is obviously cyclic.

Let $(H, *_{R_*})$ be the partial **C**-hypergroupoid defined by R_*. It is known that $(H, *) \leq (H, *_{R_*})$. (See Proposition 3.2 [1]. See also [19]) Conversely, let $x *_{R_*} y \neq \varnothing$, $x, y \in H$ and $z \in x *_{R_*} y$. Then, xR_*z, zR_*y. This means $x * x = z * z = y * y$. Then, $z \in z * z = x * x = x * y$. So $(H, *_{R_*}) \leq (H, *)$. Finally, $(H, *)$ identify with $(H, *_{R_*})$, where $R_* \in \mathfrak{R}_H^{\mathrm{refl}} \cap \mathfrak{R}_H^{cyc}$. So there is a binary relation $R = R_* \in \mathfrak{R}_H^{\mathrm{refl}} \cap \mathfrak{R}_H^{cyc}$ such that $(H, *) \equiv (H, *_R)$. □

5. Partial or non-partial *C*-hypergroupoids defined by symmetric binary relations

Let $\mathfrak{R}_H^{\mathrm{symm}} = \{R \in \mathfrak{R}_H \mid xRy \Rightarrow yRx \ \forall x, y \in H\}$. Since for all $R \in \mathfrak{R}_H^{\mathrm{symm}}$, $R = R^{-1}$, Corollary 2.6 [1] implies that

the associated (partial or not) **C**-hypergroupoid H_R is strong commutative (5.1)

Moreover, since H_R is strong commutative, for all $a, b \in H$, $a *_R b = a *_R b \cap b *_R a$ and according to Lemma 1.3 [1], $a *_R b = a *_R a \cap b *_R b$.

Therefore, we set the following proposition:

Proposition 5.1. *Let $H \neq \varnothing$ and $R \in \Re_H$. Then, the following hold:*

(i) *If the associated **C**-hypergroupoid H_R is non-partial, then,*

$$H_R \text{ is strong commutative } \Leftrightarrow R \in \Re_H^{\text{symm}}.$$

(ii) *If the associated **C**-hypergroupoid H_R is partial, then,*

$$H_R \text{ is strong commutative } \Leftrightarrow R_{*_R} \in \Re_H^{\text{symm}}.$$

(iii) *If $(H, *)$ is a strong commutative hypergroupoid (partial or not), then $R_* \in \Re_H^{\text{symm}}$.*

Proof. (i) Let H_R be strong commutative and $a, b \in H$, aRb. Moreover, for all $x \in a *_R b = b *_R a \neq \varnothing$ we have bRx, xRa, and so $b \in a *_R x = x *_R a$. Therefore, bRa. This means that $R \in \Re_H^{\text{symm}}$. The converse is implied from (5.1) of this issue.
(ii) Let H_R be strong commutative and $a R_{*_R} b$. Then, there exist $x, y \in H$, $a \in x *_R b$ or $b \in a *_R y$. Therefore, also $a \in b *_R x$ or $b \in y *_R a$. So $bR_{*_R}a$. This means that $R_{*_R} \in \Re_H^{\text{symm}}$. The converse is implied from (4.1) of this issue.
(iii) Let $a R_* b$. Then, there exist $x, y \in H$, $a \in x * b$ or $b \in a * y$. Since $(H, *)$ is strong commutative, also $a \in b * x$ or $b \in y * a$. So bR_*a, that is, $R_* \in \Re_H^{\text{symm}}$. □

Proposition 5.2. *Let $H \neq \varnothing$, $R \in \Re_H^{\text{symm}}$ and H_R be the associated partial **C**-hypergroupoid. Then, the following hold:*

(i) $R_{*_R} = R$
(ii) *The map $\varphi: \Re_H^{\text{symm}} \to \tilde{H} : R \to \varphi(R) = H_R$ is an injection.*

Proof. (i) Let aRb. Then, bRa, and so $b \in a *_R a$. Therefore, $aR_{*_R}b$. This means $R \subseteq R_{*_R}$. But then, $R = R_{*_R}$ (see Proposition 1.2 [1]).
(ii) It is obvious from (i). □

