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# How to make teaching interesting?

# Abdul Razak Salleh\*

School of Mathematical Sciences, Faculty of Science and Technology, Universiti Kebangsaan Malaysia

#### Abstract

Lectures, particularly in Mathematics, worse still in abstract courses such as Linear Algebra, Abstract Algebra and Analysis, often create boredom to some students, especially when the lectures are delivered in two continuous sessions, or in the afternoon. However interesting the lecture is, surely there must be some students who fall asleep. In this paper we give suggestions on how to make lectures or teaching interesting. Two things that should be included are the historical aspect of certain concepts to be introduced in the lectures, and some applications of the concept in our daily life. We give an illustration of the lectures in Abstract Algebra offered by the author at the School of Mathematical Sciences, Universiti Kebangsaan Malaysia. Another way of avoiding the students' boredom is to include some mathematical humours and create a language and terminology corner. This corner can help the students in their research projects and also their theses. This corner might also be helpful to those who wish to submit their papers for publication in the local journals.

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# 1. Introduction

Lectures in Mathematics courses such as Linear Algebra, Abstract Algebra and Analysis, often create boredom to some students, especially when delivered in a longer period or in the afternoon. Some students would still fall asleep even though the lecture is interesting. Suggestions on how to make lectures or teaching interesting were proposed in this article. The historical background of certain concepts should be introduced in the lectures. After introducing the concept followed by some related examples, it would be nice to discuss some applications of the concept in our daily life. An illustration of the lectures in the Abstract Algebra course once offered by the author at the School of Mathematical Sciences, Universiti Kebangsaan Malaysia was exhibited. To avoid students' boredom, some mathematical humours is included and language and terminology corner is created. The language and terminology corner can help the students in their research projects, theses and might also be helpful to those who wish to submit their papers for publication in the local journals.

\* Corresponding author. Tel.: +6- 013-311-4477; fax: +6-03-8925-4519

E-mail address: aras@ukm.my

#### 2. Historical Aspects

Pure mathematics courses such as Linear Algebra, Abstract Algebra, Analysis and Topology are often treated by students as abstract, difficult to understand and boring. It is normal and standard that the structure of these courses consists of definitions, propositions, lemmas, theorems and corollaries. Often, mathematics is presented as something whose development has been unrelated to the activities of human beings. The questions which motivated the whole theory in the first place are often simply omitted, and students are asked to appreciate the methods and the theory without context, without relevance to other mathematical or scientific activity, one might even say, without meaning. For example, how many books on group theory are there which mention the range of applications of group theory?

Even though each concept that has been introduced is followed by a few examples, students still find difficulty in understanding the examples given because they do not really understand the concept. To grasp the theorems and the proofs that follow will be more difficult. To lessen the students' difficulty and boredom, at least we make an effort to include some historical background of the subject and its applications in our daily life or culture. This might help to bring the students closer to the subject, and help them to a better understanding of the course.

Group theory started with the activity of solving algebraic equations (linear, quadratic, cubic and quartic) by radicals, since the Pharaoh. The solution of quadratic equations had been known to the Greeks (Hero, 1<sup>st</sup> century), ancient Chinese (until 3<sup>rd</sup> century) and Indian (Brahmagupta, 7<sup>th</sup> century). Muslim mathematicians such as al-Khwarizmi, Thabit ibnu Qurrah, al-Haitham, al-Karkhi and Umar Khayam, also contributed in solving the algebraic equations (from 8<sup>th</sup> until 12<sup>th</sup> century). Western mathematicians, particularly from Italy, such as Ferro, Fontana (Tartaglia), Cardano and Ferrari obtained radical solutions to cubic and quartic equations (from 15<sup>th</sup> until 16<sup>th</sup> century). The next challenge was to obtain the solution of quintic equations and those of higher degrees radically. There was no development in the next two hundred years. Suddenly in 1824 a young mathematician from Norway, Niels Henrik Abel declared that quintic equations and those of higher degrees could not be solved by radicals. In 1829 a young French mathematician, Evariste Galois, managed to link algebraic equations with what was known as a group. His discovery stated that an algebraic equation is solvable radically if its associated (Galois) group is solvable.

