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Mission decision-making method of multi-aircraft cooperatively attacking multi-target based on game theoretic framework

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Abstract Coordinated mission decision-making is one of the core steps to effectively exploit the capabilities of cooperative attack of multiple aircrafts. However, the situational assessment is an essential base to realize the mission decision-making. Therefore, in this paper, we develop a mission decision-making method of multi-aircraft cooperatively attacking multi-target based on situational assessment. We have studied the situational assessment mathematical model based on the Dempster-Shafer (D-S) evidence theory and the mission decision-making mathematical model based on the game theory. The proposed mission decision-making method of antagonized airfight is validated by some simulation examples of a swarm of unmanned combat aerial vehicles (UCAVs) that carry out the mission of the suppressing of enemy air defenses (SEAD).

1. Introduction

The team of multiple aircrafts has stronger capability than a single aircraft in detecting the targets, piercing through the defense systems, and carrying out the attack mission. Each member of the team can share the information acquired by any other one and carry out mission of cooperative attacking targets according to its position in air and the resources of fighting for a uniform airfight intention. The team of multiple aircrafts is able to form easily all kinds of vertiginous attack situation in airfight so that those opposed targets will be confronted with the defending difficulties. Thereby, the fashion that multiple aircrafts cooperatively attack targets will be the main pattern in future airfight.

In this paper, the phrase of attacking effect consists of validity, invalidity and uncertainty. The validity and invalidity of attacking effect are defined as the advantage acquired by our aircrafts (or foe's targets) and the cost paid for achieving intention by our aircrafts (or foe's targets) in antagonized airfight, respectively. Sensors aboard aircraft affect the attacking effect of aircraft due to the capability of sensors in detecting, tracking and identifying target, while weapons aboard aircraft

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affect the attacking effect due to the capability of weapons in hitting and destroying target. However, the above capabilities of sensors and weapons all rest with the distance, azimuth and pitching between one of our aircrafts and one of foe's targets. Accordingly, the fuzzy mapping function of the fighting effect of sensor and weapon is constructed by selecting the three position parameters of distance, azimuth and pitching as variables for establishing the correspondence between the position parameter and the ability of sensor and weapon. In this paper, the Dempster-Shafer (D-S) synthesize rules are used for formulating the situational assessment method.

When multiple aircrafts in our team are antagonizing some foe's targets simultaneously, one of our aircrafts is able to either detect and identify the foe's targets by sensors aboard the aircraft or receive the information of the foe's targets by wireless data link. Therefore, in this paper, we suppose that our team of multiple aircrafts has known the position and identity of all foe's targets and is able to acquire the important reasoning from the position and identity of foe's target to the capability of sensors' detecting and weapons' attack, the defending strategies and the advantage (showed by numerical value) acquired by selecting a certain defending strategy. If the above situation of antagonizing airfight is analyzed by quoting the game theory model, the situation means that our team has known the opponents and the opponents' strategies and cost function. Considering that our opponent is powerful, we think that our opponent also has known the equivalent information about our team of multiple aircrafts at least. We suppose that our team and our opponent simultaneously carry out action for equality because they all try to be the first actor. In this paper, the static non-cooperative and nonzero Nash games are used for formulating the mission decision-making method.

In recent years, a significant shift of focus has occurred in the field of autonomous unmanned combat aerial vehicles (UAVs) as researchers began to investigate problems involving multiple UAVs rather than single UAV. As a result of this focus on multiple UAVs, coordination of multiple UAVs has received significant attention.[1–3] Moreover, cooperative systems are required to operate in an adversary environment (such as suppressing of enemy air defenses (SEAD)).[4,5] Cooperative decision-making for multi-UAV or multi-Agent system is of great interest. A significant amount of current research activities focuses on a theoretic control framework for distributed cooperative decision-making for an ensemble of UAVs, and all the used research methods in this field are similar in Refs.[6–8] Additional autonomous decision-making focusing on mission planning, target assignment, or operation management of complex system can be found in Refs.[9–14] Solutions to general UAV cooperative decision-making problems in adversarial environments can be obtained by solving game problems introduced and implemented in Refs.[15–17] Related application information of game theory method appears in many sources.[18,19] A synthetic method for situation assessment based on fuzzy logic and D-S evidence theory is proposed in Refs.[20,21]

This paper develops a mission decision-making algorithm based on the game model, and then proposes a situational assessment algorithm based on the D-S evidence synthesize rules for a swarm of UCAVs in SEAD mission. In Section 2, a situational assessment algorithm of coordinated airfight is presented in detail and the D-S evidence theory is introduced simply for sustaining the mentioned situational assessment algorithm above. In Section 3, a mission decision-making algorithm is designed by formulating the strategies and cost function in the game model. Section 3 is based on Section 2. Section 4 shows an simulation example of a typical mission performed by a swarm of UCAVs. In Section 5, the simulating results in Section 4 are analyzed deeply. Section 6 summarizes the conclusions.

2. Formulating situational assessment based on evidence theory

2.1. Preliminaries

For sustaining the situational assessment algorithms mentioned in the previous section, the basic concepts of the D-S evidence theory are first introduced in the following part.

Let Θ be a set consisting of all the values that X might be and an element of set Θ is not consistent with the other elements, and then Θ is called as the discernment frame of X.

Definition 1. Let Θ be a frame of discernment, and if the function $m: 2^{\Theta} \rightarrow [0,1]$ fulfills the following conditions:

(1) $m(\varnothing) = 0$
(2) $\sum_{A \subset \Theta} m(A) = 1$

then m is called as the basic probability assignment on the frame of discernment Θ and $m(A)$ is called as the basic probability number of A. $m(A)$ denotes the believed degree of A oneself.

Definition 2. Let Θ be a frame of discernment, and if the function $m: 2^{\Theta} \rightarrow [0,1]$ is the basic probability assignment on Θ, then the function $\mathrm{Bel}: 2^{\Theta} \rightarrow [0,1]$ is called as the belief function and is defined by

$$\mathrm{Bel}(A) = \sum_{B \subset A} m(B) (\forall A \subset \Theta)$$

where $\mathrm{Bel}(A)$ denotes the believed degree of A including all of its subsets.

D-S synthesize rules. Let $\mathrm{Bel}_1, \mathrm{Bel}_2, \cdots, \mathrm{Bel}_n$ be the belief functions on the same frame of discernment $\Theta, m_1, m_2, \ldots, m_n$ are the basic probability assignments correspondingly. If $\mathrm{Bel}_1 \oplus \mathrm{Bel}_2 \oplus \ldots \oplus \mathrm{Bel}_n$ is existent and has the basic probability assignment m, then

$$\begin{cases} \forall A \subset \Theta, A \neq \varnothing, A_1, A_2, \ldots, A_n \subset \Theta \\ m(A) = K \sum_{\substack{A_1, A_2, \ldots, A_n \subset \Theta \\ A_1 \cap A_2 \cap \ldots \cap A_n = A}} m_1(A_1) m_2(A_2) \ldots m_n(A_n) \\ K = \left(1 - \sum_{\substack{A_1, A_2, \ldots, A_n \subset \Theta \\ A_1 \cap A_2 \cap \ldots \cap A_n = \varnothing}} m_1(A_1) m_2(A_2) \ldots m_n(A_n) \right)^{-1} \end{cases}$$

where K is the middle parameter, and $A_1, A_2, \ldots, A_n$ are subsets of the discernment frame element A.

The D-S synthesize rules reflect the effect of combined operations made by many evidences.

