Magnetohydrodynamic (MHD) flow of Sisko fluid near the axisymmetric stagnation point towards a stretching cylinder

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Abstract

This paper investigates the momentum and heat transfer, MHD mixed convection flow of Sisko fluid near the axisymmetric stagnation point towards a stretching cylinder. Suitable similarity variables are selected to transmute dimensional nonlinear system into non-dimensional nonlinear system. Large mesh size and high tolerance error is considered for the convergence analysis of the numerical scheme. Graphical evaluation is displayed in order to interrogate the intrinsic behavior of responsible parameters on concerning profiles. For better description of fluid flow numerical variation in local skin friction coefficient and local Nusselt number is scrutinized through graphs and tables. Appreciable growth is found in temperature profile whereas a decrease in velocity profile is noticed when the Hartmann number is augmented. Mixed convection and velocity ratio parameters serve to enhance the rate of heat transfer from the surface. Comparison of present work in a limiting case with data available in the literature has been made for the verification of the model.

Keywords:
Sisko fluid model
Magnetohydrodynamic flow (MHD)
Mixed convection
Stagnation point flow
Joule Heating
Stretching cylinder
Shooting method

Introduction

The study of electrically conducting fluids in the presence of a magnetic field is of considerable and phenomenal interest because they are encountered in many processes of engineering and applied sciences such as MHD accelerators, electrostatic filters, cooling reactors, purification of crude oil, fluid droplets and geothermal energy extraction. A magnetohydrodynamics (MHD) flow of non-Newtonian fluid was first studied by Sarpkaya [1] and then followed by many authors. Liao [2] obtained a HAM solution for a non-Newtonian fluid flow in the presence of a magnetic field over a linearly stretching surface. He analyzed that magnetic field parameter enhances the skin friction coefficient; in addition he observed that the magnetic MHD effects are more prominent in the shear-thinning region as compared to the shear-thickening region. Study of MHD stagnation point flow with combined effects of heat, mass and conjugation of a first order chemical reaction towards a porous stretching surface was discussed by Meboom et al. [3]. The problem was solved analytically by using homotopy analysis method. They noted that for all values of suction/injection parameter and Hartmann number causes a monotonically increase in local skin-friction coefficient. Malik et al. [4] employed numerical solutions of MHD flows of non-Newtonian fluid past a stretching cylinder. They used the tangent hyperbolic fluid model for their study and concluded that the fluid velocity decreases with by increasing tangent hyperbolic fluid parameter and MHD parameter. Akber et al. [5] discussed the Eyring-Powell fluid under the influence of hydromagnetic field over a stretching sheet. In the literature survey eminence of MHD boundary layer flows in various physical configurations has been interpreted in Refs. [6–24].

Convective heat transfer is one of the extensive mode of heat transfer. In addition, it is the main model of mass moving in fluids and is regarded as a direct strategy for the heat transfer. Combined effects of free and force convection is well-known as mixed convection, which occurs due to the buoyancy force. The main cause of such convection is due to difference between the surface and free stream temperature. These types of flows have been widely addressed research area due to its plentiful application in manufacturing processes comprising extrusion of plastic, hot rolling, welding, solar receivers, heat exchangers etc. Because of its unlimited applications in daily life, the study of mixed convection is being considered by numerous researchers. Gorla [25] discussed mixed convection (both buoyancy added and opposed) stagnation point flow past over a heated stretching cylinder. He noted that in buoyancy-assisted flow region both rate of heat transfer and friction factor increases as the mixed convection parameter increases.
whereas reverse behavior is noted in case of buoyancy-opposed flow region. Lok et al. [26] calculated the numerical solution of steady mixed convection stagnation point flow over a shrinking/stretching sheet. It was seen that for all values of stretching/shrinking ratio parameter buoyancy added flow solutions are possible, but there are limitations for buoyancy opposed flow solutions. Ishak [27] analyzed the numerical solution of an incompressible mixed convection flow towards a vertical cylinder with variable surface heat flux. He discussed both buoyancy assisting and opposing forces that causes the development of boundary layer. His results proved the existence of solution in reversed flow region and dual solution in the assisting flow region. Nadeem et al. [28] also examined time dependent flow of hydromagnetic nanofluid in a rotating cone. Rehman et al. [29] also studied the mixed convection flow of Eyring-Powell fluid over a stretching cylinder in a double stratified medium.

