Physics Letters B 714 (2012) 1–5

Contents lists available at SciVerse ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Constraints on $f(R)$ cosmologies from strong gravitational lensing systems

Kai Liao, Zong-Hong Zhu *

Department of Astronomy, Beijing Normal University, Beijing 100875, China

ARTICLE INFO

Article history:
Received 9 November 2011
Received in revised form 7 March 2012
Accepted 25 June 2012
Available online 28 June 2012
Editor: S. Dodelson

Keywords:
$f(R)$ gravity
Strong lensing
Cosmological constraints

ABSTRACT

$f(R)$ gravity is thought to be an alternative to dark energy which can explain the acceleration of the universe. It has been tested by different observations including type Ia supernovae (SNIa), the cosmic microwave background (CMB), the baryon acoustic oscillations (BAO) and so on. In this Letter, we use the Hubble constant independent ratio between two angular diameter distances $D = D_{ls}/D_s$ to constrain $f(R)$ model in Palatini approach $f(R) = R - \alpha H_0^2(-\frac{R}{H_0^2})^\beta$. These data are from various large systematic lensing surveys and lensing by galaxy clusters combined with X-ray observations. We also combine the lensing data with CMB and BAO, which gives a stringent constraint. The best-fit results are $(\alpha, \beta) = (-1.50, 0.696)$ or $(\Omega_m, \beta) = (0.0734, 0.696)$ using lensing data only. When combined with CMB and BAO, the best-fit results are $(\alpha, \beta) = (-3.75, 0.0651)$ or $(\Omega_m, \beta) = (0.286, 0.0651)$. If we further fix $\beta = 0$ (corresponding to ΛCDM), the best-fit value for α is $\alpha = -4.84^{+0.91}_{-0.68}(1\sigma)^{+1.63}_{-0.98}(2\sigma)$ for the lensing analysis and $\alpha = -4.35^{+0.18}_{-0.16}(1\sigma)^{+0.3}_{-0.25}(2\sigma)$ for the combined data, respectively. Our results show that ΛCDM model is within 1σ range.

1. Introduction

One of the most striking things in modern cosmology is the universe undergoing an accelerated state [1]. In order to explain this phenomenon, people have introduced new component which is known as dark energy. The simplest model is cosmological constant (ΛCDM). It is consist with all kinds of observations while it indeed encounters the coincidence problem and the "fine-tuning" problem. Besides, there are many other dark energy models including holographic dark energy [2], quintessence [3], quintom [4], phantom [5], generalized Chaplygin gas [6] and so on. Besides dark energy, the acceleration can be explained in other ways. If the new component with negative pressure does not exist, General Relativity (GR) should be modified. Until now, at least two effective theories have been proposed. One is considering the extra dimensions which is related to the brane-world cosmology [7]. The other is the so-called $f(R)$ gravity [8]. It changes the form of Einstein–Hilbert Lagrangian by $f(R)$ expression. These theories can give an acceleration solution naturally without introducing dark energy. There are two kinds of forms about the $f(R)$, the metric and the Palatini formalisms [9]. They give different dynamical equations. They can be unified only in the case of linear action (GR). For the Palatini approach, the form $f(R) = R - \alpha H_0^2(-\frac{R}{H_0^2})^\beta$ is chosen so that it can result in the radiation-dominated, matter-dominated and recent accelerating state. Furthermore, it can pass the solar system and has the correct Newtonian limit [10]. In this Letter, we consider the Palatini formalisms. Under this assumption, the $f(R)$ cosmology has two parameters. What we want to emphasize is, among the parameters $(\alpha, \beta, \Omega_m)$, only two of them are independent. Therefore, we can exhibit the constraint results on either (α, β) space or (Ω_m, β) space. Various observations have already been used to constrain $f(R)$ gravity including SNIa, CMB, BAO, Hubble parameter ($H(z)$) and so on. Among these works, parameter β has been constrained to very small value. In these papers [15], they get $\beta \sim 10^{-1}$; in [16], the matter power spectrum from the SDSS gives $\beta \sim 10^{-5}$; in [17], the β was constrained to $\sim 10^{-6}$. From these results, the $f(R)$ gravity seems hard to be distinguished from the standard theory, where $\beta = 0$. One effective way to solve this problem in astronomy is combining different cosmological probes. Strong lensing has been used to study both cosmology [18] and galaxies including their structure, formation and evolution [19]. The observations of the images combined with lens models can give us the information about the ratio between two angular diameter distances, D_{ls} and D_s. The former one is the distance between lens and source, the latter one is the distance from observer to the source. Because the angular diameter distance depends on cosmology, the D_{ls}/D_s data can be used to constrain the parameters in $f(R)$ gravity. In this Letter, we select 63 strong lensing systems from SLACS and LSD surveys assuming the singular isothermal sphere (SIS) model or the singular isothermal ellipsoid

* Corresponding author.
E-mail address: zhuzh@bnu.edu.cn (Z.-H. Zhu).