#### 3. Applications

As applications of groups, we discuss the Malay number system as an application of the group of numbers, application of the symmetry group to songket weaving and batik design, application of the permutation group or the symmetric group to kinship structure and geometric patterns, and application of the group integer modulo n to the writing of dates (the small daur) in the Malay culture, and to ISBN. The course will be more interesting if we could use computer softwares to compute or to illustrate some group concepts.

To fully understand certain mathematical rule, we could relate it to our daily life situation (Abdul Razak, 2009a). For example, take the property of group elements,  $(ab)^{-1} = b^{-1}a^{-1}$ . Some students write  $(ab)^{-1} = a^{-1}b^{-1}$  which is true only when the group is abelian. In order to understand it well, we call it *the rule of taking off the shoes*. Suppose *a* is the operation of wearing the socks, and *b* is the operation of putting on the shoes. Then *ab* will be the operation of wearing the socks followed by putting on the shoes. The reverse process or the inverse of the procedure  $(ab)^{-1}$ , is surely to take off the shoes first and then take off the socks, which is  $b^{-1}a^{-1}$ . Thus if someone writes  $(ab)^{-1} = a^{-1}b^{-1}$ , this means that he is trying very hard to take off his socks before taking off his shoes. This is ridiculous.

#### 3.1 Symmetric group

Permutation group is also known as symmetric group. Permutations can be used to study kinship structure. Permutations can also be used to get geometric patterns (Abdul Razak, 2009b). The wife's clan is determined by the *marriage permutation* 

$$P = \begin{pmatrix} 1 & 2 & \dots & n \\ \alpha(1) & \alpha(2) & \dots & \alpha(n) \end{pmatrix},$$

where 1, 2,..., *n* are the different clans of the husbands and  $\alpha(1)$ ,  $\alpha(2)$ ,...,  $\alpha(n)$  are the corresponding clans of the wives, according to the marriage rules that have been fixed. The husband that is being considered is known as *ego*. By using the composition operation on the marriage permutation *P*, we can determine the clans of the ego's brother-in-law's wife. For example,

$$P \circ P = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{pmatrix}.$$

This means that the ego's brother-in-law's wife and the ego himself are from the same clan. The clans of the children are given by the *descendant permutation*. There are two different descendant permutations, according to types of kinship relationship. The clans of the matrilineal children is given by

$$A_m = \begin{pmatrix} 1 & 2 & \dots & n \\ \varepsilon(1) & \varepsilon(2) & \dots & \varepsilon(n) \end{pmatrix},$$

where 1, 2,..., *n* are the distinct clans of the mothers and  $\varepsilon(1)$ ,  $\varepsilon(2)$ ,...,  $\varepsilon(n)$  are the corresponding clans of the matrilineal children. The clans of the patrilineal children is given by

$$A_p = \begin{pmatrix} 1 & 2 & \dots & n \\ \beta(1) & \beta(2) & \dots & \beta(n) \end{pmatrix},$$

where 1, 2,..., *n* are the distinct clans of the fathers and  $\beta(1)$ ,  $\beta(2)$ ,...,  $\beta(n)$  are the corresponding clans of the patrilineal children.

#### 3.2 The group integer modulo n

We give two applications of the group integer modulo *n*, namely in the writing of date in the Malay culture, and its application in ISBN (International Standard Book Number). In Malay culture, normally all letters, manuscripts and inscribed stones must end with a *termaktub*, that is a closing statement consisting of the date, place and time they were written (Gallop, 1994). An interesting part is the date writing using the *daur kecil* (small daur). According to al-Attas (1988), *daur kecil*, is a time cycle of eight years, consisting of the following Arabic alphabets:

# Alif, Ha, Jim, Zai, Dal, Ba, Wau and Dzal.

Al-Attas claimed that the small daur started in the Malay Archipelago, and the calculation of the cycle started from the first year of Hijrah. To determine the name of a Hijrah year, we use the arithmetic of *integer modulo* 8. The Hijrah year is divided by 8, and its remainder, which consists of integers 0 until 7, will determine that particular year (Table 1).