2.2. Situational assessment algorithm of coordinated airfight

For achieving the situational assessment of multi-aircraft cooperative attack, the relevant mathematical model is established based on the D-S evidence theory as follows.

Firstly, we establish the discernment frame of the fighting effect Θ.

Let $\Theta = \{a, b, \theta\}$ be the discernment frame of fighting effect defined in Section 1, where a, b and θ denote respectively the fighting efficacy, inefficacy and uncertainty in the three cases of single aircraft attack versus one enemy target, multi-aircraft cooperative attack versus one enemy target, and multi-aircraft cooperative attack versus multiple enemy targets.

Secondly, design the basic probability assignment on the frame of discernment Θ for our aircraft.

The basic probability assignment of the sensor distance evidence is formulated in the expression Eqs. (1)–(3), which is depicted in Fig. 1.

$$\varpi_{\mathrm{RL}}(a) = \begin{cases} a_{\mathrm{zo}} & 0 \leqslant \sigma < l_{\mathrm{zo}} \\ \frac{(a_{\mathrm{js}}-a_{\mathrm{zo}})\sigma}{l_{\mathrm{js}}-l_{\mathrm{zo}}} + a_{\mathrm{zo}} - l_{\mathrm{zo}}\frac{a_{\mathrm{js}}-a_{\mathrm{zo}}}{l_{\mathrm{js}}-l_{\mathrm{zo}}} & l_{\mathrm{zo}} \leqslant \sigma < l_{\mathrm{js}} \\ \frac{(a_{\mathrm{zj}}-a_{\mathrm{js}})\sigma}{l_{\mathrm{zj}}-l_{\mathrm{js}}} + a_{\mathrm{js}} - l_{\mathrm{js}}\frac{a_{\mathrm{zj}}-a_{\mathrm{js}}}{l_{\mathrm{zj}}-l_{\mathrm{js}}} & l_{\mathrm{js}} \leqslant \sigma < l_{\mathrm{zj}} \\ \frac{(a_{\mathrm{ks}}-a_{\mathrm{zj}})\sigma}{l_{\mathrm{ks}}-l_{\mathrm{zj}}} + a_{\mathrm{zj}} - l_{\mathrm{zj}}\frac{a_{\mathrm{ks}}-a_{\mathrm{zj}}}{l_{\mathrm{ks}}-l_{\mathrm{zj}}} & l_{\mathrm{zj}} \leqslant \sigma < l_{\mathrm{ks}} \\ \frac{(a_{\mathrm{zb}}-a_{\mathrm{ks}})\sigma}{l_{\mathrm{zb}}-l_{\mathrm{ks}}} + a_{\mathrm{ks}} - l_{\mathrm{ks}}\frac{a_{\mathrm{zb}}-a_{\mathrm{ks}}}{l_{\mathrm{zb}}-l_{\mathrm{ks}}} & l_{\mathrm{ks}} \leqslant \sigma < l_{\mathrm{zb}} \\ a_{\mathrm{zb}} & l_{\mathrm{zb}} \leqslant \sigma \end{cases} \tag{1}$$

$$\varpi_{\mathrm{RL}}(b) = \begin{cases} b_{\mathrm{zo}} & 0 \leqslant \sigma < l_{\mathrm{zo}} \\ \frac{(b_{\mathrm{js}}-b_{\mathrm{zo}})\sigma}{l_{\mathrm{js}}-l_{\mathrm{zo}}} + b_{\mathrm{zo}} - l_{\mathrm{zo}}\frac{b_{\mathrm{js}}-b_{\mathrm{zo}}}{l_{\mathrm{js}}-l_{\mathrm{zo}}} & l_{\mathrm{zo}} \leqslant \sigma < l_{\mathrm{js}} \\ \frac{(b_{\mathrm{zj}}-b_{\mathrm{js}})\sigma}{l_{\mathrm{zj}}-l_{\mathrm{js}}} + b_{\mathrm{js}} - l_{\mathrm{js}}\frac{b_{\mathrm{zj}}-b_{\mathrm{js}}}{l_{\mathrm{zj}}-l_{\mathrm{js}}} & l_{\mathrm{js}} \leqslant \sigma < l_{\mathrm{zj}} \\ \frac{(b_{\mathrm{ks}}-b_{\mathrm{zj}})\sigma}{l_{\mathrm{ks}}-l_{\mathrm{zj}}} + b_{\mathrm{zj}} - l_{\mathrm{zj}}\frac{b_{\mathrm{ks}}-b_{\mathrm{zj}}}{l_{\mathrm{ks}}-l_{\mathrm{zj}}} & l_{\mathrm{zj}} \leqslant \sigma < l_{\mathrm{ks}} \\ \frac{(b_{\mathrm{zb}}-b_{\mathrm{ks}})\sigma}{l_{\mathrm{zb}}-l_{\mathrm{ks}}} + b_{\mathrm{ks}} - l_{\mathrm{ks}}\frac{b_{\mathrm{zb}}-b_{\mathrm{ks}}}{l_{\mathrm{zb}}-l_{\mathrm{ks}}} & l_{\mathrm{ks}} \leqslant \sigma < l_{\mathrm{zb}} \\ b_{\mathrm{zb}} & l_{\mathrm{zb}} \leqslant \sigma \end{cases} \tag{2}$$

$$\varpi_{\mathrm{RL}}(\theta) = \theta \quad \sigma \geqslant 0 \ \ \mathrm{km} \tag{3}$$

where ϖ_{RL} is the basic probability assignment of the sensor distance (radar length σ) evidence, where ϖ denotes that the function is used for describing aircraft (we also define the function ϑ for describing enemy target correspondingly). The functions formulated in the expression Eqs. (1)–(3) are actually the fuzzy mapping functions of the fighting effect of the sensor distance evidence, which establishes the correspondence between the distance from the aircraft to foe's target and the sensor detecting ability of our aircraft.

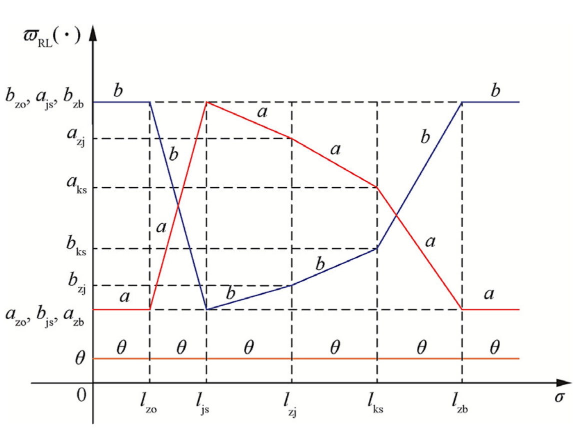

Fig. 1 Basic probability assignment function ϖ_{RL}.

In the expression Eqs. (1)–(3), the other mathematical symbols are depicted in Fig. 1.

As shown in Fig. 1, the unitary fighting effect value of the synthesis capability ($\varpi_{\mathrm{RL}}(\cdot)$, the vertical axis of the coordinates, in [0, 1] without unit) is achieved in the process of sensors when the aircraft detects, tracks and identifies foe's target. It depends on the distance (σ, the horizontal axis of the coordinates, with a unit of kilometer) between the aircraft and foe's target. $\varpi_{\mathrm{RL}}(\cdot)$ implies how much the sensors' ability being enslaved to the distance contributes to the whole attacking action. In Fig. 1, the cures with sign (a, b and θ) represent the validity, invalidity and uncertainty of the fighting effect of the sensor distance evidence, respectively; the values of the vertical axis parameters ($a_{\mathrm{zo}}, a_{\mathrm{js}}, a_{\mathrm{zj}}, a_{\mathrm{ks}}, a_{\mathrm{zb}}, b_{\mathrm{zo}}, b_{\mathrm{js}}, b_{\mathrm{zj}}, b_{\mathrm{ks}}, b_{\mathrm{zb}}$) and the horizontal axis parameters ($l_{\mathrm{zo}}, l_{\mathrm{js}}, l_{\mathrm{zj}}, l_{\mathrm{ks}}, l_{\mathrm{zb}}$) will be embodied by the capability of the actual sensors.

