# Annihilation rate of $2^{++}$charmonium and bottomonium 

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#### Abstract

Two-photon annihilation rates of $2^{+}$tensor charmonium and bottomonium up to third radial excited states are estimated in the relativistic Salpeter method. Full Salpeter equation for $2^{+}$tensor state is solved with a well defined relativistic wave function and we calculated the annihilation amplitude using the Mandelstam formalism. Our estimates of the decay widths are: $\Gamma\left(\chi_{c 2} \rightarrow 2 \gamma\right)=501 \mathrm{eV}, \Gamma\left(\chi_{c 2}^{\prime} \rightarrow\right.$ $2 \gamma)=534 \mathrm{eV}, \Gamma\left(\chi_{b 2} \rightarrow 2 \gamma\right)=7.4 \mathrm{eV}$ and $\Gamma\left(\chi_{b 2}^{\prime} \rightarrow 2 \gamma\right)=7.7 \mathrm{eV}$. We also give total decay widths of the lowest states estimated by the two-gluon decay rates, and the results are: $\Gamma_{\text {tot }}\left(\chi_{c 2}\right)=2.64 \mathrm{MeV}$, $\Gamma_{\text {tot }}\left(\chi_{b 2}\right)=0.220 \mathrm{MeV}$ and $\Gamma_{\text {tot }}\left(\chi_{b 2}^{\prime}\right)=0.248 \mathrm{MeV}$.


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## 1. Introduction

Recently, the radiative annihilation physics of $\chi_{c 0}, \chi_{b 0}\left(0^{++}\right)$, $\chi_{c 2}$ and $\chi_{b 2}\left(2^{++}\right)$become hot topics [1-11], because the annihilation amplitudes are related to the behavior of the wave functions, so the annihilation rates are helpful to understand the formalism of inter-quark interactions, and can be a sensitive test of the potential model [12].

In previous Letter [13], two-photon and two-gluon annihilation rates of $0^{++}$scalar $c \bar{c}$ and $b \bar{b}$ states are computed in the relativistic Salpeter method, good agreement of our predictions with other theoretical calculations and available experimental data is found. In our calculation, we found the relativistic corrections are large and cannot be ignored, and point out that all the calculations related to a $P$-wave state, one have to use a relativistic method, a nonrelativistic method will cause a large error, even for a heavy state $[13,14]$. In a non-relativistic calculation, the corresponding decay width is related to the derivative of the non-relativistic $P$-wave function at the origin, but in a full relativistic calculation, the relativistic corrections include not only relativistic kinematics but also the relativistic inter-quark dynamics, the decay width is related to the full behavior of $P$-wave function which can be seen in this Letter or in Ref. [15].

In this Letter, we give relativistic calculation of $2^{++}$tensor $c \bar{c}$ and $b \bar{b}$ states decaying into two photons using the instantaneous Bethe-Salpeter method [16], which is a full relativistic Salpeter method [17]. The case of tensor $2^{++}$state is special, not like other $P$-wave states, there are no decay constants in this state, and because there is the $P-F$ mixing problem, it make the physics much complicated.

[^0]The reason of the exist of $P-F$ mixing is that $P$-wave and $F$ wave state have the same parity and charge conjugate parity, they are all $J^{P C}=2^{++}$, one cannot distinguish them by the quantum number. We have found a basic method to deal with this problem [18], we begin from the quantum field theory, analyze the parity and charge conjugation of bound state, and give a formula for the wave function that is in a relativistic form with definite parity and charge conjugation symmetry, then we solve the full Salpeter equation, and obtain the corresponding state, and there are automatically the mixing between $P$ wave and $F$ wave.

The Letter is organized as following, in Section 2, we give the annihilation amplitude in Mandelstam formalism and the wave function of the $2^{+}$tensor state with a well defined relativistic form. The two-photon decay width and full width of heavy $2^{++}$ quarkonium are formulated in Section 2, we show the numerical results and give discussions in Section 3.