Year	1	2	3	4	5	6	7	8
Remainder	1	2	3	4	5	6	7	0
Name of year	Alif	Ha	Jim	Zai	Dal	Ba	Wau	Dzal

Table 1. Small daur used in the Malay world

For example, this year, 2011 is 1432*H* and 1433*H*. Dividing 1432*H* by 8 we get 179 without any remainder, which is the year of *Dzal*, and thus 1433*H* is the year of *Alif*.

Another application of integer modulo *n* is the International Standard Book Number (ISBN) that is used for every book that is published. The number is a sequence of nine digits  $a_1a_2...a_9$ , where each  $a_i$  is one of the ten numbers

0, 1, 2, ..., 9 together with a *check* or *control digit* which is one of the eleven numbers 0, 1, ..., 9, X (where X represents 10). The last (tenth) digit is inserted in order to check that the nine digits before it are correctly written. According to Humphreys and Prest (1991),

$$a_{10} \equiv 11 - (10a_1 + 9a_2 + 8a_3 + 7a_4 + 6a_5 + 5a_6 + 4a_7 + 3a_8 + 2a_9) \pmod{11}.$$

Let us check the book Gaya UKM (2008), ISBN 967-942-254-2. From the above formula we have

$$10(9) + 9(6) + 8(7) + 7(9) + 6(4) + 5(2) + 4(2) + 3(5) + 2(4)$$
  
= 90 + 54 + 56 + 63 + 24 + 10 + 8 + 15 + 8 = 328.

which implies that  $328 \div 11 = 29$  remainder 9. So the check digit is 11 - 9 = 2. The following method straight away gives the check digit:

$$1(9) + 2(6) + 3(7) + 4(9) + 5(4) + 6(2) + 7(2) + 8(5) + 9(4) = 9 + 12 + 21 + 36 + 20 + 12 + 14 + 40 + 36 = 200.$$

So  $200 \div 11 = 18$  remainder 2.

Since January 1<sup>st</sup>, 2007 ISBN is a 13 digit number. The formula for the check digit is given as follows (Wikipedia):

$$a_{13} \equiv \left[10 - (a_1 + 3a_2 + a_3 + 3a_4 + a_5 + 3a_6 + a_7 + 3a_8 + a_9 + 3a_{10} + a_{11} + 3a_{12}) \mod 10\right] \mod 10$$

Thus for the book of Abdul Razak (2009), ISBN 978-967-942-908-4

 $a_{13} \equiv [10 - \{9 + 3(7) + 8 + 3(9) + 6 + 3(7) + 9 + 3(4) + 2 + 3(9) + 0 + 3(8)\} \mod 10] \mod 10$  $\equiv [10 - (166) \mod 10] \mod 10$  $\equiv 4 \mod 10.$ 

The 10 digit ISBN consists of four components: group identity (country), press identity, number of publications and the check or control digit. On the other hand, the 13 digit ISBN consists of five components, namely code of book industry (978) and the four components are as for the 10 digit ISBN. We list some group identities (Wikipedia):

- 0 UK, USA, Australia, New Zealand and Canada 1 South Africa, Zimbabwe 2 France, Belgium, Switzerland 3 Germany, Austria, Switzerland 4 Japan 967 Malaysia (also 983) 974 Thailand 979 Indonesia
- 981 Singapore

For more information see Abdul Razak (2011).

# 4. Mathematical Humour

Normally, in a lecture, especially in Abstract Algebra or Analysis, the students get bored and sleepy. This is the time to crack some jokes or humour, if possible, mathematical jokes or mathematical humour. According to Renteln and Dundes (2005) most mathematical humour are based on words involving standard mathematical concepts. Most of the humours involve food, which indicates the existence of mathematical concepts that are difficult to digest or swallow. Consider the following humour.