Analogously, the basic probability assignments of sensor pitching evidence, sensor azimuth evidence, weapon distance evidence, weapon pitching evidence and weapon azimuth evidence can also be formulated as $\varpi_{\mathrm{R}\Theta}, \varpi_{\mathrm{R}\Psi}, \varpi_{\mathrm{ML}}, \varpi_{\mathrm{M}\Theta}$ and $\varpi_{\mathrm{M}\Psi}$, respectively.

For enemy target, by adopting the same method, the basic probability assignments of the sensor distance, sensor pitching evidence, sensor azimuth evidence, weapon distance evidence, weapon pitching evidence and weapon azimuth evidence can be formulated to be $\vartheta_{\mathrm{RL}}, \vartheta_{\mathrm{R}\Theta}, \vartheta_{\mathrm{R}\Psi}, \vartheta_{\mathrm{ML}}, \vartheta_{\mathrm{M}\Theta}$ and $\vartheta_{\mathrm{M}\Psi}$, respectively.

Finally, we achieve the evidence synthesizing of the coordinated airfight.

It should be noted that the attacking effect will be different when the swarm of aircrafts employs different strategies (the allocations of a set of aircrafts versus a number of foe's targets) to attack foe's group of targets although the positions between any of aircrafts and each foe's target are the same.

Therefore, in this paper, the situational assessment is equivalent to calculating the validity $\varpi^{\alpha_i}(a)$, invalidity $\varpi^{\alpha_i}(b)$ and uncertainty $\varpi^{\alpha_i}(\theta)$ of the certain-strategy α_i, as well as the validity $\vartheta^{\beta_j}(a)$, invalidity $\vartheta^{\beta_j}(b)$ and uncertainty $\vartheta^{\beta_j}(\theta)$ of the foe's certain-strategy β_j.

Our certain-strategy may consist of three cases:

(1) One of aircrafts attacks one of given foe's target.
(2) A set of aircrafts attack cooperatively the given foe's target.
(3) None of aircrafts attacks the given foe's target.

The foe's certain-strategy is similar to our certain strategy.

The validity of the fighting effect of k aircrafts cooperative attacking l targets may be acquired by a set of evidence synthesizing processes based on the evidence theory as follows:

$$\varpi^{\alpha_i}_{\mathrm{R}_{tp_n}} = \varpi^{\alpha_i}_{\mathrm{RL}_{tp_n}} \oplus \varpi^{\alpha_i}_{\mathrm{R}\Theta_{tp_n}} \oplus \varpi^{\alpha_i}_{\mathrm{R}\Psi_{tp_n}} \tag{4}$$

where $t = 1, 2, \ldots, l$ is the serial number of the certain foe's target meeting with attack; $p_n = 0, 1, 2, \ldots, k$ is the serial number of those aircrafts attending the action; $n = 0, 1, 2, \ldots, k$

denotes the number of aircrafts attending the action of cooperative attack against the certain foe's target.

Eq. (4) denotes that the total fighting effect (including validity, invalidity and uncertainty) of the sensor on the aircraft with the serial number p_n in the subscript of $\varpi^{\alpha_i}_{\mathrm{R}_{tp_n}}$ is acquired by synthesizing the fighting effect of the sensor distance evidences $\varpi^{\alpha_i}_{\mathrm{RL}_{tp_n}}$, the fighting effect of the sensor pitching evidences $\varpi^{\alpha_i}_{\mathrm{R}\Theta_{tp_n}}$, and the fighting effect of the sensor azimuth evidences $\varpi^{\alpha_i}_{\mathrm{R}\Psi_{tp_n}}$ based on the D-S synthesize rules in Section 2.1 when one of our certain aircraft p_n attacks one of foe's certain target t according to our certain strategy α_i.

$$\varpi^{\alpha_i}_{\mathrm{M}_{tp_n}} = \varpi^{\alpha_i}_{\mathrm{ML}_{tp_n}} \oplus \varpi^{\alpha_i}_{\mathrm{M}\Theta_{tp_n}} \oplus \varpi^{\alpha_i}_{\mathrm{M}\Psi_{tp_n}} \tag{5}$$

The meaning of Eq. (5) is similar to that of Eq. (4). The difference only lies in the fact that the weapon is described instead of the sensor.

$$\varpi^{\alpha_i}_{tp_n} = \varpi^{\alpha_i}_{\mathrm{R}_{tp_n}} \oplus \varpi^{\alpha_i}_{\mathrm{M}_{tp_n}} \tag{6}$$

Eq. (6) denotes that the total fighting effect (including validity, invalidity and uncertainty) of one of the aircrafts with the serial number p_n ($\varpi^{\alpha_i}_{tp_n}$) is acquired by synthesizing the total fighting effect of the sensor evidences $\varpi^{\alpha_i}_{\mathrm{R}_{tp_n}}$ and the total fighting effect of the weapon evidences $\varpi^{\alpha_i}_{\mathrm{M}_{tp_n}}$ based on the D-S synthesize rules in Section 2.1 when one of our certain aircrafts p_n attacks one of foe's certain target t according to the certain strategy α_i.

$$\varpi^{\alpha_i}_{t} = \varpi^{\alpha_i}_{tp_1} \oplus \varpi^{\alpha_i}_{tp_2} \oplus \ldots \oplus \varpi^{\alpha_i}_{tp_n} \tag{7}$$

Eq. (7) denotes that the total fighting effect (including validity, invalidity and uncertainty) of n our aircrafts with the serial number $(p_1, p_2, \ldots, p_n)$ $\varpi^{\alpha_i}_t$ is acquired by synthesizing the total fighting effects of all our aircraft with the serial number $(p_1, p_2, \ldots, p_n)$ cooperatively attacking the same foe's target t $(\varpi^{\alpha_i}_{tp_1}, \varpi^{\alpha_i}_{tp_2}, \ldots, \varpi^{\alpha_i}_{tp_n})$ based on the D-S synthesize rules in Section 2.1 when multiple aircrafts cooperatively attack one certain foe's target according to our certain strategy α_i.

Considering the case that none of our aircraft attacks the given foe's target, we form a special rule as follows:

$$\begin{cases} \varpi^{\alpha_i}_{t0}(a) = 0.299, \varpi^{\alpha_i}_{t0}(b) = 0.689, \varpi^{\alpha_i}_{t0}(\theta) = 0.012 \\ \varpi^{\alpha_i}_{t} = \varpi^{\alpha_i}_{tp_0} = \varpi^{\alpha_i}_{t0} \end{cases} \tag{8}$$

Eq. (8) denotes that the total fighting effect (validity, invalidity and uncertainty) of the aircraft team is set to be 0.299, 0.689 and 0.012 respectively, when none of our aircraft attack the certain foe's target t according to our certain strategy α_i.

$$\varpi^{\alpha_i} = \varpi^{\alpha_i}_1 \oplus \varpi^{\alpha_i}_2 \oplus \ldots \oplus \varpi^{\alpha_i}_t \oplus \ldots \oplus \varpi^{\alpha_i}_l \tag{9}$$

Eq. (9) denotes that the total fighting effect (including validity, invalidity and uncertainty) of the whole of our aircraft team ϖ^{α_i} is acquired by synthesizing the fighting effect of all aircrafts cooperatively attacking all l foe's targets with the serial number $(1, 2, \ldots, t, \ldots, l)$ $\varpi^{\alpha_i}_1, \varpi^{\alpha_i}_2, \ldots, \varpi^{\alpha_i}_t, \ldots, \varpi^{\alpha_i}_l$ based on the D-S synthesize rules in Section 2.1 when multiple aircrafts cooperatively attack the foe's targets according to our certain strategy α_i.