The investigation of non-Newtonian fluids towards a stretching surfaces is one of the significant phenomena during the previous few decades. Because of plentiful modern and innovative applications of non-Newtonian fluids, such as polymer handling, ink-jet printing, geographical streams etc. much attention has been paid to them. From the family of non-Newtonian fluids, the relatively straightforward and simple model named Sisko fluid exists. It is basically extension of power law fluid model and proposed in 1958 by Sisko [30]. The Sisko fluid model is the blend of Newtonian and non-Newtonian fluids. Such fluids can be easily found in nature and have numerous modern applications, flow of greases is the most relevant example of this type of fluid. In the recent past, Khan et al. [31] considered annular pipe geometry and calculated the numerical and analytic solution of steady, two dimensional flow, and the heat transfer properties of the Sisko fluid. They noticed that the velocity of the Newtonian fluids is much less than the Sisko fluid. They observed that the strong shear thickening effects becomes stronger by increasing the flow behavior index. Nadeem et al. [32] explore the characteristics of the peristaltic pumping of the Sisko fluid in a uniform tube. They observed the fluid model for different values of the flow behavior index, and found that the Newtonian fluid has the best peristaltic pumping characteristics. Munir et al. [33] investigated the favorable and non-favorable buoyancy effects for Sisko fluid over an isothermally stretched surface. They perceived that the material parameter of the Sisko fluid enhances the wall friction factor. Literature survey indicates that no analysis is available which deliberate mixed convection and stagnation point effects towards a stretching cylinder with transverse magnetic field. Hence, our motivation is to discuss the combined effect of mixed convection and axisymmetric stagnation point on the hydromagnetic flow of Sisko fluid towards a permeable stretching surface with heat transfer achieve. As the analytic solution of the governing equations are intractable to illustrate the graphical results of dimensionless parameters involving in the problem, therefore numerical calculation through shooting method is carried out to desired level of accuracy. The effect of different governing parameters on concerning profiles along with surface shear stress and heat transfer coefficient are discussed through graphs and tables.

Problem formulation

Let us consider two dimensional hydromagnetic mixed convection stagnation point flow of incompressible Sisko fluid past over a cylinder. The problem is considered under polar cylindrical coordinates \((r, \theta, x)\) with velocity components \((u, v, w)\). The cylinder is being stretched with linear velocity \(U_w(x) = cx\), where \(c > 0\) is the parameter of constant acceleration. The coordinate system is chosen such that the \(x\)-axis is along the cylinder and \(r\)-axis is normal to the cylinder. The transverse magnetic field of strength \(B_0\) is applied parallel to the \(r\)-axis. Cylinder is continuously stretched along axial direction from both sides with velocity \(U_w\) as shown in Fig. 1.

In the absence of pressure gradient along with the boundary layer approximations the continuity [34] and momentum equations [35] governing such type of flows can be written as follows.

\[
\frac{\partial (r u)}{\partial x} + \frac{\partial (r v)}{\partial r} = 0, \tag{1}
\]

\[
u \frac{\partial u}{\partial x} + r \frac{\partial u}{\partial r} = U_w \frac{\partial u}{\partial x} + \frac{a}{r} \frac{\partial}{\partial r} (r \frac{\partial u}{\partial r}) - b \frac{\partial}{\partial r} \left( r \left( \frac{\partial u}{\partial x} \right)^n \right) + g \beta (T_p - T) - \frac{\beta B_0^2}{\rho} (u - U_w), \tag{2}
\]

subject to the momentum boundary conditions

\[
u = U_w(x) = cx, \quad \nu = 0, \quad \text{at} \ r = r_0, \quad \nu = U_w(x) = dx, \quad \text{as} \ r \to \infty. \tag{3}
\]

Here \(u\) and \(v\) are the velocity components of the fluid in the \(x\) and \(r\) directions respectively, \(\rho\) is the fluid density, \(k\) is the thermal diffusivity of the fluid, \(n\) power law index, \(a\) and \(b\) are fluid parameters, \(B_0\) is the strength of the magnetic field.