- Q: What's purple and commutes?
- A: An abelian grape.

Note: In Abstract Algebra, if the binary operation on the group is commutative, then the group is known as an abelian group.

Q: Why did the mathematician name his dog "Cauchy"?

A: Because he left a residue at every pole.

Note: In Complex Analysis there is a theorem called the Cauchy Residue Theorem.

Q: Why can't you grow wheat in Z/6Z?

A: It's not a field.

Note: Z/6Z is not a field, it is an integral domain.

Q: What is grey, huge and has integer coefficients?

A: An elephantine equation.

Note: In Number Theory we have *diophantine equation*.

# Q: What do you get if you cross an elephant and a banana?

# A: elephant × banana = |elephant| |banana| $\sin\theta$

Note: In Linear Algebra the cross product of a vector **a** and a vector **b** is defined as:  $\mathbf{a} \times \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \sin\theta$ , where  $\theta$  is the angle between the two vectors.

Q: What is sour, yellow, and equivalent to the Axiom of Choice?

A: Zorn's lemon.

Q: What is yellow, linear, has a norm and complete?

A: A Bananach space.

Q: Who is a topologist?

A: Someone who cannot differentiate between a doughnut and a teacup.

**Note:** Both objects are topologically equivalent. To visualize this, imagine a doughnut in the form of plasticine, which could be transformed into a teacup in a few steps as illustrated in Figure 1.



Figure 1. From doughnut to a tea cup *Source*: Abdul Razak & Faridah 1999

Other than the mathematical humours, we can also crack other jokes which could also improve the students' command of the English Language. Among the jokes are *fried tongue, arigator, a school of whales, English descendent, I told you not to go but you wanted to go,* tongue twister "*she sells sea shells on the sea shore*", *Mary Barter, the Sikh's sixth sheep is sick, America-Indonesia,* and *are you relaxing sir?* 

#### 5. Language Corner

Universiti Kebangsaan Malaysia has been established from the aspiration of its people and to use the national language, Bahasa Melayu, as the language of teaching and knowledge and to be brought to international platform. So all the lectures in the university are conducted in Malay. The academic staffs are encouraged to write their papers in Malay which are to be published in journals. To promote the sovereignty of Bahasa Melayu as the language of knowledge, the language used must be correct grammatically and also from the terminological aspect (Abdul Razak & Khalijah, 2009). The language and terminology corner or column created in a lecture will help students to improve and strengthen their command of the language. This corner will also help students in the writing of their research projects and their theses. It is also useful for those who wish to submit their papers to the local journals.

Students often make mistakes in their writings and project presentations. The common mistakes in their writings are in the language structure, spelling and terminologies, while in the project presentations the common mistakes are their pronunciation. We now give some rules regarding the language structure, spelling, pronunciation and terminologies.

#### 5.1 Rules

In science the plural words or terminologies are *data* and *bakteria*. We did not take the singular forms *datum* and *bacterium*. The terminologies in the singular form are such as *fenomenon* (not *fenomena*), *kriterium* (not *kriteria*), *spektrum* (not *strata*). People from the arts prefer the plural form of the words.

Most people like to use the combination "antara ... dan". For instance "Antara Pasukan A dan Pasukan B". The correct combination should be "antara ... dengan".

A number of users could not differentiate between "*dari*" and "*daripada*". The word *dari* is used to indicate place and time, and also used for *aspek* and *segi*. On the other hand *daripada* is used for people, composition, comparison and for something abstract.

The phrase bergantung kepada should be replaced by bergantung pada (depends on).

Prefix	"non-noun" is matched with '	'bukan kata nama'':
	non-Malay	bukan Melayu
	non-zero	bukan sifar
Prefix	"non-adjective" is matched w	ith "tak kata sifat":
	non-empty	tak kosong
	non-linear	tak linear

Adjective derived from people's name is written with the suffix -an at the end of the name and the first letter is capital.

