

Available online at [www.sciencedirect.com](http://www.sciencedirect.com)**ScienceDirect**

Procedia Technology 15 (2014) 832 – 840

**Procedia**  
Technology

2nd International Conference on System-Integrated Intelligence: Challenges for Product and Production Engineering

## Multi-objective optimization of titanium alloy through orthogonal array and grey relational analysis in WEDM

J.B. Saedon<sup>a\*</sup>, Norkamal Jaafar<sup>b</sup>, Mohd Azman Yahaya<sup>a</sup>  
NorHayati Saad<sup>a</sup> and Mohd Shahir Kasim<sup>c</sup>

<sup>a</sup>Faculty of Mechanical Engineering, Universiti Teknologi MARA, Malaysia.

<sup>b</sup>Department of Mechanical Engineering, Polytechnic Kota Kinabalu, Sabah, Malaysia

<sup>c</sup>Faculty of Manufacturing Engineering, Universiti Teknikal Malaysia, Malaysia

---

### Abstract

Wire electro discharge machining (WEDM) operations are dynamic in nature and process responses are related with simultaneous variation of parameters during experiments. To make the process reliable and to study the effects of the process parameters on different responses such as surface roughness (Ra), cutting rate and material removal rate (MRR), attempts have been made to carry out experiments considering the WEDM process parameters. Taguchi method, which is a design of experiments (DoE) method, has been applied to the present experiment followed by Grey Relational Analysis. The signals to noise (S/N) ratio plots are analyzed to study the effect of the process parameters. The adequacy of the above analysis has been tested through the implementation analysis of variance (ANOVA). Next, multi-objective optimization of the response analysis is performed. As the Taguchi methodology is unable to perform multi-objective optimization of the response characteristics, combined approach of orthogonal array and Grey Relational Analysis (GRA) are carried out. Optimal combination of process parameters has been proposed in the machining of titanium alloy in order to achieve minimum surface roughness (Ra), higher cutting rate (CR) and higher material removal rate (MRR). Finally, confirmation tests are carried out to verify the findings.

© 2014 Published by Elsevier Ltd. This is an open access article under the CC BY-NC-ND license (<http://creativecommons.org/licenses/by-nc-nd/3.0/>).

Peer-review under responsibility of the Organizing Committee of SysInt 2014.

*Keywords:* WEDM; titanium alloy; ANOVA; grey relational analysis; optimization

---

\* Corresponding author. Tel.: +0-06 03 5543 6285; fax: +0-06 03 5543 5161.

*E-mail address:* [juri41@yahoo.com](mailto:juri41@yahoo.com)

## 1. Introduction

Wire electro discharge machining (WEDM) is one of the applications of Electro Discharge Machining (EDM) known as non-traditional machining. This machining process has the ability to machine precisely a complex and irregular shape of difficult to machine electrically conductive component [1]. The WEDM process uses a thin wire as an electrode that transforms electrical energy to thermal energy for cutting materials. With this process, titanium alloy, steel alloy, conductive ceramics and aerospace materials can be machined irrespective of their hardness and toughness [2]. The movement of the wire is controlled numerically to achieve the desired three-dimensional shape and accuracy of the work piece [3]. The usage of titanium and its alloys is increasing in many industrial and commercial applications because of these materials' excellent properties such as a high strength–weight ratio, high temperature strength and exceptional corrosion resistance. These types of alloys are extensively used in aerospace, biomedical applications and in many corrosive environments. However, titanium and its alloys are difficult to machine owing to several inherent properties of the material. Titanium alloy (TI-6AL-4V) is the most popular material among titanium alloy group. This material is widely used especially in aerospace and marine applications which are classified as difficult to machine material by conventional method due to high cutting temperature, rapid tool wear and hard metal [4].

