Super-exponential inflation from a dynamical foliation of a 5D vacuum state

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ABSTRACT

We introduce super-exponential inflation ($\omega < -1$) from a 5D Riemann-flat canonical metric on which we make a dynamical foliation. The resulting metric describes a super accelerated expansion for the early universe well known as super-exponential inflation that, for very large times, tends to an asymptotic de Sitter (vacuum dominated) expansion. The scalar field fluctuations are analyzed. The important result here obtained is that the spectral index for energy density fluctuations is not scale invariant, and for cosmological scales becomes $n_s(k < k_*) \simeq 1$. However, for astrophysical scales this spectrum changes to negative values $n_s(k > k_*) < 0$.

1. Introduction

Observations of the anisotropy in the cosmic microwave background radiation, are difficult to reconcile with a flat spectrum. For a universe dominated by stable dark matter a flat spectrum is ruled out altogether and even for a neutrino-dominated universe a spectrum of the type $\delta\rho/\rho = (M/M_0)^{-s}$, with $s > 0$, is preferable to the flat spectrum (for which $s = 0$). Unfortunately, however, a quasi-exponential inflation with $\dot{H} < 0$ leads to a spectrum with $s < 0$. This can be easily understood intuitively larger mass scales correspond to perturbations which crossed the horizon $1/H$ earlier during the quasi-exponential phase of expansion, when H was larger, which implies a larger $\delta\rho/\rho$.

It is possible to construct models in which $\dot{H} > 0$ by starting with a theory in more than four dimensions [1]. In a higher-dimensional theory of gravity containing higher-derivative terms and a cosmological constant, a period of super-exponential inflation of the physical space–time is possible, during which the Hubble parameter H increases with time, as discovered by Shafi and Wetterich [2].

In this Letter we explore this possibility from extra dimensions using the space–time–matter theory of gravity [3,4], but using a dynamical foliation $\psi \equiv \psi(t)$ of the noncompact extra dimension ψ. This topic was explored previously by Ponce de León [5]. He showed that the FRW line element can be 'reinvented' on a dynamical four-dimensional hypersurface, which is not orthogonal to the extra dimension, without any internal contradiction. This hypersurface is selected by the requirement of continuity of the metric and depends explicitly on the evolution of the extra dimension. More recently, was demonstrated that phantom [6] and warm inflationary [7] cosmological scenarios can be obtained through this mechanism from a 5D Riemann-flat. In this Letter shall find that this mechanism can be the responsible for existence of a super-exponential expansion of the early universe.

2. Dynamical foliation

In this section we are interested to study the relativistic dynamics of observers who moves on an 5D Ricci-flat space–time, such that the extra coordinate can be parameterized as a function of an effective 4D space–time.

We consider a 5D canonical metric

$$dS^2 = g_{\mu\nu}(y^\sigma, \psi)\, dy^\mu\, dy^\nu - d\psi^2. \tag{1}$$

Here the 5D coordinates are orthogonal: $y \equiv \{y^a\}$.[1] The geodesic equations for an relativistic observer are

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[1] Greek letters run from 0 to 3, and arabic letters run from 0 to 4.

$$\frac{dU^a}{dS} + \Gamma^a_{bc} U^b U^c = 0,$$

(2)

where $U^a = \frac{dy^a}{dS}$ are the velocities and Γ^a_{bc} are the connections of (1). Now we consider a parametrization $\psi(x^\alpha)$, where $x \equiv \{x^\alpha\}$ are an orthogonal system of coordinates, such that the effective line element (1), now can be written as

$$dS^2 = h_{\alpha\beta} \, dx^\alpha \, dx^\beta.$$

(3)

It is very important to notice that S will be an invariant, so that derivatives with respect to S will be the same on 5D or 4D. In other words, in this Letter we shall consider that space–time lengths that remain unaltered when we move on an effective 4D space–time. The effective 4D tensor metric is $h_{\alpha\beta} = \frac{\partial y^a}{\partial x^\alpha} \frac{\partial y^b}{\partial x^\beta} g_{ab} = e^a_\alpha e^b_\beta g_{ab}$. Furthermore, since both coordinate systems, $\{x^\alpha\}$ and $\{y^a\}$ are orthogonal and S is an invariant, hence we can define the inverse transformation $g_{ab} = \frac{\partial x^\alpha}{\partial y^a} \frac{\partial x^\beta}{\partial y^b} h_{\alpha\beta} = \bar{e}^\alpha_a \bar{e}^\beta_b h_{\alpha\beta}$, such that $e^a_\alpha \bar{e}^\beta_a = \delta^\beta_\alpha$. In other words the condition of normalization for the velocities of the observers is fulfilled in both the systems:

$$g_{ab} U^a U^b \big|_{\psi(x^\alpha)} = h_{\alpha\beta} u^\alpha u^\beta = 1,$$