## 2. Theoretical details

According to the Mandelstam [19] formalism, the relativistic transition amplitude of a quarkonium decaying into two photons (see Fig. 1) can be written as:

$$
\begin{align*}
T_{2 \gamma}= & i \sqrt{3}\left(i e e_{q}\right)^{2} \int \frac{d^{4} q}{(2 \pi)^{4}} \operatorname{tr}\left\{\chi ( q ) \left[\phi_{2} S\left(p_{1}-k_{1}\right) \phi_{1}\right.\right. \\
& \left.\left.+\phi_{1} S\left(p_{1}-k_{2}\right) \phi_{2}\right]\right\}, \tag{1}
\end{align*}
$$

where $k_{1}, k_{2} ; \varepsilon_{1}, \varepsilon_{2}$ are the momenta and polarization vectors of photon 1 and photon 2; $e_{q}=\frac{2}{3}$ for charm quark and $e_{q}=\frac{1}{3}$ for bottom quark; $p_{1}$ and $p_{2}$ are the momentum of constitute quark 1 and antiquark 2; $\chi(q)$ is the Bethe-Salpeter wave function of the corresponding meson with the total momentum $P$ and relative momentum $q$, related by


Fig. 1. Two-photon annihilation diagrams of the quarkonium.
$p_{1}=\alpha_{1} P+q, \alpha_{1} \equiv \frac{m_{1}}{m_{1}+m_{2}}$,
$p_{2}=\alpha_{2} P-q, \alpha_{2} \equiv \frac{m_{2}}{m_{1}+m_{2}}$,
where $m_{1}=m_{2}$ is the constitute quark mass of charm or bottom.
Since $p_{10}+p_{20}=M$, the approximation $p_{10}=p_{20}=\frac{M}{2}$ is a good choice for the equal mass system [7,20,21]. Having this approximation, we can perform the integration over $q_{0}$ to reduce the expression, with the notation of Salpeter wave function $\Psi(\vec{q})=$ $\int \frac{d q_{0}}{2 \pi} \chi(q)$, to

$$
\begin{align*}
T_{2 \gamma}= & \sqrt{3}\left(e e_{q}\right)^{2} \int \frac{d \vec{q}}{(2 \pi)^{3}} \operatorname{tr}\left\{\Psi ( \vec { q } ) \left[\not \phi_{2} \frac{1}{p_{1}-k_{1}-m_{1}} \phi_{1}\right.\right. \\
& \left.\left.+\phi_{1} \frac{1}{p_{1}-k_{2}-m_{1}} \not \phi_{2}\right]\right\} \tag{2}
\end{align*}
$$

The general form for the relativistic wave function of tensor $J^{P}=2^{+}$state (or $J^{P C}=2^{++}$for equal mass system) can be written as 16 terms constructed by momentum $P, q$ and Dirac matrix $\gamma$, because of the approximation of instantaneous, 8 terms with $P \cdot q$ become zero, the relativistic Salpeter wave function $\Psi(\vec{q})$ for $2^{+}$state can be written as:

$$
\begin{align*}
\Psi_{2^{+}}(\vec{q})= & \varepsilon_{\mu \nu} q_{\perp}^{\nu}\left\{q_{\perp}^{\mu}\left[f_{1}(\vec{q})+\frac{\not p}{M} f_{2}(\vec{q})+\frac{\not q_{\perp}}{M} f_{3}(\vec{q})+\frac{\not p q_{\perp}}{M^{2}} f_{4}(\vec{q})\right]\right. \\
& +\gamma^{\mu}\left[M f_{5}(\vec{q})+\not p f_{6}(\vec{q})+\not q_{\perp} f_{7}(\vec{q})\right] \\
& \left.+\frac{i}{M} f_{8}(\vec{q}) \epsilon^{\mu \alpha \beta \gamma} P_{\alpha} q_{\perp \beta} \gamma_{\gamma} \gamma_{5}\right\} \tag{3}
\end{align*}
$$

