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Galilean symmetry in noncommutative field theory

P.A. Horváthy [a], L. Martina [b], P.C. Stichel [c]

[a] *Laboratoire de Mathématiques et de Physique Théorique, Université de Tours, Parc de Grandmont, F-37200 Tours, France*
[b] *Dipartimento di Fisica dell'Università and Sezione INFN di Lecce, Via Arnesano, C.P. 193, I-73100 Lecce, Italy*
[c] *An der Krebskuhle 21, D-33619 Bielefeld, Germany*

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Abstract

When the interaction potential is suitably reordered, the Moyal field theory admits two types of Galilean symmetries, namely the conventional mass-parameter-centrally-extended one with commuting boosts, but also the two-fold centrally extended "exotic" Galilean symmetry, where the commutator of the boosts yields the noncommutative parameter. In the free case, one gets an "exotic" two-parameter central extension of the Schrödinger group. The conformal symmetry is, however, broken by the interaction.

1. Introduction

In a recent paper [1] Bak et al. consider a scalar field theory on the noncommutative plane, described by the action $S = S_0 + S^{\star}_{\text{int}} = \int d^2\vec{x}\, dt\, \mathcal{L}$,

$$\mathcal{L} = \mathcal{L}_0 - V^{\star}$$
$$= \left(i\bar{\psi}\partial_t\psi + \bar{\psi}\frac{\Delta\psi}{2} \right) - \frac{\lambda}{2}\,\bar{\psi} \star \bar{\psi} \star \psi \star \psi, \tag{1.1}$$

where the star means the Moyal product associated with the noncommutative parameter θ. Although this looks like a nonrelativistic theory, Bak et al. mention (without proof) that both the Galilean and scale invariance are lost. Our aim here is to point out that the Galilean symmetry can be restored by suitably reordering the interaction potential. Then we find that the symmetry can be implemented in *two different ways*: while the conventional one yields the usual, one-parameter central extension, another, "Moyal-type" implementation yields the "exotic" two-parameter centrally extended Galilean symmetry, found before in a point particle context [2]. We confirm that any nontrivial interaction does indeed break the scale invariance, but in the free case the symmetry actually extends to a novel type of "exotic" (i.e., two-parameter-centrally-extended) conformal (Schrödinger) symmetry.

2. Exotic Galilean symmetry

Let us start with the boosts, whose infinitesimal action on nonrelativistic space–time, $\delta_b\vec{x} = \vec{b}t$, $\delta_b t = 0$, is conventionally implemented as

$$\delta^0_b\psi = i\vec{b}\cdot\vec{x}\psi - t\vec{b}\cdot\vec{\nabla}\psi. \tag{2.1}$$

E-mail address: horvathy@univ-tours.fr (P.A. Horváthy).

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This changes the free part of the Lagrange density in (1.1) by a surface term, $\delta\mathcal{L}_0 = -t\vec{b}\cdot\vec{\nabla}\mathcal{L}_0$. In the commutative case the general potential is $V(\rho)$ with $\rho = \bar{\psi}\psi$. $\delta^0\rho = -t\vec{b}\cdot\vec{\nabla}\rho$, and hence the potential changes in the same way as the free part, $\delta^0 V = V'\delta\rho = -t\vec{b}\cdot\vec{\nabla}V$. In conclusion,

$$\delta_b^0\mathcal{L} = -t\vec{b}\cdot\vec{\nabla}\mathcal{L} \tag{2.2}$$

implying the Galilean invariance of the action. In the noncommutative case, however, the interaction potential in (1.1) is equivalent rather to

$$V^* = (\lambda/2)\rho_r\rho_l, \qquad \rho_r = \bar{\psi}\star\psi, \qquad \rho_l = \psi\star\bar{\psi}. \tag{2.3}$$

Then the relations

$$\begin{aligned} f\star(x_i g) &= x_i(f\star g) - \frac{i\theta}{2}\epsilon_{ij}\partial_j f\star g,\\ (x_i f)\star g &= x_i(f\star g) + \frac{i\theta}{2}\epsilon_{ij} f\star\partial_j g \end{aligned} \tag{2.4}$$

readily inferred from the definition of the Moyal product allow us to establish

$$\delta_b^0\rho_a = \pm\frac{\theta}{2}\vec{b}\times\vec{\nabla}\rho_a - t\vec{b}\cdot\vec{\nabla}\rho_a \tag{2.5}$$

with the plus sign for $a = r$ and the minus for $a = l$. Hence the potential changes as

$$\delta_b^0 V^* = \theta\lambda\vec{b}\times(\vec{\nabla}\rho_r\star\rho_l - \rho_r\star\vec{\nabla}\rho_l) - t\vec{b}\cdot\vec{\nabla}V^*. \tag{2.6}$$

Owing to the sign change above, the first term here is *not* a surface term. The invariance is therefore broken, as stated by Bak et al. [1].