The surface of the cylinder is maintained at uniform temperature \(T_w\), moreover it is assumed that the convecting fluid and the medium are in local thermodynamic equilibrium and energy equation [29] under above mentioned suppositions along with the boundary layer approximations is as follows

\[
u \frac{\partial T}{\partial x} + r \frac{\partial T}{\partial r} = \frac{\alpha}{r} \frac{\partial}{\partial r} (r \frac{\partial T}{\partial r}) + \frac{\beta B_0^2}{\rho c_p} u^2. \tag{4}
\]

The associated boundary conditions for temperature are given as follows

\[
T = T_w(x) \text{ at } r = r_0, \quad T \to T_\infty \text{ as } r \to \infty, \tag{5}
\]

where \(T_w\) and \(T_\infty\) are the temperatures of the fluid and surrounding respectively, \(r_0\) is the radius of the cylinder. By choosing a stream function \(\psi\) defined as

\[
u - 1 \frac{\partial \psi}{r \partial \theta} \cdot \nu = -1 \frac{\partial \psi}{r \partial \alpha}. \tag{6}
\]

The continuity Eq. (1) is satisfied. Local similarity transformations [17]

\[
\eta = \frac{r^2 - x^2}{2r^2} Re_b^{1/3}, \quad \psi = \alpha x U Re_b^{1/3} f(\eta), \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad B_0 = \frac{\mu U_{\infty} x^2}{b}, \tag{7}
\]

are incorporate in Eqs. (2) and (5) to obtain the following nonlinear differential equations.

![Fig. 1. Physical model and coordinate system.](image-url)
respectively. Which are defined as magnetic field parameter, Eckert number and Prandtl number denotes Sisko fluid parameter, mixed convection parameter, curvature of the cylinder, velocity ratio parameter (stagnation point), magnetic field parameter, Eckert number and Prandtl number respectively. Which are defined as

\[
P_r = \frac{A_r^2}{\lambda}, \quad A_r = \frac{Re_B}{\lambda}, \quad \lambda = \frac{\lambda_B(t - t_0)}{\lambda_B(\eta)}, \quad \gamma = \frac{\gamma_B(t - t_0)}{\gamma_B(\eta)}, \quad \frac{x}{\tau_B(t - t_0)}.
\]

(11)

The most important dimensionless physical quantities for the problem in manufacturing point of view are the local skin friction and local Nusselt number, which are defined as

\[
C_f = \frac{\tau_w}{\rho c^2 v^2}, \quad Nu_u = \frac{q_w}{\theta_B(1 - \theta_u)}.
\]

(12)

where \(\tau_w\) and \(q_w\) are surface shear stress and surface heat flux, which are defined as

\[
\tau_w = a \left( \frac{\partial u}{\partial r} \right)_{r = r_0} - b \left( \frac{\partial u}{\partial \eta} \right)_{r = r_0}, \quad q_w = -k \left( \frac{\partial T}{\partial r} \right)_{r = r_0}.
\]

(13)

using Eq. (5) in Eqs. (11) and (12) the dimensionless forms of skin friction and local Nusselt number are

\[
\frac{1}{2} C_f R e_b^3 = A f'(0) - (-f''(0))^n, \quad N u_u R e_b^3 = -\theta'(0).
\]

(14)

where \(Re_b\) is local Reynolds number based on the stretching velocity \(U_w(x)\).