Material removal process in WEDM is the result of spark erosion by transformation of electrical energy to thermal energy as the wire electrode is fed through the workpiece. It was found that the machining parameters such as the pulse on/off duration, peak current, open circuit voltage, servo reference voltage, electrical capacitance and table speed are the critical parameters for the estimation of the cutting rate (CR), material removal rate (MRR) and surface roughness (Ra) [5]. For the optimal selection of process parameters, the Taguchi method has been extensively used in manufacturing in order to improve processes with single performance characteristics. However, traditional Taguchi method cannot solve multi-objective optimization. Thus, the Taguchi method coupled with Grey relational analysis can assist this problem as suggested by Deng [6]. Grey Relational Analysis (GRA) is a normalization evaluation technique to solve a more complicated multi-performance characteristics optimization effectively [7]. In the Taguchi analysis, the raw data than be transformed into S/N ratio values. The criteria for the cutting rate and the material removal rate are “higher-the-better” while for the surface roughness is “lower-the-better” which can be expressed as below;

$$\text{S/N ratio} = -10 \log_{10} (1/n) \sum_{i=1}^n 1/y_{ij}^2 \quad \text{“larger-the- better”} \quad (1)$$

$$\text{S/N ratio} = -10 \log_{10} (1/n) \sum_{i=1}^n y_{ij}^2 \quad \text{“lower-the -better”} \quad (2)$$

where;  $n$  = number of replication,  $y_{ij}$  = observed response value  
for  $i = 1,2,3,\dots,m$  and  $j = 1,2,3,\dots,n$

For optimization of multi-performance characteristics, grey relational analysis is introduced [7]. The analysis is based on linear normalization of the data in the range between zero and one, which is also called the grey relational generating. **The  $y_{ij}$  is normalized** as  $Z_{ij}$  ( $0 \leq Z_{ij} \leq 1$ ) by the following formula to avoid the effect of adopting different units and to reduce the variability. The normalized experimental results  $Z_{ij}$  can be expressed as:

$$Z_{ij} = ((y_{ij} \ i=1,2,3,\dots,n) - \min (y_{ij} \ i=1,2,3,\dots,n)) / ((\max y_{ij} \ i=1,2,3,\dots,n) - \min (y_{ij} \ i=1,2,3,\dots,n)) \quad (3)$$

$$Z_{ij} = (\max (y_{ij} \ i=1,2,3,\dots,n) - (y_{ij} \ i=1,2,3,\dots,n)) / ((\max y_{ij} \ i=1,2,3,\dots,n) - \min (y_{ij} \ i=1,2,3,\dots,n)) \quad (4)$$

The larger normalized results correspond to the better performance and the best-normalized results should be equal to one. Next, the grey relational coefficient is calculated to express the relationship between the ideal (best) and actual normalized experimental results. The grey relational coefficient  $Z_{ij}$  can be expressed as:

$$\gamma = (x_{0j}, x_{ij}) = \Delta_{\min} + \zeta \Delta_{\max} / (\Delta_{ij} + \zeta \Delta_{\max}) \quad (5)$$

for  $i = 1, 2, 3, \dots, m$  and  $j = 1, 2, 3, \dots, n$

$$\begin{aligned} \text{where, } \Delta_{ij} &= |x_{0j} - x_{ij}| \\ \Delta_{\min} &= \min \Delta_{ij}, \quad i = 1, 2, 3, \dots, m; \quad j = 1, 2, 3, \dots, n \\ \Delta_{\max} &= \max \Delta_{ij}, \quad i = 1, 2, 3, \dots, m; \quad j = 1, 2, 3, \dots, n \end{aligned}$$

$\zeta$  = distinguishing coefficient,  $\zeta \in (0, 1)$

The distinguishing coefficient  $\zeta$  is taken as 0.5. Grey relational grade is the weighted sum of the grey relational coefficients for a particular experiment and it is calculated using Eq. (6),

$$\Gamma(X_0, X_i) = \sum_{j=1}^n w_j \times \gamma(x_{0j}, x_{ij}) \text{ for } i = 1, 2, 3, \dots, m \quad (6)$$

where,  $\Gamma(X_0, X_i)$  is grey relational grade between comparability sequence  $X_i$  and reference sequence  $X_0$  and  $\sum_{j=1}^n w_j = 1$ .

The estimated grey relational grade  $\hat{\gamma}$  using the optimal level of the machining parameters can be calculated as:

$$\hat{\gamma} = \gamma_m + \sum_{i=1}^n (\bar{\gamma}_i - \gamma_m) \quad (7)$$

where  $\gamma_m$  is the total mean of the grey relational grade,  $\bar{\gamma}_i$  is the mean of the grey relational grade at the optimal level, and  $n$  is the number of the machining parameters that significantly affects the multiple performance characteristics.