Now we argue that, in the Moyal context, (2.1) is *not* the correct way to act for a boost. Remember that the imaginary factor in front of ψ is in fact a[n infinitesimal] "compensating gauge transformation" which, in the present context, acts by the Moyal, rather then by the ordinary multiplication, $\psi \to g\star\psi$, $g \in U(1)_*$ [3]. (2.1) should therefore be modified as

$$\begin{aligned} \delta_b^\star\psi &= i\vec{b}\cdot\vec{x}\star\psi - t\vec{b}\cdot\vec{\nabla}\psi\\ &= i\vec{b}\cdot\vec{x}\psi - \frac{\theta}{2}\vec{b}\times\vec{\nabla}\psi - t\vec{b}\cdot\vec{\nabla}\psi,\\ \delta_b^\star\bar{\psi} &= -i\vec{b}\cdot\bar{\psi}\star\vec{x} - t\vec{b}\cdot\vec{\nabla}\bar{\psi}\\ &= -i\vec{b}\cdot\vec{x}\,\bar{\psi} - \frac{\theta}{2}\vec{b}\times\vec{\nabla}\bar{\psi} - t\vec{b}\cdot\vec{\nabla}\bar{\psi}. \end{aligned} \tag{2.7}$$

The sign change in front of the first term here is consistent with the formula $\overline{f\star g} = \bar{g}\star\bar{f}$.

Let us first investigate the free theory. The new implementation (2.7) changes $\mathcal{L}_0$ again by a surface term

$$\delta_b^\star\mathcal{L}_0 = -t\vec{b}\cdot\vec{\nabla}\mathcal{L}_0 - \frac{\theta}{2}\vec{b}\times\vec{\nabla}\mathcal{L}_0, \tag{2.8}$$

cf. (2.2), so that the free action S_0 is left invariant. Hence, the free theory admits our new type of Galilean symmetry.

The new term in (2.7) contributes to the conserved quantity associated through Noether's theorem, which says that if $\mathcal{L}$ changes as $\delta\mathcal{L} = \partial_\alpha K^\alpha$ under an infinitesimal coordinate change $\delta\vec{x}$, then

$$\int\left(\frac{\delta\mathcal{L}}{\delta(\partial_t\psi)}\delta\psi + \delta\bar{\psi}\frac{\delta\mathcal{L}}{\delta(\partial_t\bar{\psi})} - K^t\right)d^2\vec{x} \tag{2.9}$$

is a constant of the motion. For a boost, implemented as in (2.7), we get

$$\mathcal{G}_i = -\int d^2\vec{x}\,x_i|\psi|^2 + t\mathcal{P}_i + \frac{\theta}{2}\epsilon_{ij}\mathcal{P}_j, \tag{2.10}$$

where $\mathcal{P}_i = -i\int d^2x\,\bar{\psi}\partial_i\psi$ is the momentum. Our clue is that the extra piece proportional to θ changes the commutator of the boost components

$$\{\mathcal{G}_i, \mathcal{G}_j\} = \epsilon_{ij}k, \quad k \equiv \theta\int d^2x\,|\psi|^2, \tag{2.11}$$

where the Poisson bracket is

$$\{\mathcal{F}, \mathcal{G}\} = (-i)\int d^2\vec{x}\left(\frac{\delta\mathcal{F}}{\delta\psi}\frac{\delta\mathcal{G}}{\delta\bar{\psi}} - \frac{\delta\mathcal{G}}{\delta\psi}\frac{\delta\mathcal{F}}{\delta\bar{\psi}}\right). \tag{2.12}$$

Adding the energy, the angular momentum, and the mass,

$$\mathcal{H}_0 = \int d^2\vec{x}\,\frac{1}{2}|\vec{\nabla}\psi|^2, \tag{2.13}$$

$$\mathcal{J} = -i\int d^2\vec{x}\,\epsilon_{ij}x_i\bar{\psi}\partial_j\psi, \tag{2.14}$$

$$M = \int d^2\vec{x}\,\rho_r = \int d^2\vec{x}\,|\psi|^2 \tag{2.15}$$

(also derived by Noether's theorem), we obtain the "exotic" *two-fold centrally extended* Galilei algebra [2], whose commutation relations only differ from those of the usual, singly-extended Galilean algebra in that the boosts yield the second central charge k in (2.11). The usual central term is the mass M, associated with the phase invariance.

