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Research Note

The Perceived Orientation of Aliased Lines

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The use of raster display devices for the display of graphics causes problems of aliasing when edges or lines are produced. This can be significant in those psychophysical experiments where the orientational properties of the stimulus are important. We have assessed the perceived orientation of a selection of aliased lines by comparing them with the orientation of pairs of dots. It is found that the perceptual orientation is modelled well by a least-squares metric on the pixels that compose the line. Small deviations from this metric were found, and were also found in a control experiment employing anti-aliased lines. They appear to be due to range effects. Averaged across subjects, orientational acuity was only slightly lower for aliased lines.

Aliasing Orientation Psychophysics

INTRODUCTION

All graphical display screens have limited resolution. In the case of raster devices, this takes the form of the continuous plane being quantized into nominally rectangular or square pixels. This means that an ideal line or edge will be aliased (unless it is vertical or horizontal). In other words, spatial undersampling leads to the notorious "staircasing" effect seen in Fig. 1.

Several methods have been used to counter the effects of aliasing. One approach is simply to increase the number of pixels used; that is, to increase the spatial resolution. However, this increases the amount of memory required to represent the image. If the device permits multiple grey-levels to be used, then anti-aliasing algorithms (e.g. that of Gupta & Sproull, 1981; see also Foley, Van Dam, Feiner & Hughes, 1990) produce lines and edges which appear smooth. The disadvantage of using such a technique is the increased processing required to generate the appropriate grey-levels. It is not clear whether or not this effort is always justified in psychophysical experiments.

Line stimuli are ubiquitous in the psychophysical literature. The prevalence of orientationally selective neurons in the primate primary visual cortex (Hubel & Wiesel, 1959) implies that neural explanations of psychophysical results must take into account the orientational content of the stimuli. However, despite, or perhaps because of, the commonness of lines, there is almost never any discussion of their production in the psychophysical literature that employs them.

In fact, aliased lines do not have a well-defined orientation: in Fourier terms one might speak of them having a range of orientations. Orientation, strictly speaking, is a property of an ideal line segment or edge. Nevertheless, an aliased line clearly does possess a kind of perceptual orientation, although it is not clear what angle we should assign it. Dorst and Smeulders (1986) have developed optimal estimators for the properties, including orientation, of lines that are aliased as a result of the digitization of a continuous image. The problem of using aliased lines in psychophysics is a quite different one: the perceptual attributes of a sampled image may be quite different from those of the original image.

Even though vision papers rarely specify the means used to generate line stimuli, it is possible to infer from the use of one-bit displays that aliased lines have been employed (e.g. De Weerd, Vandenbussche & Orban, 1992). In particular, inspection of the figures in many published studies on texture segregation based on orientation differences suggests the widespread use of aliased line segments (e.g. Nothdurft, 1991). It is thus important to know what the perceived orientations of such lines are, and how accurately various possible metrics can reproduce them.

There are two very simple metrics that could be used as a measure of the orientation of an aliased line. These are illustrated in Fig. 2. The first is to draw an ideal line between the most distant corners of the stimulus, and take its orientation as an estimate for the underlying line.

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We will refer to this as the corner-to-corner (CC) metric. In the case of a line at 45 deg considerations of symmetry imply that this should give the perceptually correct result. In the case of a horizontal line, however, the ideal line is slanted, and clearly incorrect. The other metric takes the middle points of the faces of the ends of the aliased line (i.e. the middles of the vertical end edges), and draws a line between them, the orientation of this line again being the metric. This is the face-to-face metric (FF). By contrast with the CC metric, the FF metric appears to work perfectly with horizontal lines, but fails when applied to the 45 deg example. There is naturally an analogous FF metric which would work with vertical lines.

If experimenters were to only use aliased lines which are horizontal, vertical, and at 45 deg there would not be a problem, but as soon as other stimuli are used, it is ambiguous which if either of the above metrics is applicable. What is clear, however, is that if *just one* of these measures is used to estimate the orientations of an ensemble of lines such as those in Fig. 1, then some of the estimates must be incorrect. We propose a least-squares metric which reproduces the correct results of the FF and CC estimates, and which we compare with the perceptual orientation in the Results sections. Given the lack of discussion of these issues in the literature, it seems probable that most psychophysicists who employ aliased lines simply take the orientation of a line as being the arc tangent of the ratio of the coordinate differences used by dedicated routines to produce the lines. This metric will normally be equivalent to either FF or CC, depending on the specifications of the routine.

Least-squares metric

This orientation estimator is illustrated in Fig. 3. It was chosen because it appeared to be the simplest plausible metric available. To produce this metric one considers a given aliased line and an ideal line arbitrarily located on the plane. For such a line the sum of the squares of the perpendicular distances of the centres of the pixels from the line is calculated. The minimum of this sum over ideal line position and orientation is found, after which the orientation of the line at this least-squares minimum is taken as the estimate. Symmetry of the horizontal and 45 deg lines implies that the least-squares metric will give the desired results here. Instead of squaring the pixel distances other indices were tried, but this did not improve the fit to the perceptual data.