where the $\varepsilon_{\mu \nu}$ is the polarization tensor of the $2^{+}$state, $q_{\perp}=$ $(0, \vec{q})$. But these 8 terms wave functions $f_{i}$ are not independent, there are the further constraint from Salpeter equation [17]: $\Psi_{2^{+}}^{+-}(\vec{q})=\Psi_{2^{+}}^{-+}(\vec{q})=0$, which give the constraints on the components of the wave function, so we get the relations
$f_{1}(\vec{q})=\frac{\left[q_{\perp}^{2} f_{3}(\vec{q})+M^{2} f_{5}(\vec{q})\right]\left(\omega_{1}+\omega_{2}\right)-M^{2} f_{5}(\vec{q})\left(\omega_{1}-\omega_{2}\right)}{M\left(m_{1} \omega_{2}+m_{2} \omega_{1}\right)}$,
$f_{2}(\vec{q})=\frac{\left[q_{\perp}^{2} f_{4}(\vec{q})-M^{2} f_{6}(\vec{q})\right]\left(\omega_{1}-\omega_{2}\right)}{M\left(m_{1} \omega_{2}+m_{2} \omega_{1}\right)}$,
$f_{7}(\vec{q})=\frac{f_{5}(\vec{q}) M\left(\omega_{1}-\omega_{2}\right)}{m_{1} \omega_{2}+m_{2} \omega_{1}}$
$f_{8}(\vec{q})=\frac{f_{6}(\vec{q}) M\left(\omega_{1}+\omega_{2}\right)}{m_{1} \omega_{2}+m_{2} \omega_{1}}$.
Only four independent wave functions $f_{3}(\vec{q}), f_{4}(\vec{q}), f_{5}(\vec{q})$ and $f_{6}(\vec{q})$ been left, one can check in Eq. (3), all the terms except the two terms with $f_{2}$ and $f_{7}$ are charge conjugate parity positive, but $f_{2}$ and $f_{7}$ terms have negative charge conjugate parity, after we use the constraint relations, for equal mass system, the terms with $f_{2}$ and $f_{7}$ become zero, then the whole wave function have positive charge conjugate parity, that is $2^{++}$state. These wave functions and the bound state mass $M$ can be obtained by solving the full Salpeter equation with the constituent quark mass as input, and we will not show the details of how to solve it, we only show our result in next section.

These four independent wave functions fulfill the normalization condition:

$$
\begin{align*}
& \int \frac{d \vec{q}}{(2 \pi)^{3}} \frac{16 \omega_{1} \omega_{2} \vec{q}^{2}}{15\left(m_{1} \omega_{2}+m_{2} \omega_{1}\right)} \\
& \quad \times\left\{f_{5} f_{6} M^{2}\left[5+\frac{\left(m_{1}+m_{2}\right)\left(m_{2} \omega_{1}-m_{1} \omega_{2}\right)}{\omega_{1} \omega_{2}\left(\omega_{1}+\omega_{2}\right)}\right]\right. \\
& \quad+f_{4} f_{5} \vec{q}^{2}\left[2+\frac{\left(m_{1}+m_{2}\right)\left(m_{2} \omega_{1}-m_{1} \omega_{2}\right)}{\omega_{1} \omega_{2}\left(\omega_{1}+\omega_{2}\right)}\right] \\
& \left.-2 \vec{q}^{2} f_{3}\left(f_{4} \frac{\vec{q}^{2}}{M^{2}}+f_{6}\right)\right\}=2 M \tag{5}
\end{align*}
$$