0370-2693 © 2012 Elsevier B.V. Open access under CC BY license.
http://dx.doi.org/10.1016/j.physletb.2012.06.062

(SIE) model is right. Moreover, a sample of 10 giant arcs is also contained. Using these 73 data, we try to give a new approach to constraining $f(R)$ gravity.

This Letter is organized as follows. In Section 2, we briefly describe the basic theory about $f(R)$ gravity and the corresponding cosmology. In Section 3, we introduce the lensing data we use, the CMB data and the BAO data. The constraint results are performed in Section 4. At last, we give a summary in Section 5. Throughout this work, the unit with light velocity $c = 1$ is used.

2. The $f(R)$ gravity and cosmology

The basic theory of $f(R)$ gravity has been discussed thoroughly in history. For details, see Ref. [8]. In Palatini approach, the action is given by

$$S = -\frac{1}{2\kappa}\int d^4x\sqrt{-g}f(R) + S_m, \tag{1}$$

where $\kappa = 8\pi G$, G is the gravitational constant and S_m is the usual action for the matter. The Ricci scalar depends on the metric and the affine connection:

$$R = g^{\mu\nu}\hat{R}_{\mu\nu}, \tag{2}$$

where the generalized Ricci tensor

$$\hat{R}_{\mu\nu} = \hat{\Gamma}^{\alpha}_{\mu\nu,\alpha} - \hat{\Gamma}^{\alpha}_{\mu\alpha,\nu} + \hat{\Gamma}^{\alpha}_{\alpha\lambda}\hat{\Gamma}^{\lambda}_{\mu\nu} - \hat{\Gamma}^{\alpha}_{\mu\lambda}\hat{\Gamma}^{\lambda}_{\alpha\nu}. \tag{3}$$

The hat represents the affine term which is different from the Levi-Civita connection. The Ricci scalar is always negative. By varying the action with respect to the metric components, we can get the generalized Einstein field equations:

$$f'(R)\hat{R}_{\mu\nu} - \frac{1}{2}g_{\mu\nu}f(R) = -\kappa T_{\mu\nu}, \tag{4}$$

where $f'(R) = df/dR$ and $T_{\mu\nu}$ is the matter energy–momentum tensor. For a perfect fluid, $T_{\mu\nu} = (\rho_m + p_m)u_\mu u_\nu + p_m g_{\mu\nu}$, where ρ_m is the energy density, p_m is the pressure and u_μ is the four-velocity. Varying the action with respect to the connection gives the equation

$$\hat{\nabla}_\alpha\left[f'(R)\sqrt{-g}g^{\mu\nu}\right] = 0. \tag{5}$$

From this equation, we can obtain a conformal metric $\gamma_{\mu\nu} = f'(R)g_{\mu\nu}$ which is corresponding to the affine connection. The generalized Ricci tensor can be related to the Ricci tensor

$$\hat{R}_{\mu\nu} = R_{\mu\nu} - \frac{3}{2}\frac{\nabla_\mu f'\nabla_\nu f'}{f'^2} + \frac{\nabla_\mu\nabla_\nu f'}{f'} + \frac{1}{2}g_{\mu\nu}\frac{\nabla^\mu\nabla_\mu f'}{f'}. \tag{6}$$