For enemy target, by adopting the same method, the total fighting effect (including validity, invalidity and uncertainty) of the whole of foe's target group ϑ^{β_j} may be also acquired when multiple foe's targets cooperatively defend our multiple aircrafts according to the certain foe's strategy β_j.

3. Formulating mission decision-making based on game theory

For achieving mission decision-making of multi-aircraft cooperatively attacking multi-target, the relevant mathematical model is established based on the game theory as follows:

The game model is set as

$$G = \langle N, S_1, S_2, u_1, u_2 \rangle, \quad N = \{n_1, n_2\}$$

where N is the game members, S_1 our strategy set, and S_2 the foe's strategy set. n_1 represents our team that consists of k aircrafts and n_2 represents the foe's group that consists of l targets:

$$\begin{cases} S_1 = \{\alpha_1, \alpha_2, \ldots, \alpha_{\bar{m}}\}, S_2 = \{\beta_1, \beta_2, \ldots, \beta_{\bar{n}}\} \\ \alpha_i = \{(W_1, d_1), (W_2, d_2), \ldots, (W_l, d_l)\} \end{cases}$$

where $W_1, W_2, \ldots, W_l \subset W = \{w_1, w_2, \ldots, w_k, \varnothing\}$ and we have

$$\begin{cases} W_1 \cup W_2 \cup \ldots \cup W_l = W \\ W_1 \cap W_2 \cap \ldots \cap W_l = \varnothing \\ \beta_j = \{(D_1, w_1), (D_2, w_2), \ldots, (D_k, w_k)\} \end{cases}$$

where $D_1, D_2, \ldots, D_k \subset D = \{d_1, d_2, \ldots, d_l, \varnothing\}$ and

$$\begin{cases} D_1 \cup D_2 \cup \ldots \cup D_k = D \\ D_1 \cap D_2 \cap \ldots \cap D_k = \varnothing \end{cases}$$

where α_i $(i = 1, 2, \ldots, \bar{m})$ represents the $\bar{m}$ strategies of our k aircrafts cooperatively attacking l foe's targets, while β_j $(j = 1, 2, \ldots, \bar{n})$ the $\bar{n}$ strategies of l foe's targets cooperatively defending our k aircrafts. One among the parameters $(d_1, d_2, \ldots, d_l)$ represents the serial number of l foe's targets; one among the parameters $(w_1, w_2, \ldots, w_k)$ represents the serial number of our k aircrafts; $\varnothing$ represents nobody; (W_t, d_t) $(t = 1, 2, \ldots, l)$ represents a set of our aircrafts marked by W_t cooperatively attacking the same foe's target marked by d_t, while (D_t, w_t) $(t = 1, 2, \ldots, k)$ represents a number of foe's targets marked by D_t cooperatively defending our same aircraft marked by w_t.

When our strategy of α_i and foe's strategy of β_j are selected simultaneously, our payment function is set as

$$u_1(\alpha_i, \beta_j) = a_{ij} = \frac{\varpi^{\alpha_i}(a)\vartheta^{\beta_j}(b)}{\varpi^{\alpha_i}(b)\vartheta^{\beta_j}(a)} \tag{10}$$

Similarly, the foe's payment function is set as

$$u_2(\alpha_i, \beta_j) = b_{ij} = \frac{\varpi^{\alpha_i}(b)\vartheta^{\beta_j}(a)}{\varpi^{\alpha_i}(a)\vartheta^{\beta_j}(b)} \tag{11}$$

$\overline{\boldsymbol{A}} = [\bar{a}_{ij}]_{\bar{m}\times\bar{n}}$ and $\overline{\boldsymbol{B}} = [\bar{b}_{ij}]_{\bar{m}\times\bar{n}}$ are called our payment matrix and the foe's payment matrix, respectively.

The Nash equilibrium state of the game model G may be acquired by the certain computing method based on the matrix $\overline{\boldsymbol{A}}$ and $\overline{\boldsymbol{B}}$.

Our strategy of α_i and foe's strategy of β_j denoted by the Nash equilibrium state will be used to determine the fighting action of our aircrafts team and guess the defending action of foe's targets group, respectively.

4. Simulation

To demonstrate the performance of the proposed methods, the following part presents the simulation results by using the mission decision-making algorithms in Sections 2 and 3.

4.1. Scenario

As shown in Fig. 2, a typical scenario is simulated with our software to evaluate the performance of the proposed mission decision-making algorithm for multi-aircraft cooperative attack.

In the scenario, our one team consists of four UCAVs with the serial number $\{w_1, w_2, w_3, w_4\}$ in the air and one enemy group consists of three missile positions with the serial number $\{d_1, d_2, d_3\}$ and a crucial target with the serial number m_4 on the ground.

In our team, each UCAV has the same sensor detecting and weapon attacking ability, the flight altitude of 8000–11000 m (the value is 10000 m for this example) and the flight velocity of 200–280 m/s (the value is 240 m/s for this example). The four UCAVs take the form of trapezium (see Fig. 2). In the trapezium team, the distance between the frontal two UCAVs is 5–20 km (the value is 5 km for this example) and the distance between the latter two UCAVs is 10–50 km (the value is 10 km for this example); the distance between the frontal two UCAVs and the latter two UCAVs is 2.5–30 km (the value is 2.5 km for this example).

In enemy group, the three missile positions take the form of equilateral triangle (see Fig. 2). In the equilateral triangle team, the distance between any two missile positions is 10–30 km (the value is 10 km for this example) and the crucial target is located on the center of equilateral triangle. The distance between the center of our team and the center of enemy group is a variable that may be a value from 5 km to 70 km (the value is 10 km for this example).

The fighting ability of our UCAV depends on the suppositional sensor (SAR radar) and weapon (JDAM bomb). The ability of enemy missile position is restricted to the suppositional sensor (track homing radar) and weapon (short ground to air missile).

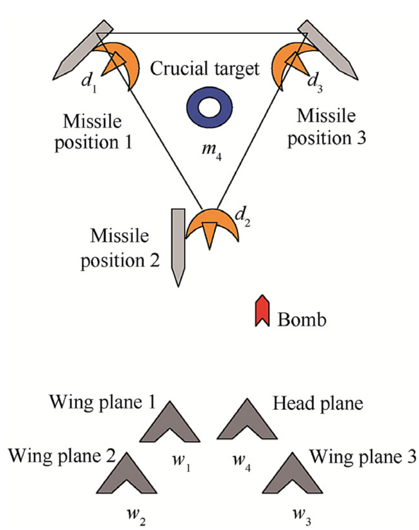

Fig. 2 A typical scenario of SEAD.