In this study, attempts have been made to find out the influences of four process parameters in order to determine the optimum parametric combination for achieving best surface roughness with optimum the cutting rate and the material removal rate. The Taguchi method combined with the gray relational analysis has been employed to directly integrate three responses which are the CR, MRR and Ra. Therefore, optimization of the complicated multiple performance characteristics can be greatly simplified to a single objective optimization problem through this approach. The analysis of variance is performed to determine which parameter has significant effect on the multi-performance characteristics. To validate the study, confirmation experiment has been carried out at optimal set of parameters, and predicted results have been found to be in good agreement with experimental findings

## 2. Experimental and Test Details

In this work, experiments were performed on a commercial WEDM Mitsubishi FX Series Machine with submerged dielectric workpiece (see Fig. 1). Deionized water was used as a dielectric fluid due to its low viscosity, rapid cooling rate, generous temperature stabilization and efficient flushing. All experimental works were carried out by pulse arc discharges generated between wire (brass) and the titanium alloy workpiece (130mm x 63mm x 6mm). The electrode wire material was brass with a diameter of 0.5 mm. The workpiece material used was titanium alloy (Ti-6Al-4V) grade 5 with material composition of Al (6%), Fe(0.25%), Ti(90%) and V(4%) while the hardness is ~36HRC. The average cutting speed (millimeters per minute) was recorded directly from the display of the machine tool. Surface roughness and kerf width were measured using Alicona Infinite Focus Microscope, IFM-Infinite Focus 2.1.5, see Fig. 2. The surface roughness profile analysis ( $L_c = 2500\mu\text{m}$  and profile length 12.5mm) with 50X magnification were performed in accordance with ISO 4288 standard. Material removal rate (kerf width) was measured in three different spots. Ten reading were taken at each spot and the average was used as the kerf width. The MRR will be calculated using Eq. (8).

$$MRR = F \times D_w \times H \quad (8)$$

where  $F$  is the machine feed rate [ $mm/min$ ],  $D_w$  is the wire diameter [ $mm$ ] and  $H$  is the workpiece thickness [ $mm$ ] and MRR is the material removal rate [ $mm^3/min$ ].

Based on the preliminary testing, an orthogonal array experimental design ( $L_9$ ) was used with variation of pulse-off time (1, 3 and 5 second), peak current (4, 8 and 12 A), wire tension (6, 10 and 16 N) and wire feed rate (4, 8 and 12 m/min), see table 1. In order to select an appropriate orthogonal array for experiments, the total degrees of freedom need to be computed. In the present study, the interaction between the machining parameters was neglected. Therefore, there were 9 degrees of freedoms with three-level machining parameters in the WEDM process. There were 27 experiments carried out ( $L_9$  repeated twice) in order to study the entire machining parameter space using the  $L_9$  orthogonal array. The experimental layout for the machining parameters using the  $L_9$  orthogonal array is shown in Table 2. All experiments were carried out in a random order. Analysis of variance (ANOVA) was used to investigate the effects of the main machining parameters on cutting rate (CR), material removal rate (MRR) and surface roughness (Ra); pulse-off time (A), peak current (B), feed rate (C) and wire tension (D).



Fig. 1. WEDM Mitsubishi FX series machine

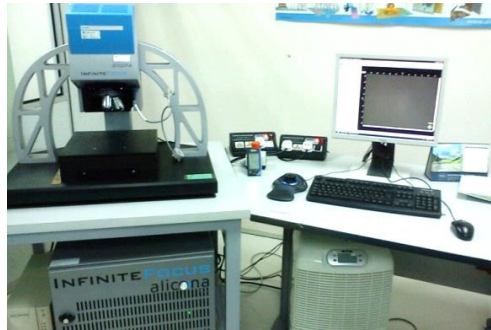


Fig. 2. Alicona Infinite Focus microscope.

Table 1. Control factors and their levels.

Parameter	Symbol	Unit	Level		
			L1	L2	L3
Pulse-off time	A	$\mu s$	1	3	5
Peak current	B	Ampere	4	8	12
Wire tension	C	N	6	10	16
Wire feed	D	mm/min	4	8	12

### 3. Results and Discussion

In the present experimental study, CR, MRR and Ra have been measured during WEDM titanium alloy Ti-Al-4V grade 5. Typically larger value of CR and MRR are desirable while for Ra smaller values indicate better surface quality, therefore, the present performances are referred to as “larger-the-better” and “lower-the-better” respectively, see Eq. 1 and Eq. 2. The average value of the experimentally obtained results for CR, MRR and Ra are tabulated in Table 2. The raw data than be transformed into S/N ratio. The corresponding S/N ratio values for experimental parametric setting according to  $L_9$  orthogonal array are shown in Table 3.