In the next, we will introduce the dynamical equations of $f(R)$ cosmology. Since all kinds of observations support a flat universe, we assume a flat FRW cosmology. The FRW metric is

$$ds^2 = -dt^2 + a(t)^2\delta_{ij}\,dx^i\,dx^j, \tag{7}$$

where the scale factor $a = (1+z)^{-1}$, z is the redshift. We choose $a_0 = 1$, the subscript "0" represents the quantity today. From Eq. (6), we can obtain the generalized Friedmann equation

$$6\left(H + \frac{1}{2}\frac{\dot{f}'}{f'}\right)^2 = \frac{\kappa(\rho + 3p)}{f'} - \frac{f}{f'}, \tag{8}$$

where the overdot denotes a time derivative. The trace of Eq. (4) can gives

$$Rf'(R) - 2f(R) = -\kappa T. \tag{9}$$

Considering the equation of state of matter is zero, Eq. (9) can give the relation between matter density and redshift

$$(1+z)^{-1} = (\kappa\rho_{m0})^{\frac{1}{3}}\left(Rf' - 2f\right)^{-\frac{1}{3}}. \tag{10}$$

Also, considering the energy conservation equation, Eq. (9) can give

$$\dot{R} = -\frac{3H\rho_M}{Rf''(R) - f'(R)}. \tag{11}$$

According to Eqs. (9), (11) and (8), we can get the Hubble quantity in term of R

$$H^2(R) = \frac{1}{6f'}\frac{Rf' - 3f}{(1 - \frac{3}{2}\frac{f''(Rf'-2f)}{f'(Rf''-f')})^2}. \tag{12}$$

This is the Friedmann equation in $f(R)$ cosmology. For each R, we can get the redshift corresponding to that time. The angular diameter distance between redshifts z_1 and z_2 is

$$\begin{aligned} D^A(z_1,z_2) &= \frac{1}{1+z_2}\int_{z_1}^{z_2}\frac{dz}{H(z)} \\ &= \frac{1}{3}\left(Rf' - 2f\right)^{-\frac{1}{3}}\int_{R_{z_1}}^{R_{z_2}}\frac{Rf'' - f'}{(Rf' - 2f)^{\frac{2}{3}}}\frac{dR}{H(R)} \\ &= D^A(R_1,R_2). \end{aligned} \tag{13}$$

The D_{ls}/D_s is given by

$$D_{ls}/D_s(z_1,z_2) = \frac{\int_{R_{z_1}}^{R_{z_2}}\frac{Rf''-f'}{(Rf'-2f)^{\frac{2}{3}}}\frac{dR}{H(R)}}{\int_{R_0}^{R_{z_2}}\frac{Rf''-f'}{(Rf'-2f)^{\frac{2}{3}}}\frac{dR}{H(R)}}. \tag{14}$$

3. Data and analysis methods

In this section, we introduce the data we use, the lensing data, CMB and BAO. These data are independent of the Hubble constant.

3.1. The D_{ls}/D_s data

Similar to Ref. [26], our data set consists of two parts. Firstly, we choose 63 strong lensing systems from SLACS and LSD surveys [20]. These systems have been measured the central dispersions with spectroscopic method. Though some of the lensing systems have 4 images, we assume the SIS or the SIE model is correct. The Einstein radius can be obtained under this assumption

$$\theta_E = 4\pi\frac{D_A(z,z_s)}{D_A(0,z_s)}\frac{\sigma^2_{SIS}}{c^2}. \tag{15}$$

It is related to the angular diameter distance ratio and stellar velocity dispersion σ_{SIS} or the central velocity dispersion σ_0 which can be obtained from spectroscopy. Secondly, the galaxy clusters can produce giant arcs, a sample of 10 galaxy clusters with redshift ranging from 0.1 to 0.6 is used under the β model [21]. Now, we have a sample of 73 strong lensing systems. There are listed in Table 2. We can fit the $f(R)$ cosmology by minimizing the χ^2 function

$$\chi^2(\mathbf{p}) = \sum_i\frac{(\mathcal{D}^{th}_i(\mathbf{p}) - \mathcal{D}^{obs}_i)^2}{\sigma^2_{\mathcal{D},i}}. \tag{16}$$