(4)

where we have used that $U^a \equiv (dy^a/dS) = e^a_\alpha u^\alpha \equiv e^a_\alpha (dx^\alpha/dS)$. If we replace this expression in (2), we obtain

$$\frac{d(e^a_\alpha u^\alpha)}{dS} + \Gamma^a_{bc} \big(e^b_\alpha e^c_\beta\big) u^\alpha u^\beta = 0,$$

(5)

and hence

$$\bar{e}^\phi_a \frac{d}{dS}\big(e^a_\alpha u^\alpha\big) + \Gamma^a_{bc}\big(e^b_\alpha e^c_\beta \bar{e}^\phi_a\big) u^\alpha u^\beta = 0.$$

(6)

Now we can use the following expression for the connection transformation

$$\bar{\Gamma}^\phi_{\alpha\beta} = \Gamma^a_{bc}\big(e^b_\alpha e^c_\beta \bar{e}^\phi_a\big) + \bar{e}^\phi_a \frac{\partial}{\partial x^\alpha}\big(e^a_\beta\big),$$

(7)

and we obtain the geodesic equations for observers on the effective 4D space–time

$$\frac{du^\phi}{dS} + \bar{\Gamma}^\phi_{\alpha\beta} u^\alpha u^\beta = \mathcal{F}^\phi,$$

(8)

where the induced extra force on the effective 4D space–time is

$$\mathcal{F}^\phi = \bar{e}^\phi_a \frac{\partial}{\partial x^\alpha}\big(e^a_\beta\big) u^\alpha u^\beta - u^\alpha \bar{e}^\phi_a \frac{d}{dS}\big(e^a_\alpha\big).$$

(9)

The extra force (9) can lead to the same result earlier obtained in [8]. In general, a nonzero $\mathcal{F}^\phi$ should be responsible for the violation of the equivalence principle on the 4D hypersurface.

2.1. Einstein equations for dynamical foliations from a 5D vacuum state

Now we consider the Einstein equations on the 5D canonical symmetrical metric

$$G_{ab} = -8\pi G T_{ab},$$

(10)

where the Einstein tensor is given by $G_{ab} = R_{ab} - \frac{1}{2}g_{ab}R$, R_{ab} being the Ricci tensor, such that the scalar of curvature is $R = g_{ab}R^{ab}$. Because we are considering a 5D Ricci-flat metric, the Einstein tensor and the Ricci scalar will be null. Using the transformation of the previous section, we obtain that

$$\bar{G}_{\alpha\beta} = \bar{R}_{\alpha\beta} - \frac{1}{2}h_{\alpha\beta}\bar{R} = -8\pi G \bar{T}_{\alpha\beta},$$

(11)

where we have used respectively the transformations

$$\bar{R}_{\alpha\beta} = e^a_\alpha e^b_\beta R_{ab},$$

(12)

$$\bar{R} = h_{\alpha\beta} \bar{R}^{\alpha\beta},$$

(13)

$$\bar{T}_{\alpha\beta} = e^a_\alpha e^b_\beta T_{ab},$$

(14)

for the effective 4D Ricci tensor, the scalar of curvature and the energy–momentum tensor.

2.2. Energy–momentum tensor from a massless scalar field in a 5D vacuum

In order to make a complete description for the dynamics of the scalar field, we shall consider its energy–momentum tensor. In order to describe a true 5D physical vacuum we shall consider that the field is massless and there is absence of interaction on the 5D Ricci-flat manifold, so that

$$T^a{}_b = \Pi^a \Pi_b - g^a{}_b \mathcal{L}[\varphi, \varphi_{,c}], \tag{15}$$

where $\mathcal{L}[\varphi, \varphi_{,c}] = \frac{1}{2} \varphi^a \varphi_a$ is the Lagrangian density for a free and massless scalar field on (1) and the canonical momentum is $\Pi^a = \frac{\partial \mathcal{L}}{\partial \varphi_{,a}}$. Notice that we are not considering interactions on the 5D vacuum, because it is related to a physical vacuum in the sense that the Einstein tensor is zero: $G^a_b = 0$. The effective 4D energy–momentum tensor will be

$$\bar{T}_{\alpha\beta} = e^a{}_\alpha e^b{}_\beta T_{ab}\big|_{\psi(x^\alpha)}. \tag{16}$$