Table 2. Different level of process parameters.

L <sub>9</sub>	Orthogonal array				Responses		
	A	B	C	D	CR (mm /min)	MRR (g/min)	Ra (μm)
1	1	1	1	1	10.044	0.74	3.807
2	1	2	2	2	9.135	0.9	3.555
3	1	3	3	3	10.790	0.98	3.326
4	2	1	2	3	10.637	0.78	3.668
5	2	2	3	1	9.635	0.98	4.254
6	2	3	1	2	10.170	0.94	3.697
7	3	1	3	2	9.094	0.78	2.688
8	3	2	1	3	9.472	0.9	2.852
9	3	3	2	1	9.822	1.08	2.989

Table 3. Signal –to- noise (S/N) ratio values.

Exp. no	η (CR)	η (MRR)	η (Ra)
1	20.038	-2.615	-11.611
2	19.214	-0.915	-11.017
3	20.661	-0.175	-10.438
4	20.536	-2.158	-11.289
5	19.677	-0.175	-12.577
6	20.146	-0.537	-11.356
7	19.175	-2.158	-8.590
8	19.529	-0.915	-9.102
9	19.844	0.668	-9.511

The S/N ratio plot of surface roughness with respect to pulse-off time, peak current, wire tension and wire feed are shown in Fig. 3. In Fig. 3(a) shows that cutting rate decreases with the increase of pulse-off time from 3 to 5 μs. However, it is found that there is no significant variation in cutting rate value at 1 to 5 μs of pulse-off time. From the same figure, it is revealed that peak current and wire feed drastically increase from 6 to 12 A and from 10 to 15 N respectively. Fig. 3(b) and 3(c) show the effect of process parameters on material removal rate and surface roughness respectively. Fig. 3(a), (b) and (c) clearly show that each response has its own parameters combination. For an optimal cutting rate, material removal rate and surface roughness, the combination of cutting parameters were (A<sub>2</sub> B<sub>3</sub> C<sub>1</sub> D<sub>3</sub>), (A<sub>3</sub> B<sub>3</sub> C<sub>2</sub> D<sub>1</sub>) and (A<sub>3</sub> B<sub>3</sub> C<sub>3</sub> D<sub>3</sub>) respectively.

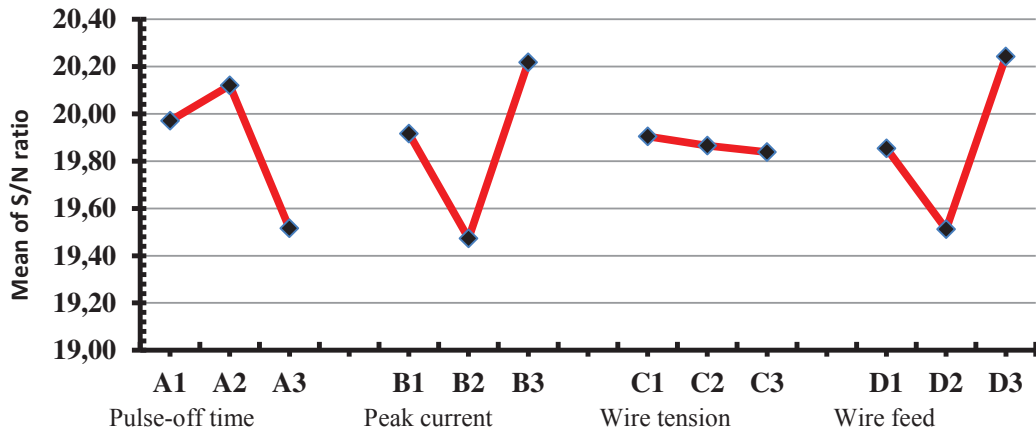


Fig. 3. (a): Effect of process parameters on cutting rate.