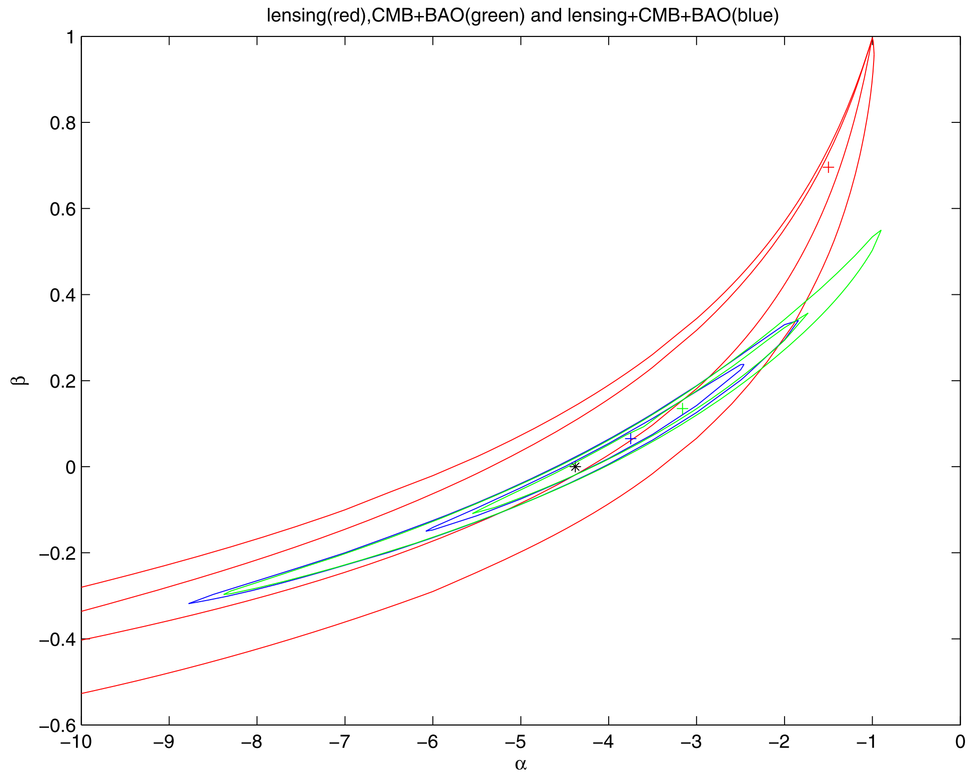

Fig. 1. The 1σ and 2σ contours for (α, β) parameter space arising from the D_{ls}/D_s data (red line), CMB + BAO (green line) and D_{ls}/D_s data + CMB + BAO (blue line). We have considered the parameter space that is not allowed. The black star represents the ΛCDM model ($\alpha = -4.38$, $\beta = 0$). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this Letter.)