With the full Salpeter wave function, the two photon decay amplitude can be written as:

$$
\begin{align*}
T_{2 \gamma}= & 4 \sqrt{3} e^{2} e_{q}^{2} \varepsilon_{\mu \nu} \int \frac{d \vec{q}}{(2 \pi)^{3}}\left\{\frac{1}{x_{1}-|\vec{q}| M \cos \theta}\right. \\
& \times\left[f_{5} M\left(k_{1}^{\mu} q_{\perp}^{v} \varepsilon_{1} \cdot \varepsilon_{2}+\varepsilon_{1}^{\mu} q_{\perp}^{v} \varepsilon_{2} \cdot q_{\perp}+\varepsilon_{2}^{\mu} q_{\perp}^{v} \varepsilon_{1} \cdot q_{\perp}\right)\right. \\
& \left.+\frac{q_{\perp}^{\mu} q_{\perp}^{v} f_{3}}{M}\left(\varepsilon_{1} \cdot \varepsilon_{2} q_{\perp} \cdot k_{1}+2 \varepsilon_{1} \cdot q_{\perp} \varepsilon_{2} \cdot q_{\perp}\right)\right] \\
& +\frac{1}{x_{1}+|\vec{q}| M \cos \theta}\left[f _ { 5 } M \left(k_{2}^{\mu} q_{\perp}^{v} \varepsilon_{1} \cdot \varepsilon_{2}+\varepsilon_{1}^{\mu} q_{\perp}^{v} \varepsilon_{2} \cdot q_{\perp}\right.\right. \\
& \left.\left.\left.+\varepsilon_{2}^{\mu} q_{\perp}^{v} \varepsilon_{1} \cdot q_{\perp}\right)+\frac{q_{\perp}^{\mu} q_{\perp}^{v} f_{3}}{M}\left(\varepsilon_{1} \cdot \varepsilon_{2} q_{\perp} \cdot k_{2}+2 \varepsilon_{1} \cdot q_{\perp} \varepsilon_{2} \cdot q_{\perp}\right)\right]\right\} \tag{6}
\end{align*}
$$

where $x_{1}=\frac{M^{2}}{4}+\vec{q}^{2}+m_{1}^{2}$, and $\theta$ is the angle between the momentum $\vec{q}$ and $\vec{k}_{1}$. Finally the decay width with first order QCD correction [22] can be written as:
$\Gamma_{2 \gamma}=\frac{1}{2!\cdot 5 \cdot 16 \pi M} \sum\left|T_{2 \gamma}\right|^{2} \cdot\left(1-\frac{16 \alpha_{s}}{3 \pi}\right)$.
Until now, only the total decay width of $\chi_{c 2}(1 P)$ is available, we can estimate the full decay width of OZI-forbidden states using the two gluon decay width, and the two gluon decay width of quarkonium can be easily obtained from the two photon decay width with a simple replacement. For the charmnium states, only the ground state $\chi_{c 2}(1 P)$ is OZI-forbidden, we have:
$\Gamma_{\text {tot }}\left(\chi_{c 2}\right) \cong \Gamma_{2 g}\left(\chi_{c 2}\right)=\Gamma_{2 \gamma}\left(\chi_{c 2}\right) \frac{9 \alpha_{s}^{2}\left(m_{c}\right)}{8 \alpha^{2}} \frac{1-\frac{2.2 \alpha_{s}\left(m_{c}\right)}{\pi}}{1-\frac{16 \alpha_{s}\left(m_{c}\right)}{3 \pi}}$.
For $2^{++}$bottomonium states, according to our estimate of mass spectra, there are two states which are below the threshold of $B \bar{B}$, and we can predict their full decay widths using their two gluon decay widths. For $\chi_{b 2}(1 P)$, we have:
$\Gamma_{\text {tot }}\left(\chi_{b 2}\right) \cong \Gamma_{2 g}\left(\chi_{b 2}\right)=\Gamma_{2 \gamma}\left(\chi_{b 2}\right) \frac{18 \alpha_{s}^{2}\left(m_{b}\right)}{\alpha^{2}} \frac{1-\frac{0.1 \alpha_{s}\left(m_{b}\right)}{\pi}}{1-\frac{16 \alpha_{s}\left(m_{b}\right)}{3 \pi}}$,
and for the first radial excited state $\chi_{b 2}^{\prime}(2 P)$, we have:
$\Gamma_{\text {tot }}\left(\chi_{b 2}^{\prime}\right) \cong \Gamma_{2 g}\left(\chi_{b 2}^{\prime}\right)=\Gamma_{2 \gamma}\left(\chi_{b 2}^{\prime}\right) \frac{18 \alpha_{s}^{2}\left(m_{b}\right)}{\alpha^{2}} \frac{1+\frac{\alpha_{s}\left(m_{b}\right)}{\pi}}{1-\frac{16 \alpha_{s}\left(m_{b}\right)}{3 \pi}}$,
where $\alpha=\frac{e^{2}}{4 \pi}$, and the QCD corrections are summarized in Ref. [22].