6. Concluding comment

In this paper we deal with the partial or non-partial **C**-hypergroupoids which are associated with special binary relations defined on H. Especially we deal with Reflexive, Symmetric, Cyclic and Transitive binary relations. Basic properties are investigated and various characterizations are given. Using the fundamental relation β^* it is proved that in the case $R \in \Re_H^{\text{refl}} \cap \Re_H^{trans}$ (or respectively $R \in \Re_H^{\text{refl}} \cap \Re_H^{cyc}$) the smallest groupoid that hides in a partial **C**-hypergroupoid is the one of the Proposition 3.5 of this issue (respectively 4.6 of this issue). We investigated Reflexive, Symmetric, Cyclic and Transitive binary relations, since they are the most common binary relations which do not necessarily lead to a total **C**-hypergroupoid. Partial or non-partial **C**-hypergroupoids which are associated with other special binary relations defined on H will be the aim of a further investigation.

References

[1] S. Spartalis, The hyperoperation relation and the Corsini's partial or not-partial hypergroupoids (A classification), Italian Journal of Pure and Applied Mathematics 24 (2008) 97–112.
[2] I. Rosenberg, Hypergroups and join spaces determined by relations, Italian Journal of Pure and Applied Mathematics 4 (1998) 93–101.
[3] P. Corsini, Binary relations and hypergroupoids, Italian Journal of Pure and Applied Mathematics 7 (2000) 11–18.
[4] P. Corsini, V. Leoreanu, Applications of Hyperstructure Theory, Kluwer Academic Publishers, Boston, Dordrecht, London, 2002.
[5] P. Corsini, V. Leoreanu, Survey on new topics of hyperstructure theory and its applications, in: Proc. of 8th Internat. Congress on AHA, 2003, pp. 1–37.
[6] P. Corsini, V. Leoreanu, Hypergroups and binary relations, Algebra Universalis 43 (2000) 321–330.
[7] J. Chvalina, Commutative hypergroups in the sense of Marty and ordered sets, in: Proc. of the Summer School on General Algebra and Ordered Sets, Olomouc (Czech Republic), 1994, pp. 19–30.
[8] J. Chvalina, Functional Graphs, Quasi-Ordered Sets and Commutative Hypergraphs, Vydavatelstvi Masarykovy Univerzity, 1995 (in Czech).
[9] M. Konstantinidou, K. Serafimidis, Sur les P-supertrillis, in: New Frontiers in Hyper-Structures and Rel. Alg, Hadronic Press, Palm Harbor U.S.A., 1996, pp. 139–148.
[10] V. Leoreanu, L. Leoreanu, Hypergroups associated with hypergraphs, Italian Journal of Pure and Applied Mathematics 4 (1998) 119–126.
[11] V. Leoreanu, Weak mutually associative hyperstructures Part II, in: Proc. of 8th Internat. Congress on AHA, 2003, pp. 183–189.
[12] K. Serafimidis, Certaines proprietes supplementaires des hypergroupoides. Elements premiers et etranges entre eux, Rendiconti dell'Istituto di Matematica dell'Università di Trieste XIX (1987) 1–11.
[13] M. De Salvo, J. Lo Faro, A new class of hypergroupoids associated to binary relations, Multiple-Valued Logic 9 (2003) 361–375.
[14] M. De Salvo, J. Lo Faro, Hypergroups and binary relations, Multiple-Valued Logic 8 (5–6) (2002) 645–657.
[15] D. Hort, A construction of hypergroups from ordered structures and their morphisms, Journal of Discrete Mathematical Sciences & Cryptography 6 (2–3) (2003) 139–150.
[16] K. Serafimidis, M. Konstantinidou, A. Kehagias, The L-fuzzy Nakano hyperlattice, in: Proc. of 8th Internat. Congress on AHA, 2003, pp. 239–250.
[17] S. Spartalis, Hypergroupoids obtained from groupoids with binary relations, Italian Journal of Pure and Applied Mathematics 16 (2004) 201–210.
[18] S. Spartalis, C. Mamaloukas, On hyperstructures associated with binary relations, Computers and Mathematics with Applications 51-1 (2006) 41–50.
[19] T. Vougiouklis, Hyperstructures and their Representations, Hadronic Press, Palm Harbor, 1994.