4.2. Game model

The mathematical model used for developing simulation software package is established based on the scenario in Section 4.1 and the algorithm designed in Section 3 as follows:

$\overline{G} = \langle \overline{N}, \overline{S}_1, \overline{S}_2, \bar{u}_1, \bar{u}_2 \rangle, \overline{N} = \{\bar{n}_1, \bar{n}_2\}$

$\bar{n}_1$ represents our team consisting of four UCAVs and $\bar{n}_2$ represents enemy group consisting of three missile positions and a crucial target. $\overline{S}_1 = \{\alpha_1, \alpha_2, \ldots, \alpha_{12}\}$ represents the primary strategy set of our four UCAVs cooperatively attacking four foe's targets (compared with other strategies, these strategies are employed with higher probability of over 50%), where

$$\begin{cases} \alpha_1 = \{(w_1, d_1), (w_2, d_2), (w_3, m_4), (w_4, d_3)\} \\ \alpha_2 = \{((w_1, w_2), d_1), (w_3, d_2), (w_4, m_4), (\varnothing, d_3)\} \\ \alpha_3 = \{((w_1, w_2, w_3), d_1), (\varnothing, d_2), (w_4, m_4), (\varnothing, d_3)\} \\ \alpha_4 = \{(w_1, d_1), ((w_2, w_3), d_2), (w_4, m_4), (\varnothing, d_3)\} \\ \alpha_5 = \{(\phi, d_1), ((w_1, w_2, w_3), d_2), (w_4, m_4), (\varnothing, d_3)\} \\ \alpha_6 = \{(w_1, d_1), ((w_2, w_3), d_2), (\varnothing, m_4), (w_4, d_3)\} \\ \alpha_7 = \{((w_1, w_2), d_1), (w_3, d_2), (\varnothing, m_4), (w_4, d_3)\} \\ \alpha_8 = \{((w_1, w_2, w_3), d_1), (w_4, d_2), (\varnothing, m_4), (\varnothing, d_3)\} \\ \alpha_9 = \{((w_1, w_2, w_3, w_4), d_1), (\varnothing, d_2), (\varnothing, m_4), (\varnothing, d_3)\} \\ \alpha_{10} = \{(\varnothing, d_1), ((w_1, w_2), d_2), (\varnothing, m_4), ((w_3, w_4), d_3)\} \\ \alpha_{11} = \{(\varnothing, d_1), ((w_1, w_2, w_3), d_2), (\varnothing, m_4), (w_4, d_3)\} \\ \alpha_{12} = \{(\varnothing, d_1), ((w_1, w_2, w_3, w_4), d_2), (\varnothing, m_4), (\varnothing, d_3)\} \end{cases}$$

where w_i $(i = 1, 2, 3, 4)$ represent the four UCAVs in our team, and $d_j (j = 1, 2, 3)$ represent the missile positions in the foe's team; m_4 represents a foe's crucial target. The term $\{(x, y), z\}$ represents that x and y attack cooperatively z, and $(\varnothing, m_4)$ represents none of our aircrafts attack the foe's target m_4.

The meaning of the strategy α_i $(i = 1, 2, \ldots, 12)$ is explained by describing the strategy α_4 as follows: α_4 represents that our UCAV w_1 attacks foe's missile position d_1, our two UCAVs w_2 and w_3 attack cooperatively foe's missile position d_2, our UCAV w_4 attacks foe's crucial target m_4, and none of our aircraft attack foe's missile position d_3.

The character of the strategies $\alpha_1 - \alpha_5$ is that foe's missile positions and crucial target are attacked comparably, while the character of the strategies $\alpha_6 - \alpha_{12}$ is that the action of attacking foe's missile positions is prior to the action of attacking foe's crucial target.

$\overline{S}_2 = \{\beta_1, \beta_2, \ldots, \beta_6\}$ represents the primary strategy set of four foe's targets cooperatively defending our four UCAVs (compared with other strategies, these strategies are employed with higher probability of over 50%), where

$$\begin{cases} \beta_1 = \{(d_1, w_1), (d_2, w_2), (d_3, w_4), (\varnothing, w_3)\} \\ \beta_2 = \{((d_1, d_2), w_1), (d_3, w_4), (\varnothing, w_2), (\varnothing, w_3)\} \\ \beta_3 = \{((d_1, d_2, d_3), w_1), (\varnothing, w_2), (\varnothing, w_3), (\varnothing, w_4)\} \\ \beta_4 = \{(d_1, w_1), (d_2, w_3), (d_3, w_4), (\varnothing, w_2)\} \\ \beta_5 = \{(d_1, w_1), ((d_2, d_3), w_4), (\varnothing, w_2), (\varnothing, w_3)\} \\ \beta_6 = \{((d_1, d_2, d_3), w_4), (\varnothing, w_1), (\varnothing, w_2), (\varnothing, w_3)\} \end{cases}$$

The meaning of the strategy $\beta_j\ (i = 1, 2, \ldots, 6)$ can be explained similarly according to the meaning of the strategy $\alpha_i\ (i = 1, 2, \ldots, 12)$.

When our strategy α_i and foe's strategy β_j are selected simultaneously, our payment function is set as

$$\begin{cases} \bar{u}_1(\alpha_i, \beta_j) = a_{ij} = \dfrac{\varpi^{\alpha_i}(a)\vartheta^{\beta_j}(b)}{\varpi^{\alpha_i}(b)\vartheta^{\beta_j}(a)} \\ i = 1, 2, \ldots, 12; \quad j = 1, 2, \ldots, 6 \end{cases} \tag{12}$$

Similarly, the foe's payment function is set as follows:

$$\begin{cases} \bar{u}_2(\alpha_i, \beta_j) = b_{ij} = \dfrac{\varpi^{\alpha_i}(b)\vartheta^{\beta_j}(a)}{\varpi^{\alpha_i}(a)\vartheta^{\beta_j}(b)} \\ i = 1, 2, \ldots, 12; \quad j = 1, 2, \ldots, 6 \end{cases} \tag{13}$$

Actually, Eqs. (12) and (13) are embodiment of Eqs. (10) and (11) after our strategies α_i and foe's strategies β_j are confirmed.

4.3. Results

The simulation examples consist of a set of computing processes as follows:

Firstly, when α_i and β_j mentioned in Section 4.2 are selected simultaneously, the actual capability of the suppositional sensor (SAR radar) and weapon (JDAM bomb) aboard the UCAV and the suppositional sensor (track homing radar) and weapon (short ground to air missile) in the missile position mentioned in Section 4.1 may be transformed respectively into the unitary fighting effect values $(\varpi^{\alpha_i}_{\text{RL}}(\cdot), \varpi^{\alpha_i}_{\text{R}\Theta}(\cdot), \varpi^{\alpha_i}_{\text{R}\Psi}(\cdot), \varpi^{\alpha_i}_{\text{ML}}(\cdot), \varpi^{\alpha_i}_{\text{M}\Theta}(\cdot)$ and $\varpi^{\alpha_i}_{\text{M}\Psi}(\cdot)$; $\vartheta^{\beta_j}_{\text{RL}}(\cdot), \vartheta^{\beta_j}_{\text{R}\Theta}(\cdot), \vartheta^{\beta_j}_{\text{R}\Psi}(\cdot), \vartheta^{\beta_j}_{\text{ML}}(\cdot), \vartheta^{\beta_j}_{\text{M}\Theta}(\cdot)$ and $\vartheta^{\beta_j}_{\text{M}\Psi}(\cdot))$ by adopting the algorithm designed in Section 2.