## 3. Numerical results and discussions

We will not show the details of Solving the full Salpeter equation, only give the final results, interested readers can find the detail technique in Ref. [23].

When solving the full Salpeter equation, we choose a phenomenological Cornell potential,
$V(\vec{q})=V_{s}(\vec{q})+\gamma_{0} \otimes \gamma^{0} V_{v}(\vec{q})$,
$V_{S}(\vec{q})=-\left(\frac{\lambda}{\alpha}+V_{0}\right) \delta^{3}(\vec{q})+\frac{\lambda}{\pi^{2}} \frac{1}{\left(\vec{q}^{2}+\alpha^{2}\right)^{2}}$,
$V_{v}(\vec{q})=-\frac{2}{3 \pi^{2}} \frac{\alpha_{s}(\vec{q})}{\left(\vec{q}^{2}+\alpha^{2}\right)}$,
and the coupling constant $\alpha_{s}(\vec{q})$ is running:
$\alpha_{S}(\vec{q})=\frac{12 \pi}{25} \frac{1}{\log \left(a+\frac{\vec{q}^{2}}{\Lambda_{\text {QCD }}^{2}}\right)}$
Here the constants $\lambda, \alpha, a, V_{0}$ and $\Lambda_{\mathrm{QCD}}$ are the parameters that characterize the potential. We found the following best-fit values of input parameters which were obtained by fitting the mass spectra for $2^{++} \chi_{c 2}: a=e=2.7183, \alpha=0.06 \mathrm{GeV}, V_{0}=-0.401 \mathrm{GeV}$, $\lambda=0.2 \mathrm{GeV}^{2}, \Lambda_{\mathrm{QCD}}=0.26 \mathrm{GeV}$ and $m_{c}=1.7553 \mathrm{GeV}$. With this parameter set, we solve the full Salpeter equation and obtain the mass spectra shown in Table 1. To give the numerical value, we need to fix the value of the renormalization scale $\mu$ in $\alpha_{s}(\mu)$. In the case of charmonium, we choose the charm quark mass $m_{c}$ as the energy scale and obtain the coupling constant $\alpha_{s}\left(m_{c}\right)=0.36$ [23].

Since the wave function Eq. (3) is general wave function for $2^{+}$ state, and either ${ }^{3} P_{2}$ and ${ }^{3} F_{2}$ can be $2^{+}$state, so the obtained

Table 1
Two-photon decay width and total width of $P$-wave $2^{++}$charmonium states, where the data of $\chi_{c 2}(1 P)$ is come from PDG [25], and data of $\chi_{c 2}^{\prime}(2 P)$ is come from Ref. [26].