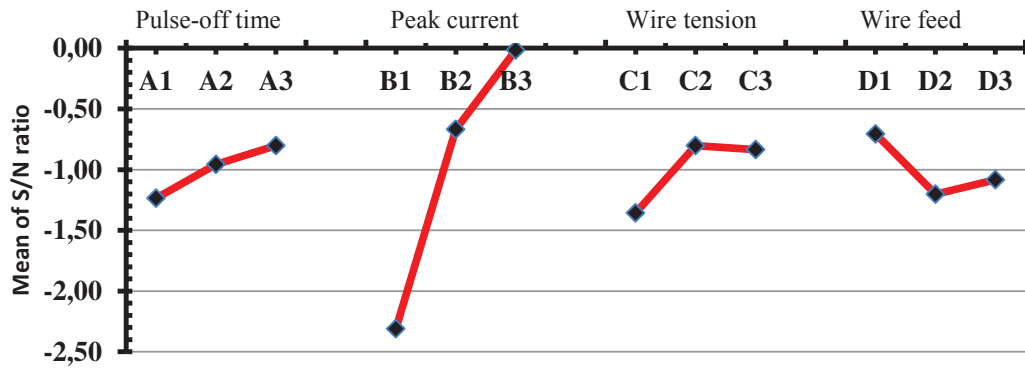


Fig. 3(b) : Effect of process parameters on material removal rate

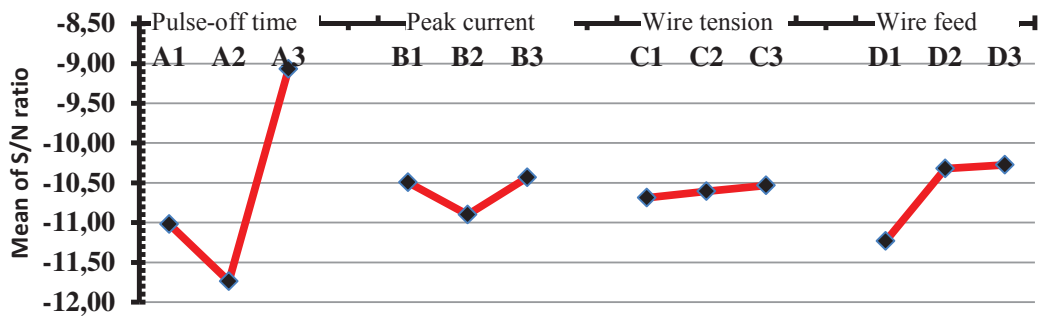


Fig. 3(c). Effect of process parameters on surface roughness.

Table 4 shows the normalized results (from Eq. 3 and Eq. 4) for cutting rate, material removal rate and surface roughness. Basically, larger normalized results correspond to a better performance and the best-normalized results should be equal to one. Next, the grey relational coefficient is calculated to express the relationship between the ideal (best) and actual normalized experimental results.

The normalized values of each of the responses for all 9 experiments are used to calculate the grey relational coefficient using Eq. (5) and the results are summarized in Table 4. The distinguishing coefficient  $\zeta$  is taken as 0.5. Grey relational grade is the weighted sum of the grey relational coefficients for a particular experiment and it is calculated using Eq. (5). The value of grey relational grade is compute using Eq. (6) and it is ranked for overall run. Thus, the multi criteria optimization problem has been transformed into a single objective optimization problem using the combination of grey relational analysis and Taguchi method. The higher value of grey relational grade shows a better characteristic of the multiple performances as the experimental result is closer to the ideal normalized value. The mean of grey relational grade for each level of WEDM process parameters is summarized and shown in Table 4. In addition, the total mean of grey relational grade for the 9 experiments run is also calculated and listed in Table 4. It is ranked in order to know the most significant parameter in the WEDM process. It is obvious from Table 4 that experiment number 3 has the highest grey relational grade value. Therefore, it can be concluded that experiment number 3 possesses best multi response characteristics among all the 9 experiments. While Fig. 4 shows the grey relational grade graph which the data is taken from Table 4. The dashed line in the figure is the value of the total mean of grey relational grade. As a result, the multi objective optimal parametric combination has been determined. Thus, the optimal factor setting becomes  $A_2 B_3 C_3 D_1$  where number indicates level of factors.