Numerical solutions

The analytical solution of ordinary differential Eqs. (8) and (9) are intractable because these equations are highly nonlinear. In order to solve Eqs. (8) and (9) combined with the associated boundary conditions in (10) numerically, shooting technique with Runge-Kutta Fehlberg method is used [34–36]. Write Eqs. (8) and (9) in the following form:

\[
f''' = f'' - 2\eta f'' + \gamma(n+1)(1+2\eta)^{-\frac{n}{2}}(-f''')^{n} - 2\eta f' + \theta - B^2 + M(f' - B),
\]

\[
\quad \frac{A(1+2\eta)}{A(1+2\eta)} + n(1+2\eta)^{\frac{n}{2}}(-f''')^{n} - 2\eta f' + \theta - B^2 + M(f' - B).
\]

(15)

\[
\theta' = \frac{-2\gamma \theta' + \theta'(2n+1)}{1+2\eta} f'' + M E c P r f''
\]

(16)

Now convert Eqs. (15) and (16) into set of five first order differential equations. Introducing the new set of dependent variables \(y_1, y_2, y_3, y_4, y_5\) and \(y_6\) as

\[
f' = y_1', \quad y_2, \quad y_3, \quad y_4, \quad y_5, \quad y_6.
\]

(17)

\[
f'' = y_2, \quad y_3, \quad y_4, \quad y_5, \quad y_6.
\]

(18)

\[
f''' = y_1', \quad y_2', \quad y_3, \quad y_4, \quad y_5, \quad y_6.
\]

(19)

\[
\theta' = y_4, \quad y_5, \quad y_6.
\]

(20)

\[
\Delta y_1 = 0, \quad y_2 = 1, \quad y_3, \quad y_4, \quad y_5 = 0, \quad \eta = 0.
\]

(21)

The boundary conditions for the system takes the form as:

\[
y_1 = 0, \quad y_2 = 1, \quad y_3 = 1, \quad \eta = 0.
\]

(22)

\[
y_2 \rightarrow B, \quad y_4 \rightarrow 0, \quad \eta \rightarrow \infty.
\]

(23)

To solve system of Eqs. (17)–(21) five initial conditions must be known, but the initial conditions at \(y_1, y_2, \) and \(y_3\) are not prescribed. Though, at \(\eta \rightarrow \infty\) the boundary conditions of \(f(\eta)\) and \(\theta(\eta)\) are prescribed. Thus, these boundary conditions are used to generate two unknown conditions. Express these two unknown conditions by \(H_1\) and \(H_2\).

\[
y_1 = 0, \quad y_2 = 1, \quad y_3, \quad y_4, \quad y_5 = 0, \quad \eta = 0.
\]

(24)

Results and discussion

This section is fascinated to a comprehensive study of numerically calculated results and effect logs of all pertinent parameters on temperature and velocity profiles. Moreover, local skin friction coefficient and local Nusselt number are presented in graphical and tabular form. The results are given to carry out a parametric study showing the effects of the dimensionless parameters namely Sisko parameter \(A\), Hartmann number \(M\), mixed convection parameter \(\lambda\), velocity ratio parameter \(B\), curvature parameter \(\gamma\) and Prandtl number \(Pr\). Table 1 provides the comparison of this analysis to the already published literature for several set values of Hartmann number \(M\) to check the accuracy of the numerical method used in present problem. Fig. 2 describes the behavior of velocity profile for various values of \(A\) and \(n\). An analysis reveals that power law index \(n\) plays a key role on the boundary layer structure. It is seen an increase in power law index \(n\) leads to a decline in the
velocity and boundary layer thickness. Because the behavior of fluids changes from shear thinning to shear thickening fluid and also as shear thickening fluid are more viscous than Newtonian and shear thinning fluids. Hence by increasing flow behavior index \( n \) velocity profile decreases. Also it is clearly shown that by increasing material parameter \( A \) velocity profile increases. By definition \( A \) is ratio of high shear rate viscosity to consistency index. Because by increasing \( A \), initial forces of fluid increases which causes a decrease in viscous forces and hence the boundary layer thickness.