In other words, using the fact that $\mathcal{L}$ is an invariant it is easy to demonstrate that

$$\bar{T}^\alpha{}_\beta = \bar{\Pi}^\alpha \bar{\Pi}_\beta - h^\alpha{}_\beta \mathcal{L}, \tag{17}$$

where $\mathcal{L}$ is an invariant of the theory

$$\mathcal{L} = \frac{1}{2} \varphi^{,a} \varphi_{,a} = \frac{1}{2} \left(e^a_\alpha \bar{\varphi}^{,\alpha} \right) \left(\bar{e}^\beta_a \bar{\varphi}_{,\beta} \right), \tag{18}$$

such that the Euler–Lagrange equations

$$\frac{\delta \mathcal{L}}{\delta \varphi} - \nabla_a \frac{\delta \mathcal{L}}{\delta \varphi_{,a}} = 0, \tag{19}$$

describes the dynamics of the field φ on the metric (1).

2.3. D'Alambertian of a scalar field for dynamical foliations

We are interested to study how is the effective 4D dynamics obtained from a dynamic foliation of a 5D Ricci-flat canonical metric. We consider a classical massless scalar field $\varphi(y^a)$ on the metric (1). The equation of motion (19) for the Lagrangian density (18) is given by $\Box \varphi = \nabla^a \varphi_{,a} = 0$, where

$$\Box \varphi = g^{ab} \left(\varphi_{,a,b} - \Gamma^c_{ab} [\varphi(y)]_{,c} \right). \tag{20}$$

We can make the transformation from $y \equiv \{y^a\}$ to $x \equiv \{x^\alpha\}$, with the parametrization $\psi(x^\alpha)$, where $x \equiv \{x^\alpha\}$. To simplify the notation we shall use the notation $\varphi(x) \equiv \bar{\varphi}$ and $\varphi(y) \equiv \varphi$, such that

$$g^{ab} \varphi_{,a,b}\big|_{\psi(x^\alpha)} = h^{\alpha\beta} \bar{\varphi}_{,\alpha,\beta}, \tag{21}$$

$$g^{ab} \Gamma^c_{ab} \varphi_{,c}\big|_{\psi(x^\alpha)} = h^{\alpha\beta} \left[\bar{\Gamma}^\gamma_{\alpha\beta} - \bar{e}^\gamma{}_c \frac{\partial}{\partial x^\alpha} \left(e^c{}_\beta \right) \right] \bar{\varphi}_{,\gamma}, \tag{22}$$

where in the last expression we have used the transformation law for the connections (7). Finally, using the expressions (21) and (22) in (20), we obtain

$$\Box \varphi\big|_{\psi(x^\alpha)} = \bar{\Box} \bar{\varphi} + h^{\alpha\beta} \bar{e}^\gamma{}_c \frac{\partial}{\partial x^\alpha} \left(e^c{}_\beta \right) \bar{\varphi}_{,\gamma}, \tag{23}$$

where $\bar{\Box} \bar{\varphi} = h^{\alpha\beta} \left(\bar{\varphi}_{,\alpha,\beta} - \bar{\Gamma}^\gamma_{\alpha\beta} \bar{\varphi}_{,\gamma} \right)$ is the effective 4D D'Alambertian on the orthogonal coordinates system $x \equiv \{x^\alpha\}$ of the scalar field $\bar{\varphi}(x^\alpha)$ and the $\bar{\Gamma}^\gamma_{\alpha\beta}$ are the Levi-Civita connections on the effective 4D metric.

We shall require that

$$h^{\alpha\beta} \bar{e}^\gamma{}_c \frac{\partial}{\partial x^\alpha} \left(e^c{}_\beta \right) \varphi_{,\gamma}\big|_{\psi(x^\alpha)} = \frac{d}{d\bar{\varphi}} V(\bar{\varphi}). \tag{24}$$

Because we are considering that S and $\mathcal{L}$ remain invariants on the effective 4D space–time, only conservative potentials will be used in (24). In particular, the equation of motion for a massive scalar field with imaginary mass (*im*), on the effective 4D space–time is

$$\bar{\Box} \bar{\varphi} - m^2 \bar{\varphi} = 0, \tag{25}$$

which describes the motion of the scalar field $\bar{\varphi}(x^\alpha)$ on the 4D hypersurface with orthogonal coordinates $x \equiv \{x^\alpha\}$. These superluminal particles are known as tachyons [8].