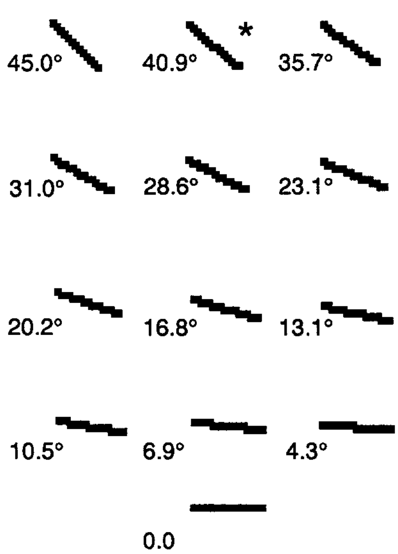

FIGURE 1. The set of aliased lines used in the experiment. The least-squares estimate of the orientation is next to each line.

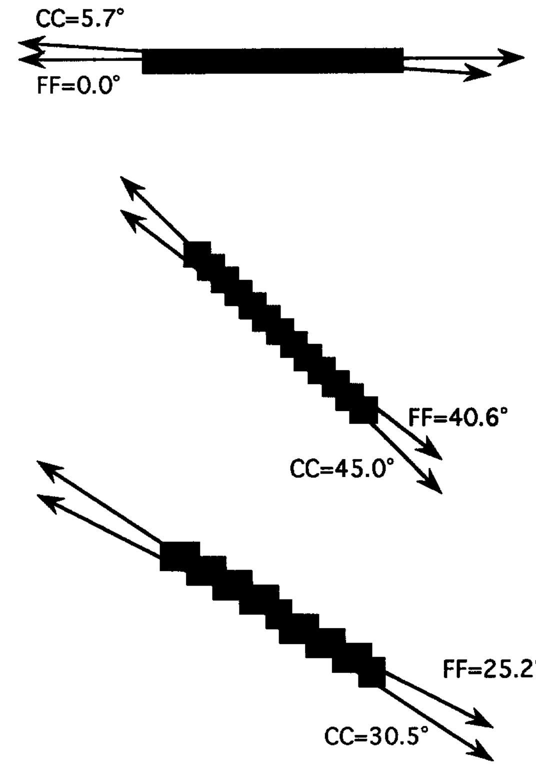

FIGURE 2. How the corner-to-corner (CC) and face-to-face (FF) metrics are produced. The least-squares estimates for these stimuli are 0, 45 and 28.6 deg.

We now consider the magnitude of errors produced by using simply the FF or CC estimates. Maximum errors for each can be seen in Fig. 4. The error for the CC metric is the deviation from horizontal when used on a horizontal line, whilst for the FF metric the difference from 45 deg when used on a 45 deg line is calculated. The width of the line in number of pixels affects the errors produced. The lines in Fig. 1 have a width of 2 pixels and are approximately 20 pixels long. It should be noted that scale invariance does not apply here: in general a 2 pixel wide line is not the same as any 1 pixel line scaled by any amount. It is clear that the length of the line is the crucial determinant of the error. In the limit as the lines become infinitely long the error tends to zero, but for short lines the error can be considerable. For the stimuli in Fig. 1 the maximal errors produced by the FF and CC metrics respectively are 4.1 and 5.4 deg—non-trivial magnitudes, particularly bearing in mind that orientation discrimination thresholds are usually lower than 1 deg (Heeley & Buchanan-Smith, 1990). In the Results section the extent

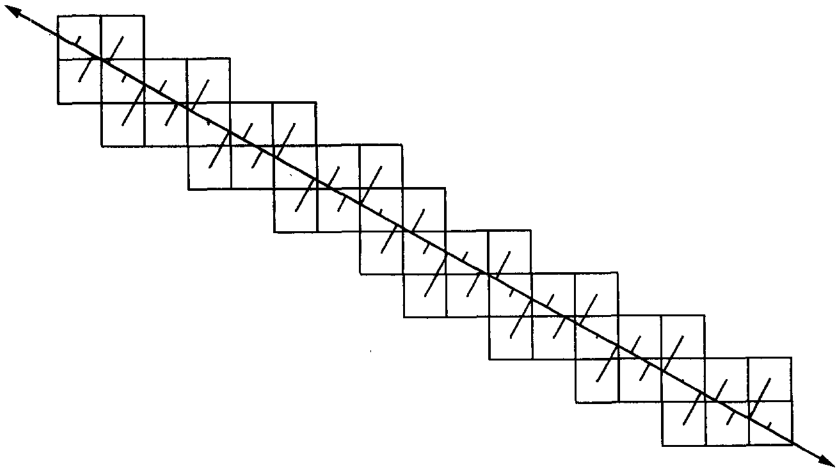

FIGURE 3. The least-squares estimate (28.6 deg) for an aliased line with the individual pixels shown. The least-squares value is produced by summing the squares of the *perpendicular* distances of the pixel centres from the ideal line under consideration. This value is then minimised to produce the estimate.

of the *mean* errors produced by these metrics will be discussed.