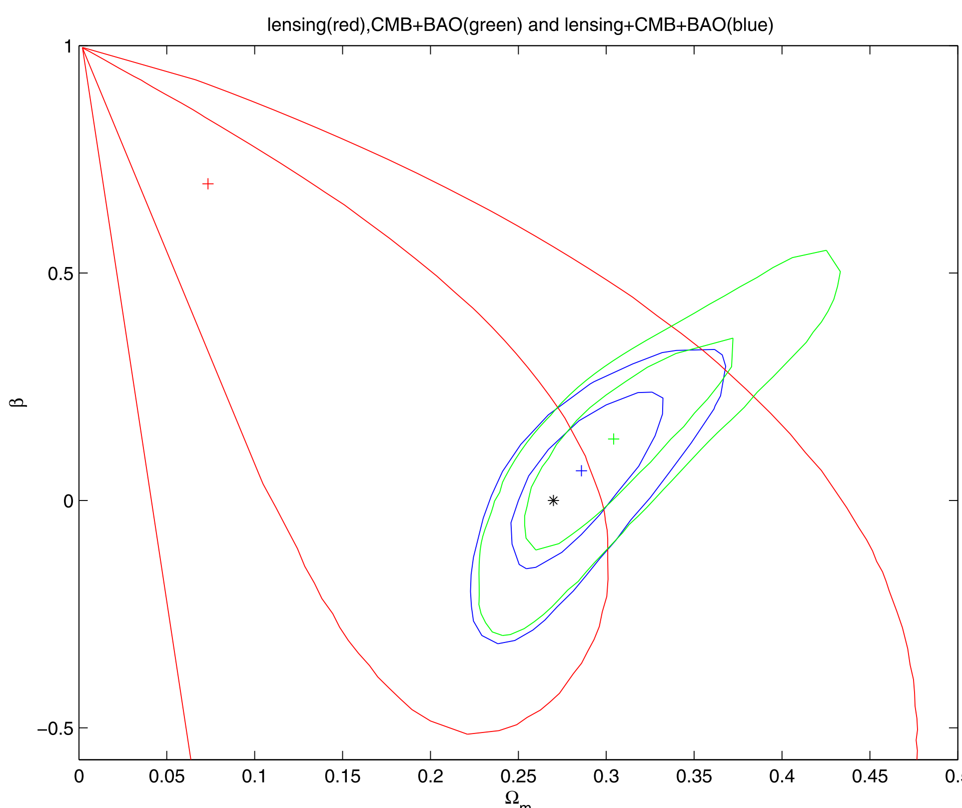

Fig. 2. The 1σ and 2σ contours for (Ω_m, β) parameter space arising from the D_{ls}/D_s data (red line), CMB + BAO (green line) and D_{ls}/D_s data + CMB + BAO (blue line). We have considered the parameter space that is not allowed. The black star represents the ΛCDM model ($\Omega_m = 0.27$, $\beta = 0$). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this Letter.)

Table 1
Best-fit values for α and β (the ΛCDM model corresponds to $\alpha = -4.38$ and $\beta = 0$).

Test	Ref.	α	β
SNe Ia (SNLS)	[12]	-12.5	-0.6
SNe Ia (SNLS) + BAO + CMB	[12]	-4.63	-0.027
SNe Ia (Gold)	[11]	-10	-0.51
SNe Ia (Gold) + BAO + CMB	[11]	-3.6	0.09
BAO	[11]	-1.1	0.57
CMB	[11]	-8.4	-0.27
SNe Ia (Union)	[14]	–	-0.99
SNe Ia (Union) + BAO + CMB	[14]	-3.45	0.12
LSS	[13]	–	-2.6
$H(z)$	[25]	-1.11	0.9
$H(z)$ + BAO + CMB	[25]	-4.7	-0.03
Lensing(D_{ls}/D_s)	This Letter	$-1.50^{+0.52}_{-12.0}$	$0.696^{+0.262}_{-1.21}$
BAO + CMB	This Letter	$-3.16^{+1.43}_{-2.39}$	$0.135^{+0.222}_{-0.244}$
Lensing(D_{ls}/D_s) + BAO + CMB	This Letter	$-3.75^{+1.29}_{-2.33}$	$0.0651^{+0.1729}_{-0.2151}$

3.2. *Cosmic microwave background and baryon acoustic oscillation*

For CMB, the shift parameter $\mathcal{R}$ is an important quantity which depends on the cosmology [22]. In $f(R)$ cosmology, it can be expressed as

$$\begin{aligned}\mathcal{R} &= \sqrt{\Omega_m H_0^2}\int_0^{z_{dec}}\frac{dz}{H(z)}\\ &= \sqrt{\Omega_m H_0^2}\int_{R_{dec}}^{R_0}\frac{a'(R)}{a(R)^2}\frac{dR}{H(R)}\\ &= \frac{1}{3^{4/3}}\left(\Omega_m H_0^2\right)^{1/6}\int_{R_0}^{R_{dec}}\frac{Rf''-f'}{(Rf'-2f)^{2/3}}\frac{dR}{H(R)},\end{aligned} \tag{17}$$

where $z_{dec} = 1091.3$ is the redshift of the recombination epoch. The 7-year WMAP gives the value $\mathcal{R} = 1.725 \pm 0.018$ [23]. The χ^2 can be defined as

$$\chi^2_{CMB} = \frac{(\mathcal{R}-1.725)^2}{0.018^2}. \tag{18}$$

For BAO, we take the A parameter which is expressed as [24]

$$A = \sqrt{\Omega_m}E(z_{BAO})^{-1/3}\left[\frac{1}{z_{BAO}}\int_0^{z_{BAO}}\frac{dz}{E(z)}\right]^{2/3}, \tag{19}$$

where $E(z) = H(z)/H_0$. The SDSS BAO measurement gives $A_{obs} = 0.469(n_s/0.98)^{-0.35} \pm 0.017$, where the scalar spectral index is taken to be $n_s = 0.963$ as measured by WMAP7 [23]. The χ^2 for BAO can be defined as

$$\chi^2_{BAO} = \frac{(A-A_{obs})^2}{\sigma_A^2}. \tag{20}$$

4. The constraint results

In the Friedmann equation (12), we can find the Ricci scalar R is always divided by H_0^2, so we can choose units so that $H_0 = 1$. For given (α, β), we can get the Ricci scalar today R_0 using the Friedmann equation. Then we can get Ω_m through Eq. (10). Now, we can get the relation between the Ricci scalar and the redshift through Eq. (10). We use the 73 D_{ls}/D_s data to constrain $f(R)$ gravity in Palatini approach. Fist, we show the (α, β) parameter space in Fig. 1. We can see the D_{ls}/D_s data is com-

Table 2
We select 73 observations where the $D_{ls}/D_s < 1$ from Ref. [26].