3. Super-exponential inflation from a 5D canonical metric

In order to investigate an example of the earlier formalism, we shall consider the Riemann-flat canonical metric [9]

$$dS^2 = \psi^2 \, dN^2 - \psi^2 e^{2N} \, dr^2 - d\psi^2, \tag{26}$$

with the transformation: $\{y^a\} \to \{x^\alpha\}$

$$x^i = y^i \psi_0, \qquad t = N \psi_0, \tag{27}$$

and the dynamic foliation: $\psi \equiv \psi(t)$. The effective 4D space–time being described by the line element

$$dS^2 = \left[\frac{\psi^2(t)}{\psi_0^2} - \dot{\psi}^2 \right] dt^2 - \frac{\psi^2(t)}{\psi_0^2} e^{2\psi_0^{-1}t} \, dR^2, \tag{28}$$

where the dot denotes the derivative with respect to t and ψ_0 is some constant. In order to consider t as a cosmic time, one must require that

$$\frac{\psi^2(t)}{\psi_0^2} - \dot{\psi}^2 = 1, \tag{29}$$

so that the foliation is described by

$$\psi(t) = \psi_0 \cosh(t/\psi_0) \quad \to \quad \dot{\psi}(t) = \sinh(t/\psi_0). \tag{30}$$

Finally, the metric (28), for a foliation (30) is described by

$$dS^2 = dt^2 - \cosh(t/\psi_0)^2 e^{2\psi_0^{-1}t} \, dR^2, \tag{31}$$

which describes an 3D (flat) spatially isotropic universe in expansion with a scale factor $a(t) = \cosh(t/\psi_0)e^{\psi_0^{-1}t}$, a scale factor $H(t) = \frac{\dot{a}}{a}$ and a deceleration parameter $q = -\frac{\ddot{a}a}{\dot{a}^2}$ given by (for $H_0 = 2/\psi_0$)

$$H(t) = \frac{H_0}{2} \left[\tanh\left(\frac{tH_0}{2}\right) + 1 \right], \tag{32}$$

$$q(t) = -\frac{2}{\tanh\left(\frac{tH_0}{2}\right) + 1}. \tag{33}$$

Notice that $\dot{H} > 0$. Furthermore the late time asymptotic derivative the Hubble parameter and the deceleration parameter, are

$$\dot{H}(t)|_{t\to\infty} \to H_0, \tag{34}$$

$$q(t)|_{t\to\infty} \to -1, \tag{35}$$

which means that the universe describe a super-exponential inflationary period with and asymptotic de Sitter (vacuum dominated) expansion.

On the other hand, the relevant components of the Einstein tensor, are

$$G^0{}_0 = -\frac{3}{4} \frac{H_0^2[\sinh(H_0t/2) + \cosh(H_0t/2)]^2}{\cosh^2(H_0t/2)} = -8\pi G\rho, \tag{36}$$

$$G^x{}_x = -\frac{H_0^2}{4} \frac{[\sinh(H_0t/2) + \cosh(H_0t/2)][5\cosh(H_0t/2) + \sinh(H_0t/2)]}{\cosh^2(H_0t/2)} = 8\pi Gp, \tag{37}$$

such that $G^x{}_x = G^y{}_y = G^z{}_z$, because the isotropy of the space. The equation of state for this model of super-exponential inflation is

$$\frac{p}{\rho} = -\frac{1}{3} \frac{[\sinh^2(H_0t/2) + 5\cosh^2(H_0t/2) + 6\sinh(H_0t/2)\cosh(H_0t/2)]}{[\sinh^2(H_0t/2) + \cosh^2(H_0t/2) + 2\sinh(H_0t/2)\cosh(H_0t/2)]}. \tag{38}$$

Notice that its asymptotic value

$$\frac{p}{\rho}\bigg|_{t\to\infty} \to -1, \tag{39}$$

corresponds to a de Sitter exponential inflationary stage, with cosmological constant

$$\Lambda_0 = 3H_0^2 = \frac{12}{\psi_0^2}. \tag{40}$$

3.1. Evolution of $\bar{\varphi}$

We consider Eq. (24). For a massive scalar field, this equation can be rewritten as

$$\ddot{\psi} \frac{\partial \varphi}{\partial \psi}\bigg|_{\psi(t)} = -m^2 \bar{\varphi}(\vec{x}, t), \tag{41}$$

so that, using the fact that $\psi_0^2 \ddot{\psi} = \psi$, we obtain the solution $\varphi(\psi) = \bar{\varphi}_0 (\psi/\psi_0)^{-A}$, where $A = m^2 \psi_0^2 > 0$ is a dimensionless parameter. The equation of motion for the modes $\bar{\varphi}_k(t)$ in Eq. (25), is

$$\ddot{\bar{\varphi}}_k + 3H(t)\dot{\bar{\varphi}}_k + \left[\frac{k^2}{a^2(t)} - m^2\right]\bar{\varphi}_k(t) = 0. \tag{42}$$