Because the appropriate value for the orientation of an aliased line is ambiguous, we conducted an experiment to determine the perceptual orientations of the lines in Fig. 1, and compared this with the least-squares metric described above.

EXPERIMENT 1

Methods

A typical stimulus for the experiment is shown in Fig. 5. An aliased line was presented at the centre of a computer monitor, with two flanking 3 × 3 pixel dots which lay on an imaginary line that passed through the centre of the monitor and hence the aliased line. The orientation of the dots varied, and subjects were required to judge whether the line was more or less clockwise than the dots. For each aliased line a psychometric function was plotted against dot orientation, and the 50% point (the point of subjective equality: pse) taken as the perceived orientation of the line. The 50% point was

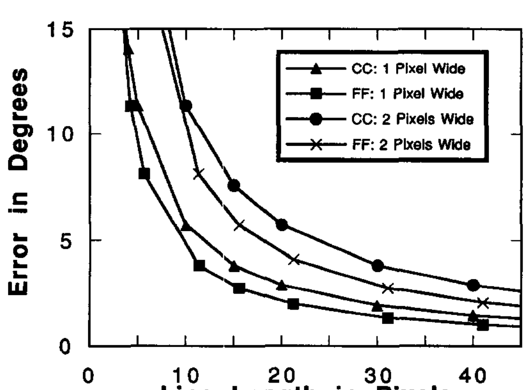

FIGURE 4. The maximum errors produced by the FF and CC metrics. These are calculated by taking the least-squares estimates of 0 and 45 deg for the horizontal and oblique lines respectively as being the correct values.

found using probit analysis (Finney, 1971), which fits a cumulative normal to the psychometric function. The standard deviation of the normal gives a measure of the threshold or orientational acuity for the task. Feedback was not provided, as there was no correct or incorrect response *per se*.

The stimuli were generated with the use of an Apple Macintosh IIcx computer. The lines were drawn using the Macintosh Toolbox routine "Line" (Apple Computer, Inc., 1985) with a width of 2 pixels. This routine presumably uses some variant of Bresenham's algorithm (Bresenham, 1965), but this is not documented. The nominal length of the lines was 20 pixels. It should be noted that the length of an aliased line is also not a well-defined quantity. We used a corner-to-corner approximation. Owing to the possibility of the particular type of screen device affecting the perceived orientation of the lines, we conducted the experiment on two different types of monitor. One was a Macintosh 13″ Colour display whilst the other was a Macintosh 21″ Monochrome display. The pixels on the latter appeared somewhat better defined than on the former.

The nominal length of the aliased stimulus line as defined above subtended 39.6′ of visual angle, whereas the notional width subtended 4.0′. The stimuli were viewed at a distance of 57.5 cm on the 21″ monitor, and 63.75 cm on the 13″ monitor. Each stimulus comprised dark pixels on a bright background, which was windowed by a circular aperture of radius 12.7 deg (for the 21″ monitor) or 5.9 deg (for the 13″ monitor). The comparison dots were each at an eccentricity of 3.0 deg, and subtended 6′. Subjects performed the experiment under dim illumination. Two stimulus durations were seen by the subjects: 495 and 105 msec. It was hoped that the use of the shorter duration would, by preventing eye movements, tap preattentive mechanisms of the kind implicated in texture perception (Julesz, 1962).

A total of 4 subjects were used. These were the authors and PL, a graduate student. All observers possessed normal or corrected-to-normal vision, and were