Cluster/galaxy	z_s	z_l	$\mathcal{D}^{obs}$	$\sigma_{\mathcal{D}}$
MS 0451.6−0305	2.91	0.550	0.785	0.087
3C220.1	1.49	0.61	0.611	0.530
CL0024.0	1.675	0.391	0.919	0.430
Abell 2390	4.05	0.228	0.737	0.053
Abell 2667	1.034	0.226	0.837	0.124
Abell 68	1.6	0.255	0.982	0.225
MS 1512.4	2.72	0.372	0.734	0.330
MS 2137.3−2353	1.501	0.313	0.778	0.105
MS 2053.7	3.146	0.583	0.968	0.209
PKS 0745−191	0.433	0.103	0.818	0.065
SDSS J0037−0942	0.6322	0.1955	0.6418	0.0501
SDSS J0216−0813	0.5235	0.3317	0.3278	0.0451
SDSS J0737+3216	0.5812	0.3223	0.3365	0.033
SDSS J0912+0029	0.324	0.1642	0.5293	0.0391
SDSS J0956+5100	0.47	0.2405	0.4532	0.0485
SDSS J0959+0410	0.5349	0.126	0.6621	0.0752
SDSS J1250+0523	0.795	0.2318	0.5319	0.0582
SDSS J1330−0148	0.7115	0.0808	0.7762	0.0796
SDSS J1402+6321	0.4814	0.2046	0.5739	0.0633
SDSS J1420+6019	0.5352	0.0629	0.851	0.0413
SDSS J1627−0053	0.5241	0.2076	0.4828	0.0426
SDSS J1630+4520	0.7933	0.2479	0.8074	0.0984
SDSS J2300+0022	0.4635	0.2285	0.4666	0.0581
SDSS J2303+1422	0.517	0.1553	0.7754	0.0916
SDSS J2321−0939	0.5324	0.0819	0.9082	0.0519
Q0047−2808	3.595	0.485	0.8872	0.1162
CFRS03−1077	2.941	0.938	0.6834	0.1035
HST 14176	3.399	0.81	0.9757	0.1307
HST 15433	2.092	0.497	0.929	0.1602
MG 2016	3.263	1.004	0.5035	0.0982
SDSS J0029−0055	0.9313	0.227	0.6356	0.0999
SDSS J0044+0113	0.1965	0.1196	0.3877	0.0379
SDSS J0109+1500	0.5248	0.2939	0.3803	0.0576
SDSS J0330−0020	1.0709	0.3507	0.8498	0.1684
SDSS J0728+3835	0.6877	0.2058	0.9477	0.0974
SDSS J0822+2652	0.5941	0.2414	0.6056	0.0701
SDSS J0841+3824	0.6567	0.1159	0.9671	0.0946
SDSS J0935−0003	0.467	0.3475	0.1926	0.0341
SDSS J0936+0913	0.588	0.1897	0.6409	0.0633
SDSS J0946+1006	0.6085	0.2219	0.6927	0.1106
SDSS J0955+0101	0.3159	0.1109	0.8571	0.1161
SDSS J0959+4416	0.5315	0.2369	0.5599	0.0872
SDSS J1016+3859	0.4394	0.1679	0.6204	0.0653
SDSS J1020+1122	0.553	0.2822	0.524	0.0669
SDSS J1023+4230	0.696	0.1912	0.836	0.1036
SDSS J1029+0420	0.6154	0.1045	0.7952	0.0833
SDSS J1032+5322	0.329	0.1334	0.4082	0.0414
SDSS J1103+5322	0.7353	0.1582	0.9219	0.1129
SDSS J1106+5228	0.4069	0.0955	0.6222	0.0617
SDSS J1112+0826	0.6295	0.273	0.5052	0.0632
SDSS J1134+6027	0.4742	0.1528	0.6687	0.0671
SDSS J1142+1001	0.5039	0.2218	0.6967	0.1387
SDSS J1143−0144	0.4019	0.106	0.8061	0.0779
SDSS J1153+4612	0.8751	0.1797	0.7138	0.0948
SDSS J1204+0358	0.6307	0.1644	0.6381	0.0813
SDSS J1205+4910	0.4808	0.215	0.5365	0.0535
SDSS J1213+6708	0.6402	0.1229	0.5783	0.0594
SDSS J1403+0006	0.473	0.1888	0.6352	0.1014
SDSS J1416+5136	0.8111	0.2987	0.8259	0.1721
SDSS J1430+4105	0.5753	0.285	0.509	0.1012
SDSS J1436−0000	0.8049	0.2852	0.775	0.1176
SDSS J1443+0304	0.4187	0.1338	0.6439	0.0678
SDSS J1451−0239	0.5203	0.1254	0.7262	0.0912
SDSS J1525+3327	0.7173	0.3583	0.6526	0.1285
SDSS J1531−0105	0.7439	0.1596	0.7628	0.0766
SDSS J1538+5817	0.5312	0.1428	0.972	0.1234
SDSS J1621+3931	0.6021	0.2449	0.8042	0.1363
SDSS J1636+4707	0.6745	0.2282	0.7093	0.0921
PG1115+080	1.72	0.31	0.7036	0.1252
MG1549+3047	1.17	0.11	0.5728	0.0908
Q2237+030	1.169	0.04	0.6685	0.1866
CY2201−3201	3.9	0.32	0.8526	0.2624
B1608+656	1.39	0.63	0.646	0.1831