The general solution is

$$\begin{aligned}
\bar{\varphi}_k(t) = {} & A_1 \big[x(t) + 1\big]^{-(1+\sqrt{1-(\frac{2k}{H_0})^2})} e^{\frac{H_0}{2}(-3+\sqrt{9+(\frac{2m}{H_0})^2})t} \mathcal{F}\big\{[a_1, b_1], [c_1], -x(t)\big\} \\
& + B_1 \big[x(t) + 1\big]^{-(1+\sqrt{1-(\frac{2k}{H_0})^2})} e^{-\frac{H_0}{2}(3+\sqrt{9+(\frac{2m}{H_0})^2})t} \mathcal{F}\big\{[a_2, b_2], [c_2], -x(t)\big\},
\end{aligned} \tag{43}$$

where $\mathcal{F}\{[a_i, b_i], [c_i], -x(t)\}$ is the hypergeometric function with argument $x(t) = e^{-H_0 t}$, $i = 1, 2$, and parameters

$$a_1 = -\sqrt{1 - \left(\frac{2k}{H_0}\right)^2} + \frac{1}{2} - \frac{1}{2}\sqrt{9 + \left(\frac{2m}{H_0}\right)^2 + \sqrt{\left(\frac{m}{H_0}\right)^2 - 4\left(\frac{2k}{H_0}\right)^2}}, \tag{44}$$

$$a_2 = -\sqrt{1 - \left(\frac{2k}{H_0}\right)^2} + \frac{1}{2} + \frac{1}{2}\sqrt{9 + \left(\frac{2m}{H_0}\right)^2 + \sqrt{\left(\frac{m}{H_0}\right)^2 - 4\left(\frac{2k}{H_0}\right)^2}}, \tag{45}$$

$$b_1 = -\sqrt{1 - \left(\frac{2k}{H_0}\right)^2} + \frac{1}{2} - \frac{1}{2}\sqrt{9 + \left(\frac{2m}{H_0}\right)^2 - \sqrt{\left(\frac{m}{H_0}\right)^2 - 4\left(\frac{2k}{H_0}\right)^2}}, \tag{46}$$

$$b_2 = -\sqrt{1 - \left(\frac{2k}{H_0}\right)^2} + \frac{1}{2} + \frac{1}{2}\sqrt{9 + \left(\frac{2m}{H_0}\right)^2 - \sqrt{\left(\frac{m}{H_0}\right)^2 - 4\left(\frac{2k}{H_0}\right)^2}}, \tag{47}$$

$$c_1 = 1 - \sqrt{9 + \left(\frac{2m}{H_0}\right)^2}, \tag{48}$$

$$c_2 = 1 + \sqrt{9 + \left(\frac{2m}{H_0}\right)^2}, \tag{49}$$

which we shall consider to be real.

3.2. Fluctuations and spectrum

For large times the hypergeometric function is irrelevant: $\mathcal{F}\{[a_i, b_i], [c_i], -x(t)\} \simeq 1$. Notice that the exponential time dependent factor of the second term in (43) decreases faster than the first one. Hence, for very large time the second term of (43) can be neglected. Furthermore, on cosmological scales the wavenumber k is very small: $(k/H_0 \ll 1)$, so that one can make the approximation $1 + \sqrt{1 - (\frac{2k}{H_0})^2} \simeq 2 - \frac{1}{2}(\frac{2k}{H_0})^2$. On the other hand, we shall consider that the mass of the scalar field is very small: $m/H_0 \ll 1$. Hence, if we chose $B_1 = 0$, we obtain that

$$\bar{\varphi}_k(t)\bar{\varphi}_k^*(t)\big|_{t/\psi_0 \gg 1} \simeq A_1 A_1^* e^{-H_0[2-(\frac{2m^2}{9H_0^2}+k^2)]t}, \tag{50}$$