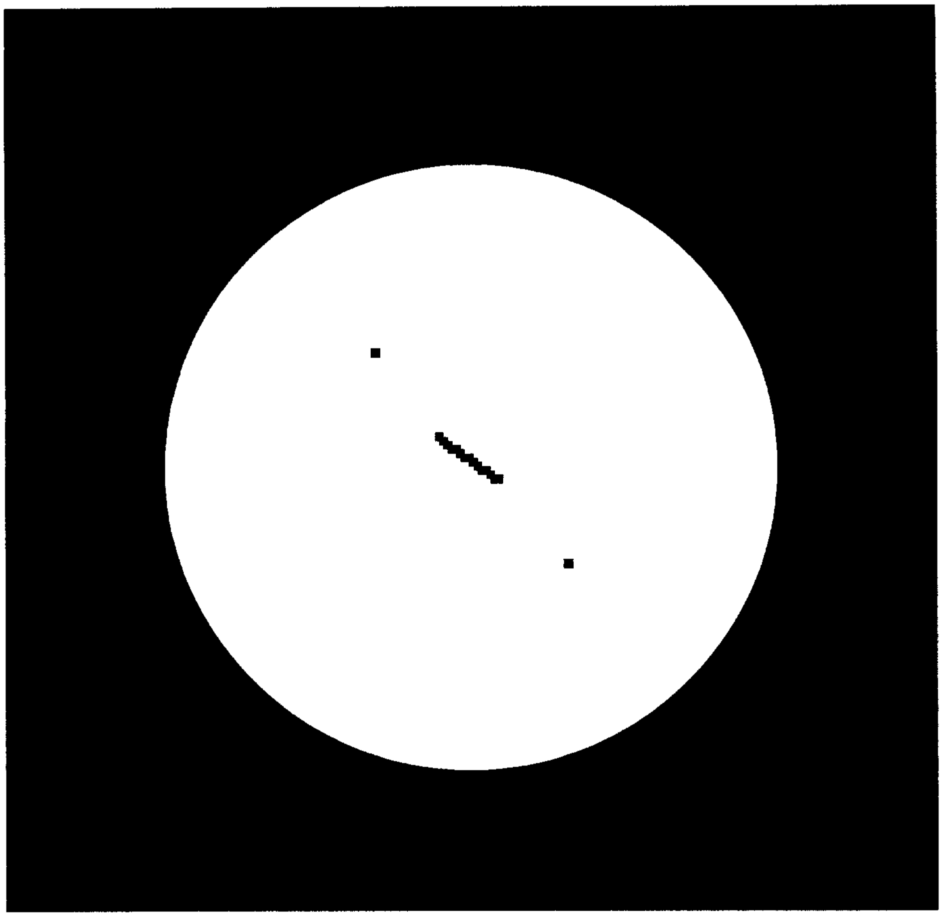

FIGURE 5. The stimulus arrangement employed in the experiment using the 13" monitor. For clarity, the central aliased line has twice its actual size relative to the circle and dots.

experienced psychophysical observers. In all, 13 aliased stimuli (shown in Fig. 1) were employed in the experiment. Each line was compared with eight different dot-pair orientations, with a steplength of 1 deg. These were centred about the least-squares estimate for aliased line orientation. The percent judged clockwise points for the psychometric functions were each taken from 20 observations. Thus every individual psychometric function comprised 160 observations. The subjects performed the experiment for a given monitor/duration combination in four blocks—five instances of every stimulus/comparison combination in each. The experiment proper was preceded by one practice block.

Results

The perceptual orientations, averaged across subjects, monitor types and duration are shown in Fig. 6. Also included are the CC and FF estimates. All three are plotted against the least-squares approximations. In general, the perceptual data lies closer to the least squares values, than to either of the other two metrics. The mean absolute deviation of the metric orientations from the pse's are as follows: CC: 3.3 deg, FF: 2.7 deg, least-squares: 0.98 deg. This demonstrates that errors due to our metric are on average only one-third of those produced using the simple CC and FF measures, which only use information concerning the ends of the stimuli. In addition, the CC and FF metrics are systematically in error in the same direction, whereas the least-squares estimator has approximately the same number of positive and negative errors. Yakimoff and Mitrani (1979),

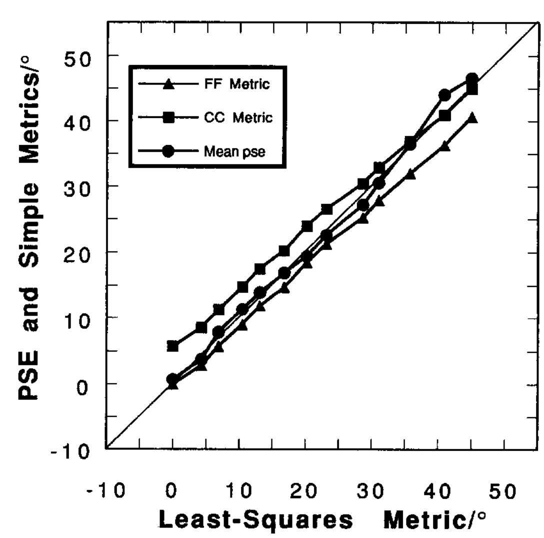

FIGURE 6. The perceptual orientations (pse = point of subjective equality), averaged over all conditions employed are plotted against the corresponding least-squares estimates. Perfect prediction produces points which lie on the diagonal. Also shown are predictions produced using the face-to-face (FF) and corner-to-corner (CC) metrics.

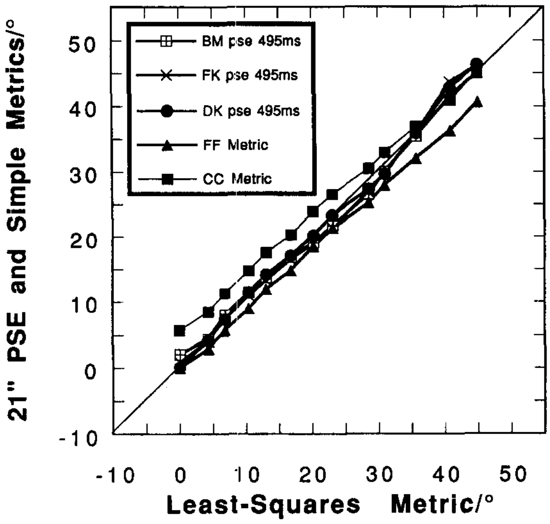

FIGURE 7. The perceptual orientations for three subjects using the 21″ monochrome monitor.

have reported that a least-squares metric effectively approximates the perceived orientation of an elliptical cloud of dots as assessed by the method of adjustment. These workers found that neither lines connecting the most distant points of their stimuli nor the axis of the determining ellipse had the correct orientation. Watt, Ward and Casco (1987) found that psychophysical results concerning deviation from straightness in lines were comprehensible in terms of deviation from a least-squares regression line rather than a line joining the line ends.