patible with the $H(z)$ data [25]. The best-fit values are $(\alpha, \beta) = (-1.50, 0.696)$. Using the D_{ls}/D_s data only cannot give a stringent constraint. After adding the CMB and the BAO data, the parameters are tightly constrained. The best-fit values are $(\alpha, \beta) = (-3.75, 0.0651)$. What we want to emphasis is the Hubble parameter should always be positive, which restricts the parameters further. We also exhibit the (Ω_m, β) parameter space in Fig. 2. The best-fit values are $(\Omega_m, \beta) = (0.0734, 0.696)$ for D_{ls}/D_s data and $(\Omega_m, \beta) = (0.286, 0.0651)$ for combination with CMB and BAO. Moreover, if we further fix $\beta = 0$, the best-fit value for α is $\alpha = -4.84^{+0.91}_{-0.68}(1\sigma)^{+1.63}_{-0.98}(2\sigma)$ for lensing data and $\alpha = -4.35^{+0.18}_{-0.16}(1\sigma)^{+0.3}_{-0.25}(2\sigma)$ for combined data respectively. From the results above, we can see the ΛCDM model which is corresponding to $(\alpha = -4.38, \beta = 0)$ or $(\Omega_m = 0.27, \beta = 0)$ is within $i\sigma$ range. In order to compare the D_{ls}/D_s data, we list some constraint results from other cosmological observations in Table 1.

5. Conclusion

In this Letter, we use D_{ls}/D_s data from lensing systems to constrain $f(R)$ gravity in Palatini approach $f(R) = R - \alpha H_0^2(-\frac{R}{H_0^2})^\beta$. Compared with references, we can see the constraint effects that D_{ls}/D_s data give can be compatible with other data (SNe Ia, $H(z)$, BAO, CMB and so on). Moreover, we find although the best-fit values of the parameters are different from various observations, the directions of the contours in (α, β) space are very similar, thus needing different observations to break the degeneracy. The D_{ls}/D_s data propose a new way to probe the cosmology [27]. As we expect, the lensing data alone cannot give a stringent constraint. There are at least three aspects that contribute to the error. First, the assumption that the lens galaxies satisfy SIS or SIE model may have some issues especially for four images. Second, the measurements of velocity dispersions have some uncertainties. Finally, the error exists due to the influence of line of sight mass contamination [28]. Combining with CMB and BAO, it gives $\beta \sim 10^{-1}$, which contains the ΛCDM model. Until now, we cannot distinguish it from the standard cosmology, where $\beta = 0$. For future lensing study, in order to improve the constraint, we hope large survey projects can find more strong lensing systems. At the same time, a better understand about the lens model and more precise measurements can give us more stringent results and more information about $f(R)$ gravity.

Acknowledgements

This work was supported by the National Natural Science Foundation of China under the Distinguished Young Scholar Grant 10825313, the Ministry of Science and Technology National Basic Science Program (Project 973) under Grant No. 2012CB821804, the Fundamental Research Funds for the Central Universities and Scientific Research Foundation of Beijing Normal University.