It is worth noting that the conventional implementation (2.1) used by Hagen [4] yields instead (2.10) without the extra piece, so that the boost components commute.[1]

We conclude that, for a free particle, both (2.1) and (2.7) act as symmetries.

Let us mention that the infinitesimal action associated with a conserved quantity $\mathcal{G}$ is

$$\delta^{\star}_{\mathcal{G}}\psi = \{\psi, \mathcal{G}\}. \tag{2.16}$$

Interestingly, choosing the equivalent expression $-i\psi\partial_t\bar{\psi}$ in the free action, the phase invariance would yield $\int d^2\vec{x}\,\rho_l$ instead of $\int d^2\vec{x}\,\rho_r$; due to the integral property $\int d^2\vec{x}\, f\star g = \int d^2\vec{x}\, fg$, this is, however, the same as our previous M. Note also that while the ρ_a are not positive definite, their integral, M, is positive.

Both densities satisfy a continuity equation

$$\partial_t\rho_a + \vec{\nabla}\cdot\vec{j}_a = 0, \qquad \vec{j}_r = \vec{\nabla}\psi\star\bar{\psi} - \psi\star\vec{\nabla}\bar{\psi},$$
$$\vec{j}_l = \bar{\psi}\star\vec{\nabla}\psi - \vec{\nabla}\bar{\psi}\star\psi. \tag{2.17}$$

This could be used as the starting point for a noncommutative hydrodynamics [6]. Note that while the coordinate-space densities have complicated commutation relations, in momentum-space they satisfy the trigonometric algebra [7]

$$\bigl\{\tilde{\rho}_a(\vec{q}), \tilde{\rho}_a(\vec{p})\bigr\} = \pm 2\sin\left(\frac{\theta}{2}\vec{q}\times\vec{p}\right)\tilde{\rho}_a(\vec{q}+\vec{p}) \tag{2.18}$$

with the positive/negative sign for $a = r$ and $a = l$, respectively.

3. Exotic conformal symmetry

The 2-parameter conformal extension of the Galilei group (called the Schrödinger group) [8] can now be considered. The new generators are the dilations and expansions, implemented infinitesimally according to

$$\begin{aligned}
&\delta_\Delta\vec{x} = \Delta\vec{x}, \qquad \delta_\Delta t = 2\Delta t,\\
&\delta^0_\Delta\psi = -\Delta\bigl[\psi + \vec{x}\cdot\vec{\nabla}\psi + 2t\partial_t\psi\bigr],\\
&\delta_\kappa\vec{x} = \kappa t\vec{x}, \qquad \delta_\kappa t = \kappa t^2,\\
&\delta^0_\kappa\psi = -\kappa\left[\left(-\frac{i}{2}x^2 + t\right)\psi + t\vec{x}\cdot\vec{\nabla}\psi + t^2\partial_t\psi\right],
\end{aligned} \tag{3.1}$$

respectively, where $\Delta > 0$ and κ is real. ($\Delta\vec{x}$ means $\vec{x}$ multiplied by Δ and Δt means t multiplied by Δ.) More generally, an infinitesimal Schrödinger transformation is of the form $\delta x_i = f_i(x,t)$, $\delta t = g(t)$ with $f_i(x,t) = F_i(t) + x_i G(t)$. When implemented conventionally

$$\begin{aligned}
\delta^0\psi(x,t) = {} & ih(x)\psi(x,t) - f_i(x,t)\partial_i\psi(x,t)\\
& + \bigl[k(t) - g(t)\partial_t\bigr]\psi(x,t),
\end{aligned} \tag{3.2}$$

where the coefficients are suitable real functions, it leaves invariant $\mathcal{L}_0$ and is therefore a symmetry for a free field. The associated conserved quantities span, w.r.t. the Poisson bracket (2.12) the one-parameter-centrally-extended Schrödinger algebra [8].