|  | Ex mass $(\mathrm{MeV})$ | Th mass $(\mathrm{MeV})$ | $\Gamma_{2 \gamma}(\mathrm{eV})$ |
| :--- | :--- | :--- | :---: |
| $\chi_{c 2}(1 P)$ | 3556.20 | 3556.6 | 501 |
| $\chi_{c 2}^{\prime}(2 P)$ | 3931 | 3967.0 | 534 |
| $\chi_{c 2}^{\prime}(1 F)$ |  | 4040.5 | 92.4 |
| $\chi_{c 2}^{\prime \prime}(3 P)$ | 4264.6 | 509 |  |
| $\chi_{c 2}^{\prime \prime}(2 F)$ | 4314.3 | 54.1 |  |
| $\chi_{c 2}^{\prime \prime \prime}(4 P)$ |  | 4498.7 | 475 |

Table 2
Two-photon decay width and total width of $P$-wave $2^{++}$bottomonium states.

|  | Ex mass $(\mathrm{MeV})$ | Th mass $(\mathrm{MeV})$ | $\Gamma_{2 \gamma}(\mathrm{eV})$ |
| :--- | :---: | :---: | :--- |
| $\chi_{b 2}(1 P)$ | 9912.21 | 9912.2 | 7.43 |
| $\chi_{b 2}^{\prime}(2 P)$ | 10268.65 | 10283 | 7.69 |
| $\chi_{b 2}^{\prime 2}(F)$ |  | 10364 | 1.76 |
| $\chi_{b 2}^{\prime \prime}(3 P)$ | 10561 | 7.19 |  |
| $\chi_{b 2}^{\prime \prime}(2 F)$ | 10616 | 1.43 |  |
| $\chi_{b 2}^{\prime \prime}(4 P)$ |  | 10786 | 6.60 |

states of $\chi_{c 2}^{\prime}(1 F)$ and $\chi_{c 2}^{\prime \prime}(2 F)$ are not pure $F$ wave, they are $P-F$ mixing state, but a $F$ wave dominate state, and $\chi_{b 2}^{\prime}(1 F), \chi_{b 2}^{\prime \prime}(2 F)$ in Table 2 are also $P-F$ mixing, but $F$ wave dominate state, we will discuss the detail of mixing in other paper [24].

Our prediction of the mass for $\chi_{c 2}^{\prime}(2 P)$ is 3967.0 MeV , which is a little larger than the first observation by BELLE Collaboration, their data is 3931 MeV . And our prediction of the first $F$ wave dominate state, we list as $\chi_{c 2}^{\prime}(1 F)$, whose mass is 4040.5 MeV , and the second one, $\chi_{c 2}^{\prime \prime}(2 F)$, whose mass is 4314.3 MeV .

With the obtained wave function and Eq. (3), we calculate the two-photon decay width of $c \bar{c} 2^{++}$states, the results are also shown in Table 1. Not similar to the $S$ wave case, where the two photon decay width of ground state $\Gamma\left(\eta_{c} \rightarrow 2 \gamma\right)$ is much larger then the first radial excited state $\Gamma\left(\eta_{c}^{\prime} \rightarrow 2 \gamma\right)$, almost twice [15], the ground state decay width is a little smaller than the first radial excited state, and from the table we obtain the conclusion that the decay widths with successive radial excitation fall very slowly. The decay width of the $F$ wave dominate state, $\Gamma\left(\chi_{c 2}^{\prime}(1 F) \rightarrow 2 \gamma\right)=$ 92 eV and $\Gamma\left(\chi_{c 2}^{\prime \prime}(2 F) \rightarrow 2 \gamma\right)=54 \mathrm{eV}$ are much smaller than the $P$ wave dominate state.

For the case of $b \bar{b}$, our best fitting parameters are $V_{0}=$ $-0.459 \mathrm{GeV}, \Lambda_{\mathrm{QCD}}=0.20$ and $m_{b}=5.13 \mathrm{GeV}$, other parameters are same as in the case of $c \bar{c}$. With this set of parameters, the coupling constant at scale of bottom quark mass is $\alpha_{s}\left(m_{b}\right)=0.232$. The corresponding mass spectra, two-photon decay widths are shown in Table 2. Our mass prediction of $\chi_{b 2}^{\prime}(2 P)$ is 10283 MeV , a little higher than the data. And the two photon decay widths are much smaller than the case of charmonium, their value is only about several eV.