Fig. 3 demonstrates the effect of velocity ratio parameter \( B \) on the dimensionless velocity profile. It is perceived that for \( B < 1 \) boundary layer thickness decreases but velocity profile increases, for \( B < 1 \) the boundary thickness has opposite effects. But for \( B = 1 \) there is no information of boundary. Fig. 4 reveals that velocity exceeds with the increment of curvature parameter \( \gamma \) and be inferior for flow behavior index \( n \). These curves prescribed that far away from the surface the velocity of the fluid decreases and disappears asymptotically. It is examined that with the increase of curvature parameter \( \gamma \) velocity field increases, because by increasing curvature parameter \( \gamma \) radius of curvature decreases due to which the contact area between fluid and cylinder decreases. Thus less resistance is offered by the surface of cylinder to fluid motion. Furthermore, boundary layer is thicker for larger values of curvature parameter \( \gamma \). Fig. 5 demonstrates the behavior of mixed convection parameter \( \lambda \) and power index \( n \) on dimensionless velocity profile. It is perceived that with an increase in mixed convection parameter \( \lambda \) velocity profile increases. Also it is witnessed that for greater values of mixed convection parameter \( \lambda \) momentum boundary layer thickness increases. Fig. 6 specifies the imposition of Hartmann number \( M \) on velocity profile. Since increase in Hartmann number \( M \) leads to an increase in the drag force called Lorentz force. This force has the propensity to slow down the flow. Strong Lorentz forces yields more opposition to the momentum transport, thus for higher values of Hartmann number \( M \), causes decrease in hydrodynamics boundary layer thickness. Fig. 7 shows the influence of curvature parameter \( \gamma \) on temperature profile for pertinent values of power index \( n \). It is observed that the temperature and the corresponding thermal boundary layer thickness decreases as the values of power index \( n \) are increased. Also it is
evidently observed that for larger values of curvature parameter near the surface thermal boundary layer thickness decreases whereas increases away from the surface. Physically, by enlarging the curvature parameter causes an increase in local Nusselt number due to which temperature profile drops close to the surface, but it is responsible in the enhancement of the temperature profile away from the surface. Fig. 8 is plotted to see the impact of Prandtl number Pr on dimensionless temperature field for various values of power law index n. It is perceived that by enlarging the values of Prandtl number Pr thermal boundary layer thickness becomes thinner which causes reduction on temperature profile. The temperature distributions of Newtonian and non-Newtonian fluids are plotted in Fig. 9 for different values of Eckert number Ec. Since Eckert number Ec is the ratio between kinetic energy and enthalpy. As Ec increases it increase the kinetic energy, so fluid particle collides frequently with each other and they dissipate the energy i.e. the kinetic energy is converted into the thermal energy, this yields rise in the temperature of the fluid. Fig. 10 shows the influence of velocity ratio parameter B, material parameter A and flow behavior index n on local skin friction coefficient. It is noted that local skin
friction coefficient decays with a growth in velocity ratio parameter $B$, because boundary layer thickness progressively decreases with an increase in velocity ratio parameter $B$. Coefficient of skin friction enhances with an increase in material parameter $A$. Since material parameter $A$ increases boundary layer thickness which causes increase in skin friction coefficient. Fig. 11 demonstrates the graphical representation of skin friction coefficient by varying curvature parameter $\gamma$, material parameter $A$ and flow behavior index $n$. It is perceived that absolute value of skin friction coefficient increases by increasing material parameter $A$ and curvature parameter $\gamma$, while decreases by increasing flow behavior index $n$. Since curvature parameter $\gamma$ has an inverse relation with Reynolds number and Reynolds number has inverse relation with skin friction coefficient. So by increase in curvature parameter causes a decrease in Reynolds number which ultimately enhances the values of local skin friction coefficient. Finally by increasing flow behavior index $n$ skin friction decreases because fluid behavior changes from shear thinning to Newtonian and then shear thickening fluid, which ultimately cause a decrease in boundary layer thickness and increases the viscosity of the fluid. As a result local skin friction coefficient reduces by increase in $n$. Fig. 12 depicts the graphical interpretation of curvature parameter $\gamma$, Prandtl number $\text{Pr}$ and flow behavior index $n$ on local Nusselt number. It is evident that by increasing curvature parameter $\gamma$ local Nusselt number increases. Because with the increase of curvature parameter $\gamma$, temperature near to the surface of cylinder decreases as a result thermal boundary layer becomes thin which causes an enhancement in temperature gradient. From this figure it is also cleared that with the increase in Prandtl number $\text{Pr}$ local Nusselt number increases. Prandtl number $\text{Pr}$ causes a decrease in temperature which increase difference between surface of cylinder and fluid temperature. Hence it leads to increase the rate of heat transfer from surface of cylinder i.e. local Nusselt number increases. Furthermore this graph also shows that by increasing flow behavior index $n$ local Nusselt number increases. Fig. 13 explained the behavior of Eckert number $E_c$ and Prandtl number $\text{Pr}$ on Nusselt number. For larger values of Eckert number $E_c$ the temperature difference $T_w - T_o$ decreases. So heat transfer rate decreases which causes decrease in the Nusselt number. Also Prandtl number $\text{Pr}$ increases the values of Nusselt number due to the fact that for larger values of Prandtl number $\text{Pr}$ conductivity of the fluid decreases. Table 2 shows the impact of flow parameters $\gamma$, $A$, $\lambda$, and $B$ on skin friction coefficient for different values of flow behavior index $n$. From table it can be shown that numeric values of skin friction coefficient are increasing for larger values of $n$. Also this table shows that larger values of three parameters $\gamma$, $A$ and $M$ enhances the computed values of skin friction coefficient in absolute sense. This is due to fact that when all these parameters increases the boundary layer thickness also increases so the viscous forces near the surface become stronger. Hence it increases the values of skin friction coefficient. Whereas from table it is noticed that with the increase of $\lambda$ and $B$ the skin drag decreases. Table 3 depicts the variations in Nusselt number for change in physical parameters $\gamma$, $\text{Pr}$, $\lambda$, $B$, $E_c$ and $M$. When flow parameters $\gamma$, $\text{Pr}$, $\lambda$, and $B$ increases it reduces the thermal boundary layer. Thus values of $-\theta''(0)$ enhances when larger values are awarded to parameters $\gamma$, $\text{Pr}$, $\lambda$, and $B$. On the other hand if values of physical parameters $E_c$ and $M$ increases it causes reduction in local Nusselt number. This is physically valid because when larger values of all flow parameters $E_c$ and $M$ are considered the thermal boundary layer become thick. Hence/So wall heat flux reduces.
Concluding remarks