Since the experimental design is orthogonal, it is then possible to separate out the effect of each machining parameter on the grey relational grade at different levels. For example, the mean of grey relational grade for the pull-off time at levels 1, 2 and 3 can be calculated by averaging the grey relational grade for the experiments 1 to 3, 4 to 6 and 7 to 9, respectively (Table 2). The mean of grey relational grade for each level of the other machining parameters also can be computed in the similar manner. The mean of grey relational grade for each level of the machining parameters is summarized and shown in Table 5. In addition, the total mean of grey relational grade for the 9 experiments is also calculated and listed in Table 5.

Fig. 4 shows the grey relational grade graph and the dash line indicated in Fig. 2 is the value of the total mean of grey relational grade. Basically, the larger the grey relational grade, the better is the multiple performance characteristics. However, the relative importance among the machining parameters for the multiple performance characteristics still needs to be known so that the optimal combinations of the machining parameter levels can be determined more accurately by analysis of variance (ANOVA).

Table 4. Grey relation co-efficient and grey relational grade values.

Exp. no	Orthogonal array $L_9 (3^4)$				Grey relational coefficient			Grey relational grade	Order
	A	B	C	D	CR	MRR	Ra		
1	1	1	1	1	0.5440	0.3333	0.6737	0.5170	6
2	1	2	2	2	0.3393	0.5090	0.5610	0.4698	7
3	1	3	3	3	1.0000	0.6605	0.4824	0.7143	1
4	2	1	2	3	0.8566	0.3674	0.6075	0.6105	4
5	2	2	3	1	0.4302	0.6605	1.0000	0.6969	2
6	2	3	1	2	0.5909	0.5765	0.6203	0.5959	5
7	3	1	3	2	0.3333	0.3674	0.3333	0.3447	9
8	3	2	1	3	0.3962	0.5090	0.3646	0.4233	8
9	3	3	2	1	0.4764	1.0000	0.3940	0.6235	3

Table 5. Influence of parameters on grey relational grade.

Parameters				Max-min
	Level 1	Level 2	Level 3	
Pulse-off time	0.5670	<b>0.6344</b>	0.4638	0.1706
Peak current	0.4907	0.5300	<b>0.6446</b>	0.1538
Wire tension	0.5121	0.5679	<b>0.5853</b>	0.0732
Wire feed	<b>0.6125</b>	0.4701	0.5827	0.1423

Total mean value of the Grey Relational Grade = 0.5551

\*Optimal level are indicated by bold type.

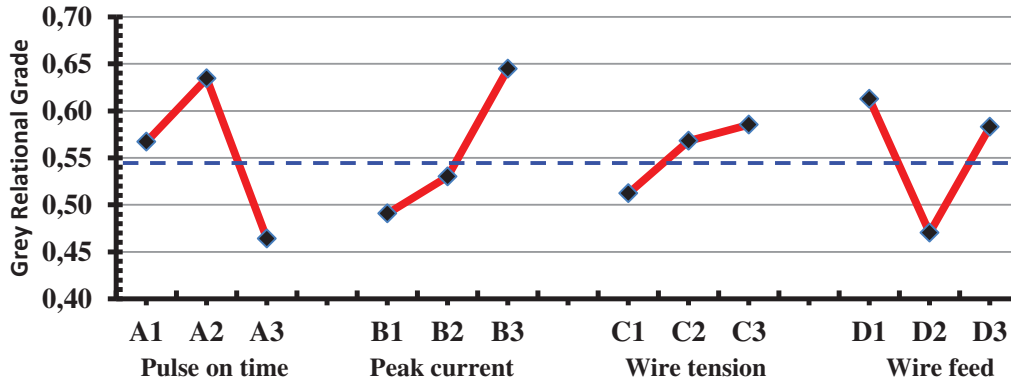


Fig. 4. Variation of grey relational grade on various levels of process parameters.

In order to investigate the most significant WEDM process parameters, analysis of variance (ANOVA) is applied which can be measured by the sum of squared deviations from the total mean of grey relational grade. Thus, the analysis of variance (ANOVA) test has been performed for all responses i.e., cutting rate, material removal rate and surface roughness. Table 6 shows the ANOVA results of the performance criteria with calculated F-values of the respective control factors. Results of analysis of variance (Table 6) indicate that pulse-off time is the most significant machining parameter (35% contribution) that affects the multiple performance characteristics. The other factors such as peak current, wire feed and wire tension contribute about 31%, 27% and 7 % respectively. Based on the above discussion, the optimal machining parameters are pulse-off time at level 2, peak current at level 3, wire tension at level 3 and wire feed at level 1.