Secondly, the total fighting effect (including validity, invalidity and uncertainty) of the whole of our aircrafts team $\varpi^{\alpha_i}(\cdot)$ and the total fighting effect of the whole of foe's target group $\vartheta^{\beta_j}(\cdot)$, mentioned in Eqs. (12) and (13) in Section 4.2, can be acquired by a set of computing processes based on the algorithms designed in Section 2 and the unitary fighting effect values acquired by the above computing.

As a result, our payment matrix $\overline{\boldsymbol{A}} = [\bar{a}_{ij}]_{m\times n}$ and foe's payment matrix $\overline{\boldsymbol{B}} = [\bar{b}_{ij}]_{\bar{m}\times\bar{n}}$ mentioned in Section 3 can be acquired by computing Eqs. (12) and (13) and shown as follows:

$\overline{\boldsymbol{A}} = [\bar{a}_{ij}]_{12\times 6} =$

	β_1	β_2	β_3	β_4	β_5	β_6
α_1	1.120	3.692	9.216	1.115	3.690	8.687
α_2	2.314	7.631	19.05	2.305	7.626	17.96
α_3	0.126	0.416	1.038	0.126	0.416	0.978
α_4	12.56	41.41	103.4	12.51	41.38	97.43
α_5	2.724	8.984	22.43	2.714	8.979	21.14
α_6	2.408	7.940	19.82	2.399	7.934	18.68
α_7	0.444	1.463	3.653	0.442	1.462	3.443
α_8	0.012	0.039	0.098	0.012	0.039	0.092
α_9	0.009	0.029	0.071	0.009	0.029	0.067
α_{10}	0.096	0.318	0.793	0.096	0.317	0.747
α_{11}	0.522	1.723	4.301	0.521	1.722	4.054
α_{12}	0.113	0.374	0.933	0.113	0.374	0.880

,

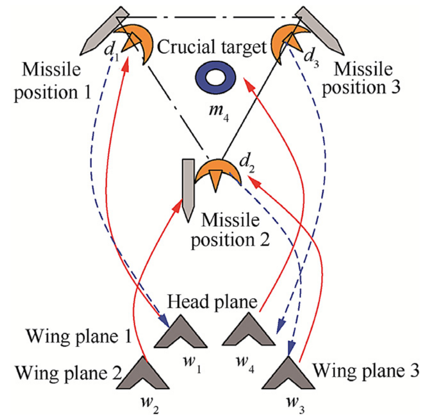

Fig. 3 Our No. 4 strategy versus foe's No. 4 strategy.

$\overline{\boldsymbol{B}} = [\bar{b}_{ij}]_{12\times 6} =$

	β_1	β_2	β_3	β_4	β_5	β_6
α_1	0.893	0.271	0.109	0.897	0.271	0.115
α_2	0.432	0.131	0.052	0.434	0.131	0.056
α_3	7.931	2.405	0.963	7.960	2.406	1.022
α_4	0.080	0.024	0.010	0.080	0.024	0.010
α_5	0.367	0.111	0.045	0.368	0.111	0.047
α_6	0.415	0.126	0.050	0.417	0.126	0.054
α_7	2.254	0.683	0.274	2.262	0.684	0.290
α_8	84.39	25.59	10.25	84.71	25.61	10.88
α_9	115.4	34.99	14.02	115.8	35.01	14.87
α_{10}	10.38	3.149	1.262	10.42	3.151	1.338
α_{11}	1.914	0.580	0.233	1.921	0.581	0.247
α_{12}	8.820	2.675	1.071	8.853	2.676	1.137

Finally, the Nash equilibrium state of the game model $\overline{G}$ mentioned in Section 4.2 may be acquired by solving the matrix $\overline{\boldsymbol{A}}$ and $\overline{\boldsymbol{B}}$ based on the Scarf arithmetic in the Gambit software package.

In the situation of multi-aircraft cooperatively attacking multi-target enacted in Section 4.1, the Nash equilibrium state of the game model $\overline{G}$ is that our strategy α_4 and foe's strategy β_4 are selected simultaneously.

Our strategy of α_4 and foe's strategy of β_4 will be regarded as the mission decision-making of our team antagonizing foe's group (Fig. 3) and will be used to determine the fighting action of our aircraft team and to guess the defending action of foe's target group, respectively.

5. Analysis

For validating the rationality of the studying results in Section 4, on the one hand, the other two strategies α_5 and α_6 are selected to antagonize foe's strategy β_4, because the fighting effect difference between the two strategies and strategy α_4 is the least (Figs. 4 and 5); on the other hand, the other two strategies β_2 and β_6 are selected to antagonize our strategy α_4, because the fighting effect difference between the two strategies and strategy β_4 is the least (Figs. 6 and 7).

The results of comparing the fighting effect of our strategies $(\alpha_4, \alpha_5$ and $\alpha_6)$ and the fighting effect of foe's strategy β_4 are

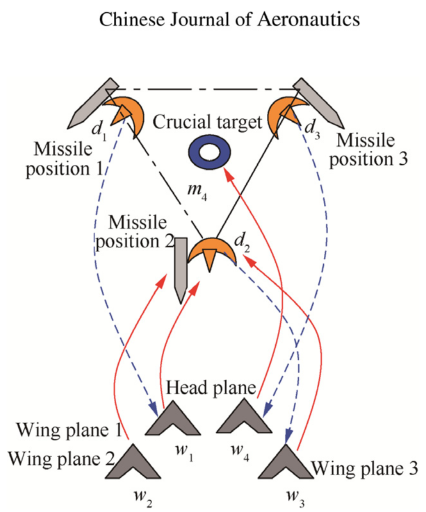

Fig. 4 Our No. 5 strategy versus foe's No. 4 strategy.

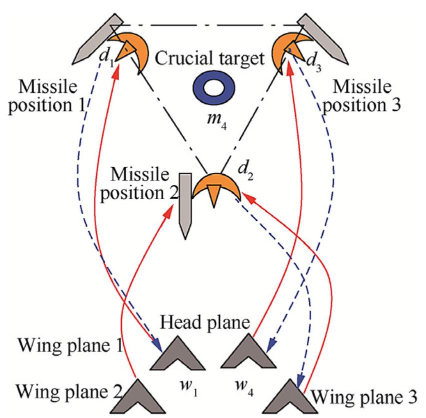

Fig. 5 Our No. 6 strategy versus foe's No. 4 strategy.

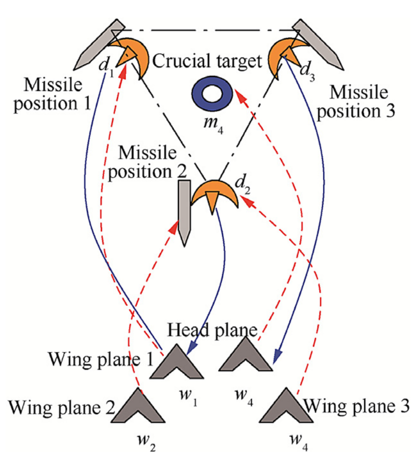

Fig. 6 Foe's No. 2 strategy versus our No. 4 strategy.

shown in Table 1, while the results of comparing the fighting effect of foe's strategies (β_2, β_4 and β_6) and the fighting effect of our strategy α_4 are shown in Table 2.

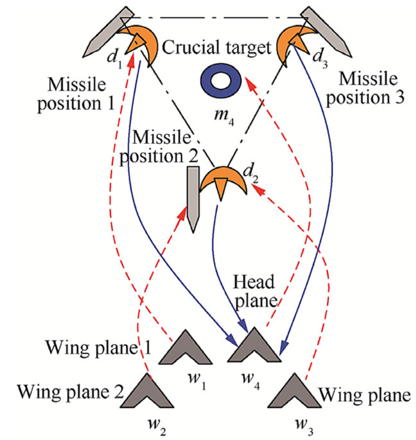

Fig. 7 Foe's No. 6 strategy versus our No. 4 strategy.