Results for individual subjects are plotted in Fig. 7 (21″ monitor) and Fig. 8 (13″ monitor). Although most of the individual data points are very similar to the mean results shown in Fig. 6, there is one rather striking exception. Subject PL at 105 msec assessed the orientation of the stimulus marked "*" in Fig. 1 as being radically different to all three metrics, and all other subject/monitor/duration combinations. The probit analysis on the corresponding psychometric function

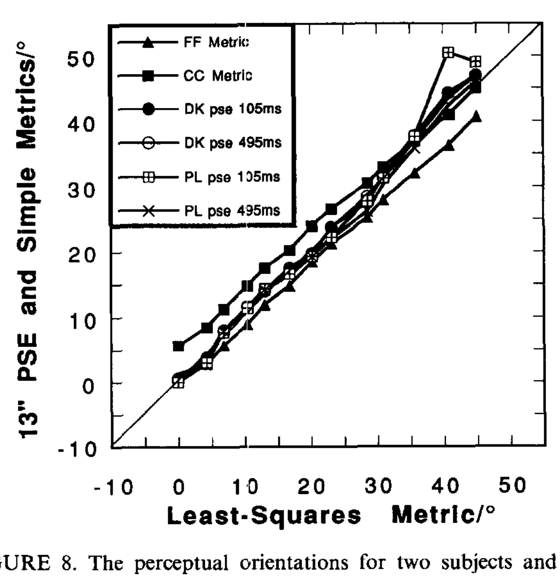

FIGURE 8. The perceptual orientations for two subjects and two durations using the 13″ colour monitor.

produced a pse of 50.5 deg, as compared with a mean of 44.0 deg for all other pse's at this angle. Inspection of the psychometric function reveals that the proportion judged to be anticlockwise of the comparison dots never fell below 75%, and that the pse thereby generated must be viewed with caution as to its accuracy. Because of the anomalous nature of this data point, PL was required to perform a further experiment using just the "*" stimulus at two viewing distances with a presentation time of 105 msec. One viewing distance was the standard 63.75 cm and the other double this: 127.5 cm. The rationale for using the longer distance was that reducing the visual angle of the line would make it harder to resolve individual pixels of the line, thus helping to remove distracting local features from the stimulus. The range of dot orientations was expanded to 14.5 deg—more than adequate to cover the full range of this subject's performance. Surprisingly, the results from this experiment were rather similar to the average pse from the other subjects for this line of 44.0 deg. For PL the perceived orientations at 63.75 and 127.5 cm were 43.1 and 42.3 deg respectively. We must therefore conclude that, for this subject at least, the perceptual orientation is context dependent. In fact, PL reported that several of the lines, including "*", appeared curved to him. The line "*" arguably has the largest deviation from linearity, or "bump" of any of the line cohort used. Watt *et al.* (1987) concluded that line segments are seen as not straight if they contain a bump with an area larger than 0.3 arc min^2, the axis of reference being the least-squares regression line. That study used very thin lines, making direct comparison with our results difficult, but it is worth noting that a single pixel in our standard results had a visual angle of 4 arc min^2: many times the Watt *et al.* straightness threshold. What appears to be happening is that in some circumstances perceptible local features on our stimuli can greatly distort the subjective orientation. Even for other subjects the least-squares metric is at its worst for this stimulus. It is therefore possible that if these aliased stimuli were used in different psychophysical tasks further anomalies would manifest themselves. In order to make the most parsimonious estimates of the accuracy of our least-squares metric, we have retained the initial peculiar data point seen in Fig. 8 when comparing the overall effectiveness of the estimators at the beginning of this section.

In Fig. 9 we display the mean acuities generated for each stimulus at the different durations. These are produced in the probit analysis and, if the psychometric function is an unbiased cumulative Gaussian, correspond to the angular difference between the perceptual orientation and the orientation of the dot pair at which 84% angular discrimination is achieved. It can be seen that the acuity at short durations is approx. 45′ poorer than at long durations, and that the ubiquitous meridional anisotropy (e.g. Heeley & Buchanan-Smith, 1992) is evident. The best acuity is about 1.1 deg—somewhat worse than the record orientational acuity of 0.24 deg reported by Westheimer (1981), but well within the range of acuities for various tasks summarised by Heeley and

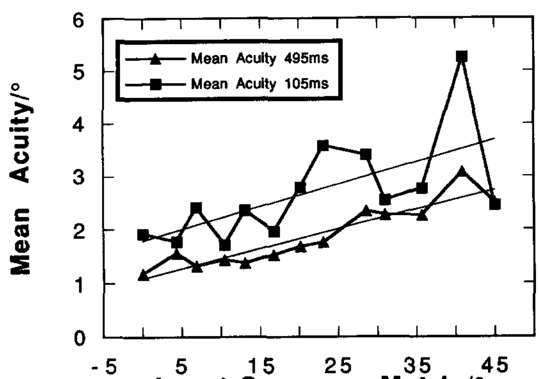

FIGURE 9. Orientational acuities for two stimulus durations averaged over all subject/monitor conditions. Regression lines are also shown. Comparisons between durations for individual subjects DK and PL showed very similar trends.