References

[1] A.G. Riess, et al., ApJ 501 (1998) 61;
S. Perlmutter, et al., ApJ 517 (1999) 565;
A.C. Pope, et al., ApJ 607 (2004) 655.
[2] A. Cohen, D. Kaplan, A. Nelson, PRL 82 (1999) 4971;
M. Li, PLB 603 (2004) 1.
[3] P.J.E. Peebles, B. Ratra, ApJL 325 (1988) 17;
P.J.E. Peebles, B. Ratra, PRD 37 (1988) 3406.
[4] Z.K. Guo, Y.S. Piao, Y.Z. Zhang, PLB 608 (2005) 177;
B. Feng, X. Wang, X. Zhang, PLB 607 (2005) 35.
[5] R. Caldwell, PLB 545 (2002) 23.
[6] M.C. Bento, O. Bertolami, A.A. Sen, PRD 66 (2002) 043507;
M.C. Bento, O. Bertolami, A.A. Sen, PRD 70 (2002) 083519.
[7] L. Randall, R. Sundrum, PRL 83 (1999) 3370;
J.S. Alcaniz, PRD 65 (2002) 123514;
C. Deffayet, et al., PRD 65 (2002) 044023;
V. Sahni, Y. Shtanov, JCAP 0311 (2003) 014.
[8] R. Kerner, Gen. Rel. Gra. 14 (1982) 453;
J.D. Barrow, S. Cotsakis, PLB 214 (1988) 515;
S.M. Carroll, et al., PRD 70 (2004) 043528;
S. Capozziello, et al., PRD 71 (2005) 043503;
S. Nojiri, S.D. Odintsov, Gen. Rel. Gra. 36 (2004) 1765.
[9] S. Capozziello, M. Francaviglia, Gen. Rel. Gra. 40 (2008) 357;
T.P. Sotiriou, V. Faraoni, Rev. Mod. Phys. 82 (2008) 451.
[10] T.P. Sotiriou, PRD 73 (2006) 063515.
[11] M. Amarzguioui, et al., A&A 454 (2006) 707.
[12] S. Fay, et al., PRD 75 (2007) 063509.
[13] T. Koivisto, PRD 76 (2007) 043527.
[14] J. Santos, et al., PLB 669 (2008) 14.
[15] T.P. Sotiriou, Gen. Rel. Gra. 23 (2006) 1253;
A. Browne, et al., PRD 74 (2006) 043502;
X.J. Yang, D.M. Chen, MNRAS 394 (2009) 1449.
[16] T. Koivisto, PRD 73 (2006) 083517.
[17] B. Li, J.D. Barrow, PRD 75 (2007) 084010.
[18] K.-H. Chae, MNRAS 346 (2003) 746;
K.-H. Chae, et al., ApJ 607 (2004) L71;
Z.-H. Zhu, et al., A&A 483 (2008) 15.
[19] Z.-H. Zhu, X.-P. Wu, A&A 324 (1997) 483;
E.O. Ofek, et al., MNRAS 343 (2003) 639.
[20] T. Treu, et al., ApJ 640 (2006) 662;
M. Biesiada, et al., MNRAS 406 (2010) 1055;
E. Newton, et al., ApJ 734 (2011) 104;
L.V.E. Koopmans, T. Treu, ApJ 568 (2002) L5;
L.V.E. Koopmans, T. Treu, ApJ 583 (2003) 606;
T. Treu, L.V.E. Koopmans, ApJ 611 (2004) 739.
[21] H. Yu, Z.-H. Zhu, RAA 11 (2011) 776;
N. Ota, K. Mitsuda, A&A 428 (2004) 757;
M. Bonamente, et al., ApJ 647 (2006) 25;
G. Covone, et al., A&A 456 (2006) 409;
J. Richard, et al., ApJ 662 (2007) 781.
[22] J.R. Bond, et al., MNRAS 291 (1997) L33.
[23] E. Komatsu, et al., ApJS 192 (2011) 18.
[24] D.J. Eisenstein, et al., ApJ 633 (2005) 560.
[25] F.C. Carvalho, et al., JCAP 0809 (2008) 008.
[26] S. Cao, et al., JCAP 3 (2012) 16.
[27] M. Biesiada, A. Piorkowska, B. Makec, MNRAS 406 (2010) 1055.
[28] N. Dalal, et al., ApJ 622 (2005) 99;
I. Momcheva, et al., ApJ 641 (2006) 169.