Table 1 Fighting effect comparison of our strategies.

Item	a	b	θ
$\varpi^{\alpha_4}(\cdot)$	0.988	0.011	0.001
$\vartheta^{\beta_2}(\cdot)$	0.669	0.330	0.001
$\vartheta^{\beta_4}(\cdot)$	0.869	0.130	0.001
$\vartheta^{\beta_6}(\cdot)$	0.467	0.532	0.001

Table 2 Fighting effect comparison of foe's strategies.

Item	a	b	θ
$\vartheta^{\beta_4}(\cdot)$	0.869	0.130	0.001
$\varpi^{\alpha_4}(\cdot)$	0.988	0.011	0.001
$\varpi^{\alpha_5}(\cdot)$	0.947	0.052	0.001
$\varpi^{\alpha_6}(\cdot)$	0.940	0.059	0.001

Table 1 indicates that the fighting effects (validity) of our strategies α_4, α_5 and α_6 are $\varpi^{\alpha_4}(a) = 0.988, \varpi^{\alpha_5}(a) = 0.947$ and $\varpi^{\alpha_6}(a) = 0.940$, respectively. As a result, it can be explained why our strategy α_4 is regarded as the fighting action that our aircraft team should carry out when our team is antagonizing foe's group in the case described in Section 4.1.

Similarly, Table 2 indicates that the fighting effects (validity) of foe's strategies β_2, β_4 and β_6 are $\vartheta^{\beta_2}(a) = 0.669, \vartheta^{\beta_4}(a) = 0.869$ and $\vartheta^{\beta_6}(a) = 0.467$, respectively. As a result, it can also be explained why foe's strategy β_4 is regarded as the defending action that foe's target group may carry out in the same case.

To further evaluate the results of the proposed mission decision-making method in Section 4.3, we study the situation analyzing process of our equilibrium strategy α_4 in comparison with the strategy α_5. Tables 3 and 4 list the fighting effect values which are obtained by using Eqs. (6)–(9) in Section 2.

According to the description in Section 4.3, the equilibrium results of the game model depend on our payment matrix $\overline{\boldsymbol{A}}$ and the foe's payment matrix $\overline{\boldsymbol{B}}$. Each element in the payment matrix $\overline{\boldsymbol{A}}$ and matrix $\overline{\boldsymbol{B}}$ can be computed by the payment

Table 3 Situation analyzing process of our No. 4 strategy.

Item	a	b	θ
$\varpi_{R_{11}}^{\alpha_4}(\cdot)$	0.655	0.082	0.263
$\varpi_{M_{11}}^{\alpha_4}(\cdot)$	0.206	0.659	0.135
$\varpi_{11}^{\alpha_4}(\cdot)$	0.503	0.432	0.065
$\varpi_{1}^{\alpha_4}(\cdot)$	0.503	0.432	0.065
$\varpi_{R_{22}}^{\alpha_4}(\cdot)$	0.670	0.080	0.250
$\varpi_{M_{22}}^{\alpha_4}(\cdot)$	0.563	0.182	0.255
$\varpi_{22}^{\alpha_4}(\cdot)$	0.827	0.096	0.077
$\varpi_{R_{23}}^{\alpha_4}(\cdot)$	0.670	0.080	0.250
$\varpi_{M_{23}}^{\alpha_4}(\cdot)$	0.563	0.182	0.255
$\varpi_{23}^{\alpha_4}(\cdot)$	0.827	0.096	0.077
$\varpi_{2}^{\alpha_4}(\cdot)$	0.964	0.028	0.006
$\varpi_{30}^{\alpha_4}(\cdot)$	0.299	0.689	0.012
$\varpi_{3}^{\alpha_4}(\cdot)$	0.299	0.689	0.012
$\varpi_{R_{44}}^{\alpha_4}(\cdot)$	0.701	0.062	0.237
$\varpi_{M_{44}}^{\alpha_4}(\cdot)$	0.572	0.188	0.240
$\varpi_{44}^{\alpha_4}(\cdot)$	0.846	0.085	0.069
$\varpi_{4}^{\alpha_4}(\cdot)$	0.846	0.085	0.069
$\varpi^{\alpha_4}(\cdot)$	0.988	0.011	0.001

Table 4 Situation analyzing process of our No. 5 strategy.

Item	a	b	θ
$\varpi_{10}^{\alpha_5}(\cdot)$	0.299	0.689	0.012
$\varpi_{1}^{\alpha_5}(\cdot)$	0.299	0.689	0.012
$\varpi_{R_{21}}^{\alpha_5}(\cdot)$	0.263	0.319	0.418
$\varpi_{M_{21}}^{\alpha_5}(\cdot)$	0.151	0.485	0.364
$\varpi_{21}^{\alpha_5}(\cdot)$	0.241	0.575	0.184
$\varpi_{R_{22}}^{\alpha_5}(\cdot)$	0.670	0.080	0.250
$\varpi_{M_{22}}^{\alpha_5}(\cdot)$	0.563	0.182	0.255
$\varpi_{22}^{\alpha_5}(\cdot)$	0.827	0.096	0.077
$\varpi_{R_{23}}^{\alpha_5}(\cdot)$	0.670	0.080	0.250
$\varpi_{M_{23}}^{\alpha_5}(\cdot)$	0.563	0.182	0.255
$\varpi_{23}^{\alpha_5}(\cdot)$	0.827	0.096	0.077
$\varpi_{2}^{\alpha_5}(\cdot)$	0.938	0.059	0.003
$\varpi_{30}^{\alpha_5}(\cdot)$	0.299	0.689	0.012
$\varpi_{3}^{\alpha_5}(\cdot)$	0.299	0.689	0.012
$\varpi_{R_{44}}^{\alpha_5}(\cdot)$	0.701	0.062	0.237
$\varpi_{M_{44}}^{\alpha_5}(\cdot)$	0.572	0.188	0.240
$\varpi_{44}^{\alpha_5}(\cdot)$	0.846	0.085	0.069
$\varpi_{4}^{\alpha_5}(\cdot)$	0.846	0.085	0.069
$\varpi^{\alpha_5}(\cdot)$	0.947	0.052	0.001

functions Eqs. (12) and (13). Therefore, the items $\varpi^{\alpha_4}(a), \varpi^{\alpha_4}(b), \varpi^{\alpha_5}(a)$ and $\varpi^{\alpha_5}(b)$ in the payment functions Eqs. (12) and (13) mainly determine that our equilibrium strategy α_4 has better results than the strategy α_5.

Table 3 presents the obtaining process of the items $\varpi^{\alpha_4}(a)$ and $\varpi^{\alpha_4}(b)$, which shows the quantitative analysis and computing process of the combat situation that our multiple aircrafts cooperatively attack multi-object with strategy α_4. Similarly, Table 4 presents the obtaining process of the items $\varpi^{\alpha_5}(a)$ and $\varpi^{\alpha_5}(b)$, which shows the quantitative analysis and computing process of the combat situation that our multiple aircrafts cooperatively attack multi-object with strategy α_5.

In the following part, the numerical results in Tables 3 and 4 are analyzed to obtain the underlying reason why strategy α_4 has better results than strategy α_5.