Buchanan-Smith (1990). The 3 deg eccentricity of the comparison dots no doubt degrades performance, but it is not possible to say from these results whether or not the aliased nature of the stimulus further diminishes acuity.

It is evident from Figs 7 and 8 that aside from the single exception just described, neither monitor type, stimulus duration nor subject makes any systematic difference to the perceived orientation. However, there is some small but systematic variation of the perceptual orientation with respect to the least-squares metric: the mean curve in Fig. 6 snakes smoothly about the diagonal of perfect prediction. This could be due to the presence or absence of various local features which subtly distort the perception from the least-squares approximation. Or alternatively, some factor unrelated to aliasing may be at work.

EXPERIMENT 2

We decided to test whether the small but systematic departures from the least-squares prediction seen in Fig. 6 was due to aliasing by repeating Expt 1, but using anti-aliased lines. Because some time had elapsed since Expt 1 was performed, and the equipment used was different, we also tested subjects on aliased lines again.

Methods

The stimuli were produced with the use of an Apple Macintosh IIfx computer and a Macintosh 13″ Colour monitor, the anti-aliased lines being drawn using the algorithm of Gupta and Sproull (1981). They were 39.6′ long (as before), and had a width of 2′ (half that of the aliased lines). The orientation of the line is that of an ideal line, low-pass filtered and sampled. The stimuli were displayed for 105 msec. Authors DK and FK were again subjects, along with SR, a graduate student who was naive to the purpose of the experiment. The spacing between the comparison dot orientations was again 1 deg for most of the lines, but was increased to 1.5 deg for the four lines with orientations closest to 45 deg. All other conditions were the same as in Expt 1.

Results

The residual differences between the perceived orientation of the lines and the metric orientation (least-squares for aliased lines and algorithm input orientation for anti-aliased lines) are presented in Fig. 10. This pattern of errors is broadly similar to that seen in Expt 1. There is very little significant difference between the residuals for aliased and anti-aliased lines. We can thus

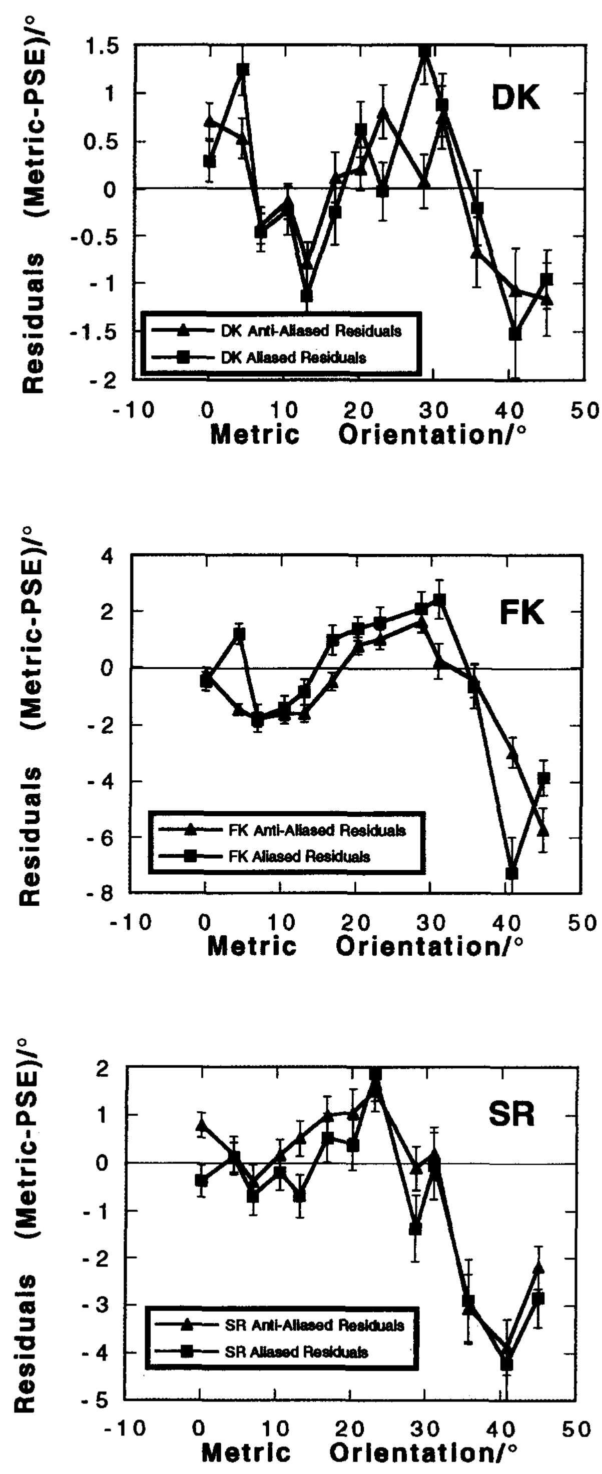

FIGURE 10. The residual difference between the perceived orientation (pse) and the metric orientation as a function of the metric orientation. Results are shown for both aliased and anti-aliased lines. These results are from Expt 2.

conclude that the deviations of the perceptual orientations from the least-squares metric are not caused by aliasing, or, to put it a different way, the least-squares metric is not substantially defective. The cause of the deviations is likely to be high-level in origin.