With hindsight to the noncommutative case, let us consider instead[2]

$$\begin{aligned}
\delta^\star\psi(x,t) = {} & ih(x)\star\psi(x,t) - f_i(x,t)\partial_i\psi(x,t)\\
& + \bigl[k(t) - g(t)\partial_t\bigr]\psi(x,t).
\end{aligned} \tag{3.3}$$

The "exotic" dilatations act in the same way as the conventional ones in (3.1); for the "exotic" expansions we find

$$\begin{aligned}
\delta^*_\kappa\psi = {} & -\kappa\left[\left(-\frac{i}{2}x^2 + t\right)\psi + t\vec{x}\cdot\vec{\nabla}\psi + t^2\partial_t\psi\right]\\
& - \kappa\left[\frac{\theta}{2}\vec{x}\times\vec{\nabla}\psi + \frac{\theta^2}{4}\partial_t\psi\right].
\end{aligned} \tag{3.4}$$

The new transformation is readily seen to be still a symmetry that extends the "exotic" Galilei algebra by adding the two conformal generators[3]

$$\begin{aligned}
&\mathcal{D} = -2t\mathcal{H}_0 + \frac{1}{2i}\int d^2\vec{x}\, x_i\bigl(\bar{\psi}\partial_i\psi - (\partial_i\bar{\psi})\psi\bigr),\\
&\mathcal{K} = t^2\mathcal{H}_0 + t\mathcal{D} - \frac{1}{2}\int d^2\vec{x}\,\vec{x}^2|\psi|^2 + \frac{\theta}{2}\mathcal{J} - \frac{\theta^2}{4}\mathcal{H}_0.
\end{aligned} \tag{3.5}$$

Unlike in the commutative case, $\mathcal{D}, \mathcal{H}_0$ and $\mathcal{K}$ do *not* close to an $\mathrm{o}(2,1)$ [8] but yield

$$\begin{aligned}
&\{\mathcal{D}, \mathcal{H}_0\} = 2\mathcal{H}_0 \qquad \{\mathcal{D}, \mathcal{K}\} = -2\mathcal{K} + \theta\mathcal{J} - \theta^2\mathcal{H}_0,\\
&\{\mathcal{H}_0, \mathcal{K}\} = \mathcal{D}.
\end{aligned} \tag{3.6}$$

[1] This corrects an error in calculating the commutator, committed in [5].

[2] According to [9] one should take $-(1/2)(f_i\star\partial_i\psi + \partial_i\psi\star f_i)$ for the second term. But when f_i is at most linear in $\vec{x}$, this reduces to our expression. For $\bar{\psi}$ the first term becomes $-i\bar{\psi}\star h(x)$, cf. (2.7).

[3] Note that (3.1) is also consistent with (2.16).

When added to the Galilean generators we get a closed algebra, though, since the nonvanishing brackets read

$$\{\mathcal{K},\mathcal{G}_i\}=\theta\epsilon_{ij}\mathcal{G}_i,\qquad \{\mathcal{K},\mathcal{P}_i\}=\mathcal{G}_i,$$
$$\{\mathcal{D},\mathcal{G}_i\}=-\mathcal{G}_i+\theta\epsilon_{ij}\mathcal{P}_j. \tag{3.7}$$

The relations (2.11), (3.6), (3.7) define a new, two-parameter central extension of the Schrödinger algebra which seems to have escaped attention so far. We call it the "exotic Schrödinger algebra".

4. Symmetry properties of the potential

We now turn our attention to the potential. Let us first consider a boost, implemented "exotically" i.e., as in (2.7). It follows readily (cf. (2.5)) that the right and left densities transform differently

$$\delta^\star_b\rho_r=-t\vec{b}\cdot\vec{\nabla}\rho_r,$$
$$\delta^\star_b\rho_l=-t\vec{b}\cdot\vec{\nabla}\rho_l-\theta\vec{b}\times\vec{\nabla}\rho_l. \tag{4.1}$$

Hence, for $V^*=\rho_r\rho_l$ we get

$$\delta^\star_b V^\star=-t\vec{b}\cdot\vec{\nabla}V^*_r-\theta\vec{b}\times(\rho_r\star\vec{\nabla}\rho_l). \tag{4.2}$$

Here the term proportional to θ is not exact. Therefore, the Galilean invariance is broken by the potential $V^\star$ even for the new implementation (2.7).

