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Influence of Joule Heating on MHD Peristaltic Flow of a Nanofluid with Compliant Walls

M. Ganeswara Reddy* and K. Venugopal Reddy

Department of Mathematics, Acharya Nagarjuna University Campus, Ongole 523 001, India

Abstract

The present study investigates the effects Joule heating and MHD peristaltic transport of nanofluid in a channel with compliant walls. The transport equations involve the combined effects of Brownian motion and thermophoretic diffusion of nanoparticles. The mathematical modeling is carried out by utilizing long wavelength and low Reynolds number assumptions. The closed form solution for stream function is computed and resulting coupled nonlinear equations are solved numerically by using shooting technique through computational software Mathematica. Numerical results are graphically discussed for various values of sundry parameters on the flow. Also, the trapping phenomenon is examined with respect to emerging parameters of interest.

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Keywords: Peristalsis; MHD; Nanofluid; Joule heating; Compliant walls

1. Introduction

The nanofluids are an innovative class of resolutions proposed by scattering nanometer-sized materials (nanoparticles, nanofibers, nanotubes, nanowires, nanorods, nanosheet, or droplets) in base fluids. Peristalsis in connection with nanofluids has application in biomedicines, i.e. cancer treatment radiation therapy, etc. Choi [1] was the first to introduce the word nanofluid that represent the fluid in which nanoscale particles (diameter less than 50 nm) are suspended in the base fluid. Kuznetsov and Nield [2] have studied the natural convective boundary-layer

* Corresponding author. Tel.: +91-9490636769; fax: 08592224842.
E-mail address: mgrmaths@gmail.com

flow of a nanofluid past a vertical plate analytically. They used a model in which Brownian motion and thermophoresis effects were taken into account. Moreover, Khan and Pop [3] used the same model to study the boundary layer flow of a nanofluid past a stretching sheet with a constant surface temperature. Very recently, Gnaneswara Reddy [4] analyzed the influence of magnetohydrodynamic and thermal radiation boundary layer flow of a nanofluid past a stretching sheet. Akbar et al. [5] discussed the slip effects on the peristaltic transport of nanofluid in an asymmetric channel. The effects of endoscope on the peristaltic transport of nanofluids have been studied by Akbar and Nadeem [6]. Recently, the influence of wall properties on the peristaltic flow of a nanofluid is discussed by Mustafa et al. [7].

Joule heating describes the process where the energy of an electric current is converted into heat as it flows through a resistance. There are many practical uses of Joule heating such as Electric stoves and other electric heaters Soldering irons and cartridge heaters Electric fuses Electronic cigarettes usually work by Joule heating, vaporizing propylene glycol and vegetable glycerine, thermistors and resistance thermometers.

The peristaltic flow of nanofluid model in literature regarding the effects of wall properties channel is still not detonated. In the present investigation highlighted MHD peristaltic motion of nanofluid in a channel with compliant walls with joule heating. The governing equations of motion, heat and nanoparticles concentration are simplified using the assumptions of long wavelength and low Reynolds number approximations. The exact solution of stream function is computed and using the stream function in the temperature and nanoparticles concentration, the resulting equations can be solved numerically by using shooting technique. The importance of pertinent flow parameters entering into the flow modeling is discussed. Also, the trapping bolus phenomenon is also elaborated through streamlines.

2. Mathematical formulation of problem

Consider the peristaltic nanofluid flow of an incompressible through a two-dimensional channel of uniform thickness $2d$. A uniform magnetic field B_0 is applied along the y -axis. The lower and upper walls of the channel are maintained at constant temperatures T_0 and T_1 , and at constant concentrations C_0 and C_1 respectively. Fig. 1 shows the geometry of the present flow problem. The motion in a channel are induced by imposing moderate amplitude sinusoidal waves on the compliant walls of the channel and thus the walls are defined by

$$y = \pm \eta(x, t) = \pm \left[d + a \sin \frac{2\pi}{\lambda} (x - ct) \right] \quad (1)$$

where d is the mean half width of the channel, a is the amplitude, λ is the wavelength, t is the time and c is the phase speed of the wave respectively.