We turn to the mean absolute deviations of the pse's from the metric orientations for the aliased and anti-aliased stimuli. They are, respectively, for FK 1.99 and 1.54; for DK 0.71 and 0.57; and for SR 1.25 and 1.16. The aliased pse's are thus, on average, 0.23 deg further from the metric than those of the anti-aliased lines. This is the most reasonable estimate of the deficit in the least-squares metric that can be obtained from our data. The acuities for the three subjects and two conditions may be seen in Fig. 11. Subject DK shows no difference in acuity between the two conditions, whilst FK shows a small (average = 1.8 deg), but consistent superiority for the anti-aliased lines and SR shows a small superiority for anti-aliased lines around the oblique. Overall, then, aliased lines seem to be only marginally inferior to anti-aliased lines in the specifiability of their perceived orientations.

EXPERIMENT 3

Although we have established that the deviations in perceived orientation from the least-squares metric are not due to aliasing, their source remains an open question. We conjectured that they could be due to range effects caused by having blocks containing 13 different line orientations. Subjects might tend to be influenced by the deviation of a particular presentation from the mean orientation of the line cohort (21.2 deg). This would predict that stimuli close to 45 deg would be seen as further from the mean than otherwise expected. Our results in Fig. 10 show this effect, although of course there is quite a lot of other structure present in the residuals.

We tested our conjecture by performing an experiment with anti-aliased lines where the orientations of the lines varied from 20.2 to 69.8 deg—that is, the orientations were symmetrical about 45 deg, and the range of orientations was similar to that used before. In the previous experiments the orientation ranged from 0 to 45 deg. In each block 14 orientations were presented, as opposed to the previous 13, but otherwise the conditions were identical to Expt 2. The resulting residuals, together with the anti-aliased residuals from those orientations in Expt 2 which overlapped with those used in this experiment are shown in Fig. 12. For subjects SR and DK the change in stimulus orientation range from 0–45 deg to 20.2–69.8 deg has clearly shifted the residual data such that the point of least error is now at approx. 45 deg, whereas formerly this was the region of *greatest* error. In FK's results no such change is seen. The fact that we can, for two out of three subjects, disrupt the pse for the orientation of a line by changing the stimulus set in which it appears is strong evidence that a rather complex cognitive range effect is the cause of the deviations from veridicality seen in these experiments.

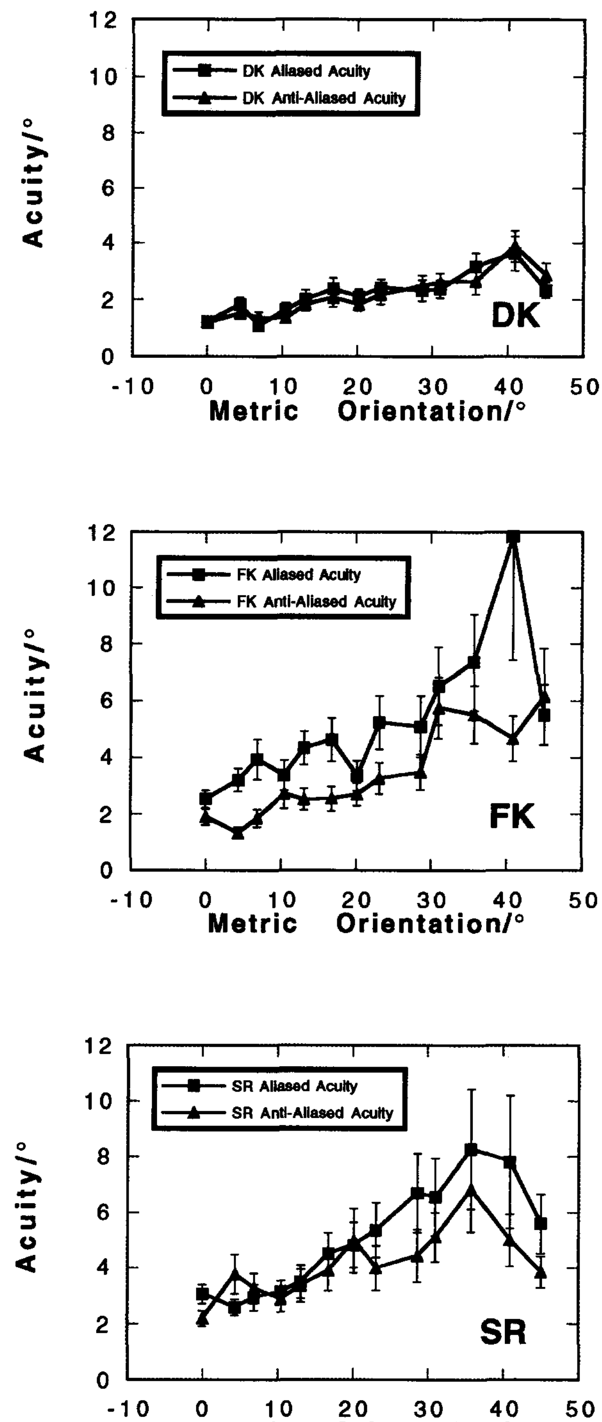

FIGURE 11. Orientational acuities for both aliased and anti-aliased lines from Expt 2.