Comparing the data in Tables 3 and 4, we can see that the fighting effects against the foe's missile position 3 and the high value target m_4 are the same in strategy α_4 and strategy α_5. The distinctions are delineated as follows:

(1) In strategy α_4, our aircraft 1 is deployed to attack the foe's missile position 1, and the computed fighting effect value $\varpi_1^{\alpha_4}(a)$ is 0.503; in strategy α_5, our aircraft is not deployed to attack the foe's missile position 1, and according to Eq. (8), the computed fighting effect value $\varpi_1^{\alpha_5}(a)$ is 0.299.

(2) In strategy α_4, our aircraft 2 and aircraft 3 are deployed to cooperatively attack the foe's missile position 2, and the computed fighting effect value $\varpi_2^{\alpha_4}(a)$ is 0.964; in strategy α_5, our aircraft 1, aircraft 2 and aircraft 3 are deployed to cooperatively attack the foe's missile position 2, and the computed fighting effect value $\varpi_2^{\alpha_5}(a)$ is 0.938.

In light of the above reasons (1) and (2), the fighting effect value $\varpi^{\alpha_4}(a)$ (0.988) with strategy α_4 is larger than the fighting effect value $\varpi^{\alpha_5}(a)$ (0.947) with strategy α_5.

In strategy α_5, why does the fighting effect value decrease when our aircraft 1 is added to attack foe's missile position 2? In Table 4, it can be found that the fighting effect value $\varpi_{21}^{\alpha_5}(a)$ is 0.241 when our aircraft 1 is deployed to attack foe's missile position 2. In other words, the effectiveness turns bad when our aircraft 1 is added to attack foe's missile position 2, which has a negative impact on the original strategy of our aircraft 2 and aircraft 3 cooperatively attacking foe's missile position 2. Therefore, the overall fighting effect value against foe's missile position 2 decreases when our aircraft 1 is added. It also demonstrates that "more is not necessarily better".

In the scenario of Section 4.1, the horizontal distance between our aircraft 1 and the foe's missile position 2 is 4582 m. The pitch angle and azimuth angle are 65.8° and 57.3°, respectively.

To analyze the essential reason why our aircraft 1 should not be deployed to attack foe's missile position 2, the situation analysis is performed according to Eqs. (4)–(6). The detailed results are listed in Table 5.

Table 5 Situation analyzing process of our aircraft 1 attacking foe's missile position 2.

Item	a	b	θ
$\varpi_{RL_{21}}^{\alpha_5}(\cdot)$	0.725	0.265	0.010
$\varpi_{R\Theta_{21}}^{\alpha_5}(\cdot)$	0.100	0.890	0.010
$\varpi_{R\Psi_{21}}^{\alpha_5}(\cdot)$	0.744	0.246	0.010
$\varpi_{R_{21}}^{\alpha_5}(\cdot)$	0.263	0.319	0.418
$\varpi_{ML_{21}}^{\alpha_5}(\cdot)$	0.681	0.309	0.010
$\varpi_{M\Theta_{21}}^{\alpha_5}(\cdot)$	0.100	0.890	0.010
$\varpi_{M\Psi_{21}}^{\alpha_5}(\cdot)$	0.100	0.890	0.010
$\varpi_{M_{21}}^{\alpha_5}(\cdot)$	0.152	0.485	0.363
$\varpi_{21}^{\alpha_5}(\cdot)$	0.241	0.575	0.184

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According to the effective range and angle of the sensor (SAR radar) and weapon (JDAM bomb), the horizontal distance between our aircraft 1 and foe's missile position 2 is proper, which supports the attack missions. In Table 5, it can be found that both the fighting effect value with the sensor $\varpi^{\alpha_5}_{\mathrm{RL}_{21}}(a)$ (0.725) and that with the weapon $\varpi^{\alpha_5}_{\mathrm{ML}_{21}}(a)$ (0.681) are larger than 0.5.

The azimuth angle between our aircraft 1 and foe's missile position 2 (57.3° > 45°) has a large difference with the desired direction. Because the lateral detection is the main direction for the sensor (SAR radar), a large azimuth angle is near the main detection direction, which can improve the sensor performance and better support the attack mission. In Table 5, it can be found that the fighting effect value with the sensor azimuth angle $\varpi^{\alpha_5}_{\mathrm{R}\Psi_{21}}(a)$ (0.744) is larger than 0.5. The weapon (JDAM bomb) is the unpowered gliding weapon, and therefore, the yaw control of the weapon cannot be performed before its landing if the azimuth angle is too large at some given altitude (i.e., given gliding time). The hit probability may also decrease, which cannot support the attack mission. Table 5 shows that the fighting effect value with the weapon azimuth angle $\varpi^{\alpha_5}_{\mathrm{M}\Psi_{21}}(a)$ (0.100) is smaller than 0.5.

The pitch angle (65.8° > 45°) between our aircraft 1 and foe's missile position 2 has a large overlook range. The offset of the sensor (SAR radar) antenna depends on the inner state of aircraft and has a limited range. Therefore, it is hard to achieve a large pitch angle, which blocks the performance of the sensor and the attack mission. In Table 5, it can be found that the fighting effect value with the sensor pitch angle $\varpi^{\alpha_5}_{\mathrm{R}\Theta_{21}}(a)$ (0.100) is smaller than 0.5. The weapon (JDAM bomb) is the unpowered gliding weapon, and therefore, the flight path must be adjusted if the relative pitch angle is too large at some given altitude (i.e., given gliding time). Otherwise, the weapon cannot hit the target and support the attack mission before its landing. Table 5 shows that the fighting effect value with the weapon azimuth angle $\varpi^{\alpha_5}_{\mathrm{M}\Theta_{21}}(a)$ (0.100) is smaller than 0.5.

In a word, the fighting effect value with the sensor pitch angle does not support the attack mission ($\varpi^{\alpha_5}_{\mathrm{R}\Theta_{21}}(a) = 0.100$), which prevents the sensor evidence from supporting the attack mission ($\varpi^{\alpha_5}_{\mathrm{R}_{21}}(a) = 0.263$). In addition, both the fighting effect value with the weapon pitch angle and that with azimuth angle do not support the attack mission ($\varpi^{\alpha_5}_{\mathrm{M}\Theta_{21}}(a) = \varpi^{\alpha_5}_{\mathrm{M}\Psi_{21}}(a) = 0.100$), and therefore, the weapon evidence cannot support the attack mission ($\varpi^{\alpha_5}_{\mathrm{M}_{21}}(a) = 0.152$).

Under the condition that both the sensor and weapon evidences do not support the attack mission, it is easy to conclude that, in the air combat scenario, our aircraft 1 should not be deployed to attack foe's missile position 2.

6. Conclusions

By using the above numerical simulations, the proposed mission decision-making method of multi-aircraft cooperatively attacking multi-object based on situation analysis can be concluded as follows:

(1) The general rule of the real combat process can be demonstrated on the basis of the equilibrium results, which are obtained by selecting the horizontal distance, pitch angle, and azimuth angle between our aircraft and foe's target, determining the fighting effect values of the sensor and weapon, and performing the quantitative analysis of the combat situation that our multiple aircrafts cooperatively attack multi-object by D-S evidence theory.

(2) Based on the complete information static game theory, the mission decision-making model of the multi-aircraft cooperatively attacking multi-object is established. The payment function can also be constructed, which realizes the parameter transfer between the quantitative analysis of air combat situation and optimal mission decision-making. The equilibrium results of the game model are computed by using the static dual matrix game algorithm. The air combat strategy can be determined by the Nash equilibrium solution which reflects the real air combat situation and antagonizing ability between two sides.

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