The equations governing the motion for the present problem are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (2)$$

$$\rho_f \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\frac{\partial p}{\partial x} + \mu \left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right] - \sigma B_0^2 u \quad (3)$$

$$\rho_f \left(\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = -\frac{\partial p}{\partial y} + \mu \left[\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right] \quad (4)$$

$$\left(\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = \alpha \left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right] + \frac{v}{c_p} \left[4 \left(\frac{\partial u}{\partial x} \right)^2 + \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right)^2 \right]$$

$$+\tau \left[D_B \left(\frac{\partial C}{\partial x} \frac{\partial T}{\partial x} + \frac{\partial C}{\partial y} \frac{\partial T}{\partial y} \right) + \frac{D_T}{T_m} \left\{ \left(\frac{\partial T}{\partial x} \right)^2 + \left(\frac{\partial T}{\partial y} \right)^2 \right\} + \frac{\sigma B_0^2 u^2 \mu}{\rho_f c_p} \left[2 \left(\left(\frac{\partial u}{\partial x} \right)^2 + \left(\frac{\partial v}{\partial y} \right)^2 \right) + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 \right] \right] \tag{5}$$

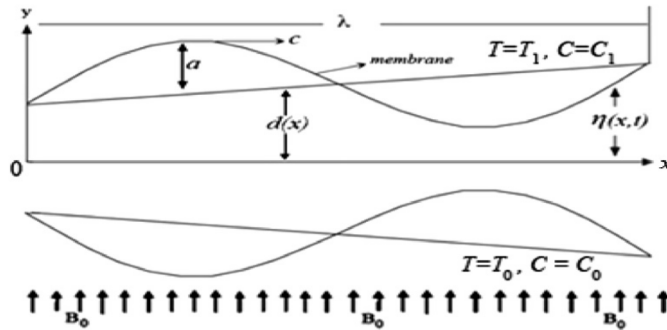


Fig.1. Geometry of the problem.

$$\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_m \left[\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} \right] + \frac{D_T}{T_m} \left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right] \tag{6}$$

where u, v are the components of velocity along x -and y directions, p is the pressure, μ is the coefficient of viscosity of the fluid, σ is the electrical conductivity of the fluid, α is the thermal diffusivity, ν is the kinematic viscosity, ρ_f is the density of the fluid, c_p is the specific heat at constant pressure, D_B is the Brownian motion coefficient, D_T is the thermophoretic diffusion coefficient, τ is the ration of effective heat capacity of the nanoparticle material to heat capacity of the fluid, T is the temperature, T_m is the mean temperature and C is the concentration.

The boundary conditions for the velocity, temperature and nanoparticles concentration conditions at the wall interface are given by

$$u = 0, T = \begin{bmatrix} T_1 \\ T_0 \end{bmatrix} \text{ and } C = \begin{bmatrix} C_1 \\ C_0 \end{bmatrix} \text{ at } y = \pm \eta \tag{7}$$

where T_1 and C_1 are the temperature and concentration at the upper wall and T_0 and C_0 are the temperature and concentration at the lower wall respectively.

Introducing the stream function $\psi(x, y, t)$ such that

$$u = \frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial x} \tag{8}$$

and defining the following non-dimensional quantities

$$x' = \frac{x}{\lambda}, y' = \frac{y}{d}, \psi' = \frac{\psi}{cd}, t' = \frac{ct}{\lambda}, \eta' = \frac{\eta}{d}, p' = \frac{d^2 p}{c\lambda\mu}, \theta = \frac{T - T_0}{T_1 - T_0}, \phi = \frac{C - C_0}{C_1 - C_0}, \varepsilon = \frac{a}{d}, \delta = \frac{d}{\lambda},$$

$$Re = \frac{\rho cd}{\mu}, M = \sqrt{\frac{\sigma}{\mu}} B_0 d, Pr = \frac{\rho \nu \zeta}{k}, Nb = \frac{\tau D_B (C_1 - C_0)}{\nu}, Nt = \frac{\tau D_T (T_1 - T_0)}{T_m \nu}, \tag{9}$$

$$Ec = \frac{c^2}{\zeta(T_1 - T_0)}, E_1 = \frac{-\tau d^3}{\lambda^3 \mu c}, E_2 = \frac{m_1 c d^3}{\lambda^3 \mu}, E_3 = \frac{c d^3}{\lambda^2 \mu}, Sc = \frac{\nu}{D_B}$$