Archimedean	Arkhimedesan	
abelian	Abelan	
Hamiltonian	Hamiltonan	
Lagrangean	Lagrangean	
Pythagorean	Pithagorasan	

#### 5.2 Spelling

Spelling mistakes often occur in the writing of students and also lecturers. According to the latest development of the Malay language, the word *baru* (new) is now spelt *baharu*, and the phrase *di kalangan* (among) is now being replaced by *dalam kalangan*. The frequent mistakes are as follows:

pemetakan (plot)	not	pemetakkan
idea	not	ide
lapor (report)	not	lapur

<i>mi</i> (noodle)	not	mee
perabot (furniture)	not	perabut
insurans (insurance)	not	insuran
kohort (cohort)	not	kohot
respons (response)	not	respon
trend	not	tren
merekrut (recruiting)	not	mengrekrut
pengezonan (zoning)	not	pengzonan

Other mono syllable words are: bom, cam, cat, dop, had, kod, lap, sah, syor, zat, zip.

The following terminologies should be two words:

atur cara (program)	not	aturcara
kenal pasti (identify)	not	kenalpasti
hasil tambah (sum)	not	hasiltambah
isi padu (volume)	not	isipadu
pemboleh ubah (variable)	not	pembolehubah
sama ada (whether)	not	samada (does not exist)
reka bentuk (design)	not	rekabentuk
segi tiga (triangle)	not	segitiga
uji kaji (experiment)	not	ujikaji
urus setia (secretariat)	not	urusetia
Pro-Canselor (Pro-Chancellor)	not	Pro Canselor
inframerah (infrared)	not	infra merah
<i>juruukur</i> (surveyor)	not	juru ukur
mikrogelombang (microwave)	not	mikro gelombang/gelombang mikro
prasiswazah (undergraduate)	not	pra siswazah
tatakaedah (methodology)	not	tata kaedah
ultralembayung (ultraviolet)	not	ultra lembayung

#### 5.3 Pronounciation

In a lecture or students' presentation, special attention should be given to the correct pronounciation according to the rules of the Malay language. For example;  $sin \theta$ ,  $kos \theta$  dan  $tan \theta$  should be pronounced sinus (not sain) teta, kosinus (not kosain) teta, and tangen (ta-ngen, or tan-gen, but not tenjen) teta respectively; plus (not plas) and minus (not maines) as for "halus".  $\pi$  should be pronounced as pi Yunani (Greek p, so as not to be confused with the letter p, and not pai). In Universiti Kebangsaan Malaysia people often pronounce DECTAR (Dewan Canselor Tun Abdul Razak) as DEKTAR, such that the acronym DEKTAR was written on a directory board in the campus.

#### 5.4 Terminology

In the terminology book, two points have been listed, namely synonymous (same meaning) terminologies indicated by a stroke (/), and terminologies for different context, indicated by a semi-colon (;). For example:

# Synonym

dimension	matra/dimensi
transformation	penjelmaan/transformasi
adjoint	dampingan/adjoin
unique	bitara/unik
operator	pengoperasi/operator

not

We should popularise the terminologies in Malay.

gabungan	
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kombinasi

pengelasan	not	klasifikasi
zarah	not	partikel

#### **Different context**

binary	dedua; perduaan
distributive	taburan; agihan
square	segi empat sama; kuasa dua; persegi
table	jadual; sifir

The user has to be very careful in selecting the correct terminology. For example, *operasi dedua* (not *operasi perduaan*) is used for "binary operation" and "binary system" is *sistem perduaan* (not *sistem dedua*).

It is well known, in the Malay language we have more words (terminologies, depending on the context) to match a single entry. For example, for the entry "carry" we have *angkat* (lift), *jinjing* (carry in one hand), *bimbit* (carry in the palm), *pikul* (carry on the shoulder), *junjung* (carry on the head) etc, and for the entry "rice" we have *beras* (uncooked rice) and *nasi* (cooked rice).