Table 6. ANOVA results for CR, MRR and Ra.

Factor	Level 1	Level 2	Level 3	DOF	Sum of squares	Mean square	F value	Contribution (%)
A	0.5670	0.6344	0.4638	2	0.0443	0.0222	0.8349	35%
B	0.4907	0.5300	0.6446	2	0.0383	0.0192	0.7222	31%
C	0.5121	0.5679	0.5853	2	0.0088	0.0044	0.1655	7%
D	0.6125	0.4701	0.5827	2	0.0338	0.0169	0.6371	27%
Error					0.1252			
Total				8	0.2123	0.0265		

Total mean value of the Grey Relational Grade = 0.555



Table 7. Comparison between initial level and optimal level.

	Initial machining parameters		Optimal machining parameters	
	Orthogonal Array	Prediction by Grey Relational Analysis	Confirmation Experiment	% Improvement
Setting level	A <sub>3</sub> B <sub>1</sub> C <sub>3</sub> D <sub>2</sub>	A <sub>2</sub> B <sub>3</sub> C <sub>3</sub> D <sub>1</sub>	A <sub>2</sub> B <sub>3</sub> C <sub>3</sub> D <sub>1</sub>	
Cutting rate (mm/min)	9.094		10.637	17%
Material removal rate (g/min)	0.78		0.98	26%
Surface roughness (μm)	2.688		2.575	4%
Grey relational grade	0.3447		0.730	
Improvement of the grey relational grade =0.3853				

Once the optimal machining parameters are selected, the final step is to verify the improvement of the cutting performance. Based on Eq. (8), the estimated grey relational grade using the optimal machining parameters can then be obtained. Table 7 shows the results of the confirmation experiment using the optimal machining parameters. As shown in Table 7, cutting rate and material removal rate are accelerated from 9.094 to 10.637 mm/min and from 0.78 to 0.98 g/min respectively. On the other hand surface roughness is improved from 2.688 to 2.575 μm. It is clearly shown that the multiple performance characteristics in the WEDM process are greatly improved.

#### 4. Conclusion

The use of the orthogonal array with grey relational analysis to optimize the WEDM process with the multiple performance characteristics has been reported in this paper. The grey relational analysis of the experimental results of cutting rate (CR), metal removal rate (MRR) and surface finish (Ra) can convert optimization of the multiple performance characteristics into optimization of a single performance characteristic called the grey relational grade. As a result, optimization of the complicated multiple performance characteristics can be greatly simplified through this approach. The optimal machining parameters of titanium alloy in order to achieve minimum surface roughness (Ra), higher cutting rate (CR) and higher material removal rate (MRR) are ; the pulse-off time at 3μs, peak current at 12A, wire tension at 16 N and wire feed at 4 mm/min. It is shown that the performance characteristics of the WEDM process such as cutting rate (CR), metal removal rate (MRR) and surface finish (Ra) are improved by using the method proposed in this study.

#### Acknowledgements

The authors would like to thank the Faculty of Mechanical Engineering (AMTEX), Research Management Institute (RMI-600-RMI/FRGS 5/3 (20/2013)), Universiti Teknologi MARA and Ministry of Education, Malaysia for the research funding under Fundamental Research Grant Scheme (FRGS).

#### References

- [1] Ho KH, Newman ST, Rahimifard S, Allen RD. International Journal of Machine Tools and Manufacture 2004;44:1247-1259.
- [2] Rohit G. National Institute Of Technology; Kurukshetra 2010; PhD thesis. 2010.
- [3] Hascalik A, Caydas U. Applied Surface Science 2007;253:9007-9010.
- [4] Datta S, Mahapatra SS. International Journal of Engineering; Science and Technology 2010;2:162-183.
- [5] Saedon JB, Jaafar N, Jaafar R, Saad RH, Kasim MS. Applied Mechanics and Materials 2014;510:123-129.
- [6] Deng JL. Introduction to Grey system theory. J. Grey Syst. 1989;1:1-24.
- [7] Lin JL, Wang KS, Yan BH, Tarang YS. J Grey Syst 1998;10:355-370.