Let us remember, however, how the potential comes about [11]. One starts with the second-quantized expression for the interaction

$$\int d^2\vec{x}\,d^2\vec{x}'\,\bar{\psi}(\vec{x})\bar{\psi}(\vec{x}')U(\vec{x}-\vec{x}')\psi(\vec{x})\psi(\vec{x}'), \tag{4.3}$$

where U is a two-body potential. Choosing the contact interaction $U=(\lambda/2)\delta(\vec{x}-\vec{x}')$ yields a quartic $V=(\lambda/2)\rho^2$.

Promoting the commutative theory into a noncommuting one requires to replacing the ordinary products by Moyal products. This requires particular care, though. For example, putting naively Moyal stars between the various factors in (4.3) while keeping the original order would lead to inconsistency: expressions of the form $\psi(\vec{x})\star\psi(\vec{x}')$ that should be defined due to associativity, would require us to redefine the Moyal product. Our clue is that this procedure is *ambiguous*, as the order of the factors is irrelevant in the commutative theory, but not in its Moyal version. Rearranging as, e.g.,

$$\int d^2\vec{x}\,d^2\vec{x}'\,\bar{\psi}(\vec{x})\star\psi(\vec{x})\star U(\vec{x}-\vec{x}')\star\bar{\psi}(\vec{x}')\star\psi(\vec{x}') \tag{4.4}$$

(where the various products *are* well-defined) would yield, instead of V^* in (1.1),

$$\widetilde{V}^*=\frac{\lambda}{2}\bar{\psi}\star\psi\star\bar{\psi}\star\psi \tag{4.5}$$

equivalent to $(\lambda/2)\rho_r^2$ or to $\widetilde{\widetilde{V}}{}^*=(\lambda/2)\rho_l^2$ (which could also be obtained by a suitable reordering). Remarkably, it is this expression that had been used by Lozano et al. in their noncommutative nonrelativistic Chern–Simons vortex construction [12], and also by Langmann et al. [13] in their recent exact scalar field solution in a background magnetic field.

The important fact for us is that the new interaction *is* Galilei invariant, since, by (4.1)

$$\delta^*_b\widetilde{V}^*=-t\vec{b}\cdot\vec{\nabla}\widetilde{V}^*. \tag{4.6}$$

Similarly, for $\widetilde{\widetilde{V}}{}^*$ we get $\delta^*_b\widetilde{\widetilde{V}}{}^*=-t\vec{b}\cdot\vec{\nabla}\widetilde{\widetilde{V}}-\theta\vec{b}\times\vec{\nabla}\widetilde{\widetilde{V}}{}^*$ which is again a surface term, so that the Galilean symmetry is again restored. Clearly, the same statement holds for any "pure" function of ρ_r or of ρ_l alone. For a mixture, $\rho=\epsilon_r\rho_r+\epsilon_l\rho_l$ where the ϵ_i are real coefficients, the cross term would break, however the symmetry, whenever $\epsilon_r\epsilon_l\neq 0$.

We failed to find a natural way to reproduce the potential $V^*=\bar{\psi}\star\bar{\psi}\star\psi\star\psi$ in (1.1), proposed by Bak et al. [1].[4] Another argument in favor of our choice (4.5) is that it is, unlike that of Bak et al., V^*, renormalizable as well as invariant w.r.t. deformed $U(1)$ transformations [14, p. 25]. It is, therefore, (4.5) that we shall adopt in what follows.

Interestingly, the modified potential $\widetilde{V}^*$ also allows the conventional symmetry (2.1).[5] (2.5) indeed implies that

$$\delta^0_b\rho_a^2=\pm\frac{\theta}{2}\vec{b}\times\vec{\nabla}\rho_a^2-t\vec{b}\cdot\vec{\nabla}\rho_a^2, \tag{4.7}$$

$a=r,l$. Finally, the *conventional* interaction in terms of ρ is also δ^*-invariant, proving the Galilean symmetry also in this case. In conclusion, any of the "pure"

[4] Their potential could be obtained by inserting further δ factors.

[5] But this is not a surprise as (2.1) and (2.7) just differ by a space-translation term.

expressions $V^*(\rho_a)$ as well as the standard potential $V = \rho^2$ provide us with a theory which is *Galilei-invariant in two ways*: the conventional implementation yields the usual one-parameter extension with commuting boosts, and the "star-implementation" yields the exotic two-parameter extension with non-commuting boosts.