and the squared φ-fluctuations are

$$\langle \varphi^2 \rangle = \frac{-A_1 A_1^*}{64\pi^2} \frac{H_0}{t^{3/2}} e^{-\frac{2t}{9H_0}(9H^2 - m^2)} \left\{ 4t^{1/2} \epsilon k_0 e^{\frac{4\epsilon^2 k_0^2}{H_0}t} + \sqrt{\pi H_0}\, \mathbf{Erf}\left[\frac{2\epsilon k_0 \sqrt{t}}{\sqrt{H_0}}\right] \right\}, \tag{51}$$

where $\epsilon \simeq 10^{-3}$, $\mathbf{Erf}[x] = (2/\sqrt{\pi}) \int_0^x e^{-x'^2} dx'$ is the error function and $k_0(t)$ is the wavenumber that separates the horizon is

$$k_0(t) = a(t) \left[\frac{9}{4}H^2(t) + \frac{3}{2}\dot{H} + m^2\right]^{1/2}. \tag{52}$$

The spectrum for the squared φ-fluctuations corresponds with a k-dependent spectral index $n_s(k)$

$$n_s(k) = 3 - 2\left(\frac{H_0 t_*}{\ln(k_*/H_0)}\right)\left[1 - \left(\frac{m^2}{18H_0^2} + \frac{k^2}{H_0^2}\right)\right], \tag{53}$$
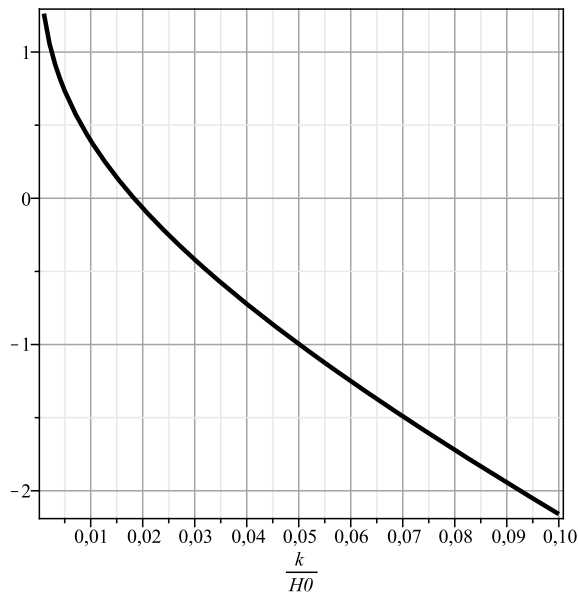
Fig. 1. Spectral index $n_s(k)$ as a function of k/H_0.

such that t_* is defined as the time when super-inflation ends:

$$k_* = H_0 e^{-H_0 t_*}, \tag{54}$$

and inflation begins. The spectral index $n_s(k)$, is

$$n_s(k) = 5 - 2\left(\frac{m^2}{18H_0^2} + \frac{k^2}{H_0^2}\right). \tag{55}$$

In Fig. 1 we have plotted n_s for $0.001 < k/H_0 < 0.1$. We have used the values $H_0 = 6 \times 10^{-9} M_p$ and $m = 10^{-9} M_p$, $M_p = 1.2 \times 10^{19}$ GeV being the Planckian mass. Notice that for cosmological scales the spectral index is close to the unity, but for astrophysical scales its value becomes negative. This is an important result that agrees with observational date and it is not possible to be recovered in an standard inflationary model for a single scalar field.

4. Final comments

Starting from a 5D canonical metric on which we make a dynamical foliation, in this Letter we have studied a model for super-exponential inflation. Along the super-accelerated expansion $\dot{\omega} > 0$, so that for very large times the equation of state tends to an asymptotic de Sitter (vacuum dominated) expansion with $\omega|_{t \to \infty} \to 1$. We have obtained the effective 4D dynamics of the scalar field, which drives the expansion of the universe, from a dynamical foliation of a 5D Riemann-flat canonical metric, on which this field is considered as a test massless field. However, the scalar field acquires dynamics and (imaginary) mass on the effective 4D FRW and can be considered as a tachyon field on this 4D hypersurface which was obtained from a dynamical foliation of this space the 5D Riemann-flat space–time.

The scalar field fluctuations on very large scales were analyzed. The important result here obtained is that the spectral index for energy density fluctuations ($\delta\rho/\rho \sim \langle\varphi^2\rangle$), are scale invariant for cosmological scales becomes $n_s(k < k_*) \simeq 1$, but for more shorten scales this spectrum changes to take negative values $n_s(k > k_*) < 0$, until take values close to -2, in agreement with observations.

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