Our predictions of the total decay width for the ground state:
$\Gamma_{\text {tot }}\left(\chi_{c 2}\right) \cong \Gamma_{2 g}\left(\chi_{c 2}\right)=2.64 \mathrm{MeV}$,
is little larger then the PDG data $\Gamma_{\text {tot }}\left(\chi_{c 2}\right)=2.03 \pm 0.12 \mathrm{MeV}$. For $2^{++}$bottomonium states, we have no data in hand, our theoretical predictions are:

$$
\Gamma_{\text {tot }}\left(\chi_{b 2}\right) \cong \Gamma_{2 g}\left(\chi_{b 2}\right)=0.220 \mathrm{MeV}
$$

and
$\Gamma_{\text {tot }}\left(\chi_{b 2}^{\prime}\right) \cong \Gamma_{2 g}\left(\chi_{b 2}^{\prime}\right)=0.248 \mathrm{MeV}$.
We compare our predictions with recent other theoretical relativistic calculations and experimental results in Table 3. Except the total decay width of $\Gamma_{\text {tot }}\left(\chi_{c 2}\right)$, all the values of listed in the table consist with each other.

In summary, by solving the relativistic full Salpeter equation with a well defined form of wave function, we estimate twophoton decay rates: $\Gamma\left(\chi_{c 2} \rightarrow 2 \gamma\right)=501 \mathrm{eV}, \Gamma\left(\chi_{c 2}^{\prime} \rightarrow 2 \gamma\right)=$ $534 \mathrm{eV}, \quad \Gamma\left(\chi_{b 2} \rightarrow 2 \gamma\right)=7.4 \mathrm{eV}$ and $\Gamma\left(\chi_{b 2}^{\prime} \rightarrow 2 \gamma\right)=7.7 \mathrm{eV}$, and the total decay widths: $\Gamma_{\text {tot }}\left(\chi_{c 2}\right)=2.64 \mathrm{MeV}, \Gamma_{\text {tot }}\left(\chi_{b 2}\right)=$ 0.220 MeV and $\Gamma_{\text {tot }}\left(\chi_{b 2}^{\prime}\right)=0.248 \mathrm{MeV}$.

Table 3
Recent theoretical and experimental results of two-photon decay width and total width.

|  | $\Gamma_{2 \gamma}^{\chi} \chi_{\text {c2 }}(\mathrm{keV})$ | $\Gamma_{\text {tot }}^{\chi}{ }_{\text {c }}(\mathrm{MeV})$ | $\Gamma_{2 \gamma}^{\chi_{c 2}^{\prime}}(\mathrm{keV})$ | $\Gamma_{2 \gamma}^{\chi_{02}}(\mathrm{eV})$ | $\Gamma_{\text {tot }}^{\chi}(\mathrm{MeV})$ | $\Gamma_{2 \gamma}^{\chi_{b 2}^{\prime}}(\mathrm{eV})$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Ours | 0.50 | 2.64 | 0.53 | 7.4 | 0.22 | 7.7 |
| Gupta [6] | 0.57 | 1.20 |  | 8 | 0.22 |  |
| Ebert [8] | 0.50 |  | 0.52 | 8 |  | 6 |
| Münz [9] | $0.44 \pm 0.14$ |  | $0.48 \pm 0.16$ | $5.6 \pm 0.6$ |  | $6.8 \pm 1.0$ |
| CLEO [27] | $0.53 \pm 0.15 \pm 0.06 \pm 0.22$ |  |  |  |  |  |
| CLEO [28] | $0.56 \pm 0.06 \pm 0.05 \pm 0.04$ |  |  |  |  |  |
| CLEO [29] | $0.60 \pm 0.06 \pm 0.06$ |  |  |  |  |  |
| PDG [25] | 0.493 | $2.03 \pm 0.12$ |  |  |  |  |

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