Let us the record for completeness the equation of motion associated with our potential (4.5): either by variation or using the Hamiltonian structure, we get the "Moyalized" nonlinear Schrödinger equation

$$i\partial_t\psi = -\frac{\Delta}{2}\psi + \lambda\rho_l \star \psi = -\frac{\Delta}{2}\psi + \lambda\psi \star \rho_r. \qquad (4.8)$$

For comparison, for the choice V^* of Bak et al., the nonlinear term is $(\lambda/2)(\rho_r \star \psi + \psi \star \rho_l)$. The commutative counterpart of (4.8) is known to be non-integrable; coupling our system to an external magnetic field, yields exact solutions [13], however.

The "ordinary" conformal symmetry is also consistent with adding a quartic potential $V(\psi) = \rho^2$. In fact, using (3.1) we find that $\delta^0_\Delta\rho = \Delta(2\rho - \vec{x}\cdot\vec{\nabla}\rho - 2t\partial_t\rho)$ and $\delta^0_\kappa\rho = \kappa(2\rho - \vec{x}\cdot\vec{\nabla}(t\rho) - t^2\partial_t\rho)$, so that

$$\begin{aligned}\delta^0_\Delta\rho^2 &= -\vec{\nabla}\cdot\big(\Delta\vec{x}\rho^2\big) - \partial_t\big(\Delta 2t\rho^2\big) \quad \text{for a dilatation,}\\ \delta^0_\kappa\rho^2 &= -\vec{\nabla}\cdot\big(\kappa t\vec{x}\rho^2\big) - \partial_t\big(\kappa t^2\rho^2\big) \quad \text{for an expansion.}\end{aligned} \qquad (4.9)$$

The associated conserved quantities, that still form a representation of the one-parameter centrally extended Schrödinger algebra, only differ from the free expressions in that $\mathcal{H}_0$ is replaced by $\mathcal{H} = \mathcal{H}_0 + V$ [10].

As the "exotic" action of a dilatation is the same as the conventional one in (3.1) we find, using (2.4),

$$\begin{aligned}\delta^*_\Delta\rho_r = \delta^0_\Delta\rho_r &= \Delta\{2\rho_r - \vec{x}\cdot\vec{\nabla}\rho_r - 2t\partial_t\rho_r\}\\ &\quad - \Delta i\theta\epsilon_{ij}\partial_i\bar{\psi} \star \partial_j\psi\end{aligned} \qquad (4.10)$$

which differs from $\delta^*_\Delta\rho$ in the second term behind θ. Thus, owing precisely to this term, ρ_r^2 cannot change by a surface term. The scaling symmetry is therefore broken, as stated in [1]. (The same statement holds also for ρ_l^2 and $\rho_r\rho_l$.) More generally, (3.3) readily implies that

$$\begin{aligned}\delta^\star\rho_r &= (2k - g\partial_t)\rho_r - f_i\partial_i\rho_r\\ &\quad - \big[(f_i\partial_i\bar{\psi}) \star \psi + \bar{\psi} \star (f_i\partial_i\psi)\big],\end{aligned}$$

$$\begin{aligned}\delta^\star\rho_l &= (2k - g\partial_t)\rho_l - f_i\partial_i\rho_l\\ &\quad - \big[(f_i\partial_i\psi) \star \bar{\psi} + \psi \star (f_i\partial_i\bar{\psi})\big].\end{aligned} \qquad (4.11)$$

As the second brackets involve here expressions of the form

$$x_i\partial_i\rho \pm i\theta\epsilon_{ij}\partial_i\bar{\psi}\partial_j\psi + O\big(\theta^2\big)$$

the additional terms are not given by a surface term. that never vanish unless $G(t) = 0$. In conclusion, any potential made of products of ρ_r and ρ_l necessarily breaks the conformal invariance.

5. Discussion

An interesting feature of the model studied here is the two-way Galilean symmetry, and one can be puzzled how this can happen. Let us consider the "Moyalized" counterpart of $\mathcal{L}_0$, namely

$$\mathcal{L}_0^\star = i\bar{\psi} \star \partial_t\psi - \frac{1}{2}\vec{\nabla}\psi \star \vec{\nabla}\bar{\psi}. \qquad (5.1)$$

Now the conventional implementation (2.1) of the boosts is natural for $\mathcal{L}_0$, as is (2.7) for $\mathcal{L}_0^\star$. But the integral property $\int f \star g = \int fg$ implies that $\mathcal{L}_0$ and $\mathcal{L}_0^\star$ are equivalent, so one can use either of them to describe a free field. This resolves the paradox which says that noncommutativity does not alter the free theory.

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