## 5.5 Translation

Students' translation work in their research project is not encouraging. Most of them used Google to find the terminologies and as a dictionary. We obtained the following in their work.

untuk tidak membuktikan (for not to prove: prove is bukti, and "im" is considered as a prefix) for "to improve",

musim selsema for "cold season",

terjemahan which should have been translasi for "translation",

di mana is often taken for "where".

In the Malay language *di mana* is normally used for question and should end with a question mark (?). Instead of *di mana* we can use the word *yang* or *dengan* or by answering *Di sini* (Here) .... Similarly with the words *bila* and *apabila*. The word *bila* should be used for question.

*masing-masing* for "respectively" should be placed in the correct position, not at the end of a sentence as in English. For example:

Let  $\mu_A$  and  $\mu_B$  denote the membership functions of the fuzzy sets A and B, respectively.

Andaikan  $\mu_A$  dan  $\mu_B$  masing-masingnya melambangi fungsi keahlian bagi set kabur A dan B.

The phrase *is called* should not be translated as "dipanggil". In Malay we use "dipanggil" for something that could respond to the call. We should use the words "dinamai" or "disebut" or "dikenali sebagai".

Some students and academic staffs translate "orthogonal to", "parallel to", "perpendicular to" and "isomorphic to" as *ortogon kepada*, *selari kepada*, *serenjang kepada* and *berisomorfisma kepada*, respectively, since "to" is translated as *kepada*. The appropriate and correct terminologies are *berortogon dengan*, *selari dengan*, *serenjang dengan* and *berisomorfisma dengan*, respectively. Now the word "online" is translated as *atas talian*, which should have been *dalam talian*.

## 5.6 Confusion/"Bunyi serupa"

Some mathematical sciences students could not differentiate between *semesta* (universe) and *semester*. Some of the postgraduate students could not see the difference between "symmetry group" and "symmetric group".

# 6. Conclusion

We hope that the suggestions that have been discussed in this paper could be applied to other courses and the language corner or column could help students and members of the academic staffs to improve and strengthen their command of the Malay language which in turn execute the country's hope in promoting the sovereignty of the national language, Bahasa Melayu, and strengthen the command of the English language (MBMMBI).

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#### References

Abdul Razak Salleh. (2009a). How to make the teaching of Abstract Algebra interesting? Paper presented at The 5<sup>th</sup> IMT-GT International Conference on Mathematics, Statistics and Their Applications. The Hills, Bukittinggi, Sumatera Barat, Indonesia, 9-11 June.

Abdul Razak Salleh. (2009b). Matematik dalam budaya Melayu. Professorial Inaugural Lecture. Bangi: Penerbit Universiti Kebangsaan Malaysia.

Abdul Razak Salleh. (2011). Popularisation of Mathematics. Journal of Quality Measurement and Analysis 7(1): 1-16.

Abdul Razak Salleh & Faridah Mohd Yusof. (1999). Penjelmaan kuih keria kepada cawan kopi. Ceramic.

Abdul Razak Salleh & Khalijah Mohd Salleh. (2009). *Memperkasakan Bahasa Melayu di UKM dan di Persada Antarabangsa*. Proceedings Congress of Teaching and Learning, Universiti Kebangsaan Malaysia: 1-10.

Al-Attas Syed Muhammad Naquib. (1988). The Oldest Known Malay Manuscript: A 16<sup>th</sup> Century Malay Translation of the 'Aqa'id of al-Nasafi. Kuala Lumpur: Department of Publications, Universiti Malaya.

Gallop, A.T. (1994). Warisan Warkah Melayu. London: The British Library.

Humphreys, J.F. & Prest, M.Y. (1991). Numbers, groups and codes. Cambridge: Cambridge University Press.

Renteln, P. & Dundes, A. (2005). Foolproof: a sampling of mathematical folk humour. Notices Amer. Math. Soc. 52(1): 24-34.

Wikipedia. International standard book number. http://en.wikipedia.org/wiki/International\_Standard\_Book\_Number [23 July 2008]