SUMMARY AND CONCLUSION

(1) The perceived orientation of an aliased line is well described by a least-squares metric. This metric defines a notional line which minimises the sum of the squared deviations of the centres of the pixels perpendicular to it. Two other metrics which considered only the location of features at the end of the aliased line were found to be relatively poor descriptors of perceived orientation, and were

shown on *a priori* grounds to be particularly inappropriate for short line lengths.

(2) When the least-squares metric was used to define the orientation of an aliased line, its perceived orientation was only marginally different from an anti-aliased line whose orientation was defined by the algorithm which generated it. The orientational acuities of the aliased lines were found to be on average only slightly worse than the anti-aliased lines, and this was not systematic across subjects.

(3) We did find small but significant departures in the perceived orientation of the aliased lines from that predicted by the least-squares metric. A subsequent experiment using anti-aliased lines revealed a virtually identical pattern of departures, suggesting that they were not due to aliasing *per se*. Finally, when we shifted the range of orientations tested to a different mean orientation, the pattern of departures was quite different in two out of the three subjects, implying that the departures from least-sqares prediction were of a high level cognitive origin.

These results imply that the effort involved in generating anti-aliased stimuli, or in developing the code for a reasonable metric for the orientation of aliased lines, is justified when conducting accurate psychophysical experiments—a fact that had not to our knowledge been previously demonstrated.

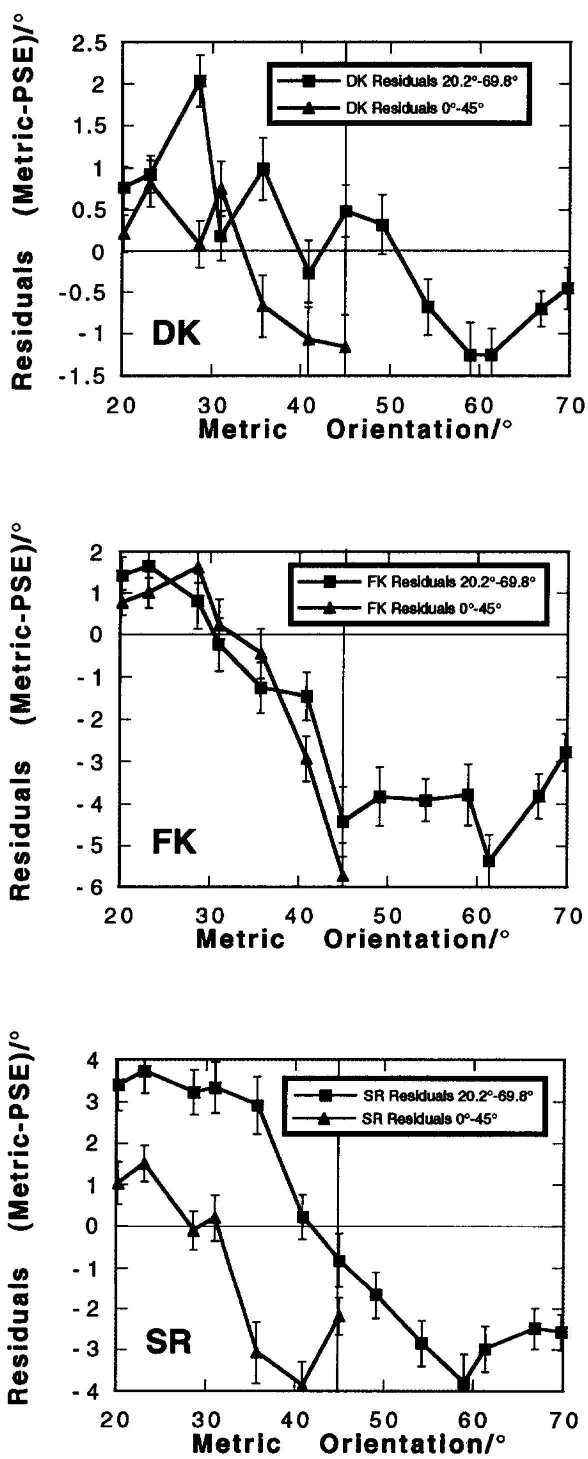

FIGURE 12. The residual difference between the perceived orientation (pse) and the metric orientation as a function of the metric orientation. Results for anti-aliased lines are shown. Different ranges of orientations were used in the two conditions shown. The data from the 0–45 deg range are from Expt 2, whereas the data from the 20.2–69.8 deg range are from Expt 3.

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