A theoretical study is presented to explore physical features of MHD mixed convection flow of Sisko fluid near the axisymmetric stagnation point towards a linearly stretching cylinder. The resulting differential equations were solved numerically using a shooting technique together with Runge-Kutta-Fehlberg integration system to express the effects log of different embedded parameters on flow and heat transfer graphically and quantitatively. The following observations are worth mentioning:

- Strength of magnetic field parameter $M$ reduces the velocity profile but causes increase in temperature profile due to Joule Heating effect.
- There is a direct relation of curvature parameter $\gamma$ with velocity and temperature profiles.
- The material parameter $A$ enhances the momentum boundary layer thickness.
- Increase in the mixed convection parameter $\lambda$ enhances the fluid velocity, while it reduces the temperature profile.
- The power law index $n$ reduces both velocity and temperature profiles. But impacts of the power law index $n$ are opponent on the wall friction coefficient, and local Nusselt number.
- Impact of Prandtl number $Pr$ on temperature profile is opposite.
- Velocity ratio parameter $B$ reduces the thermal resistance but causes increase in velocity profile.
- An enhancement in the skin friction coefficient is noticed against the magnetic field parameter $M$.
- Magnetic parameter $M$ and the Eckert number $Ec$ both impart the considerable decrease in local Nusselt number.

References