Now using the above non-dimensional quantities in Eqs. (3) – (6) and eliminating the pressure gradient from the resulting momentum equations (after dropping primes) and under the assumptions of long wavelength and low Reynolds number, we get

$$\frac{\partial^4 \psi}{\partial y^4} - M^2 \frac{\partial \psi}{\partial y} = 0 \tag{10}$$

$$\frac{\partial^2 \theta}{\partial y^2} + Pr Nb \frac{\partial \theta}{\partial y} \frac{\partial \phi}{\partial y} + Pr Nt \left(\frac{\partial \theta}{\partial y} \right)^2 + Br \left(\frac{\partial^2 \psi}{\partial y^2} \right)^2 + Br M^2 \left(\frac{\partial \psi}{\partial y} + 1 \right)^2 = 0 \tag{11}$$

$$\frac{\partial^2 \phi}{\partial y^2} + \frac{Nt}{Nb} \frac{\partial^2 \theta}{\partial y^2} = 0 \tag{12}$$

The corresponding boundary conditions are

$$\frac{\partial \psi}{\partial y} = 0 \quad \theta = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \text{ and } \phi = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \text{ at } y = \pm \eta \tag{13}$$

$$\frac{\partial^3 \psi}{\partial y^3} - M^2 \frac{\partial \psi}{\partial y} = \left[E_1 \frac{\partial^3}{\partial x^3} + E_2 \frac{\partial^3}{\partial x \partial t^2} + E_3 \frac{\partial^2}{\partial t \partial x} \right] (\eta) \text{ at } y = \pm \eta \tag{14}$$

where $Br (= Pr Ec)$, M , Pr , Ec , Sc are the Brinkman number Hartman number, Permeability parameter, Prandtl number, Eckert number, Schmidt number and E_1, E_2, E_3 are the non-dimensional wall compliant parameters.

The closed form solution for the equation (10) with the boundary conditions (13) and (14) is

$$\psi = \frac{8\epsilon\pi^3}{M^2} \left[\frac{E_3}{2\pi} \sin 2\pi(x-t) - (E_1 + E_2) \cos 2\pi(x-t) \right] \left[\frac{\sinh My}{M \cosh M\eta} - y \right] \tag{15}$$

3. Numerical Method

Now substituting the expression ψ from equation (15) into Eq. (11), then resulting equation and (12) with corresponding boundary condition Eq. (13) can be solved numerically by employing the built in routine for solving nonlinear boundary value problems with shooting method through the command NDSolve of the symbolic computational software Mathematica 9. Further our obtained results in the limiting case ($M = 0$) is in a very good agreement with the previous study [7].

4. Graphical results and discussion

In the present study following default parameter values are adopted for computations: $x = 0.2, t = 0.1, \epsilon = 0.1, M = 0.5, Pr = 1.0, Br = 0.5, Nb = 0.2, Nt = 0.2, E_1 = 0.02, E_2 = 0.01,$ and $E_3 = 0.01$. All graphs therefore correspond to these values unless specifically indicated on the appropriate graph.

In order to test the accuracy of our results, compared our results of temperature and nanoparticles concentration profiles with those of Mustafa et al. [7] when neglect the effect of M . The comparison show good agreement as presented in Figs. 2 & 3.

The effect of the M on the velocity is illustrated in Fig. 4. It is evident that the velocity decreases with

increase the values of M . This is because of the presence of the transverse magnetic field creates a resistive force similar to the drag force that acts in the opposite direction of the fluid motion, thus causing the velocity of the fluid to decrease. The variation of the velocity with E_1 , E_2 and E_3 are shown in Fig. 5. It is notice that the velocity increases with increasing E_1 and E_2 , whereas it decreases with increasing E_3 .

The temperature profiles for different values of joule heating parameter is plotted in Fig. 6. The contribution of M on temperature appears through the viscous dissipation term. The applied magnetic field has direct effect on the temperature. It is found that the temperature increases in presence of Joule heating effect. Also, this increase in temperature becomes more prominent for $M > 2$. Fig.7 examines the effect of Brinkman number Br on the temperature. It reveals that the temperature increases by increasing the Brinkman number. The influence of Brownian motion parameter on the temperature is shown Fig. 8. There is a substantial increase in the temperature with an increase in Nb . Fig.9. shows the temperature distribution of fluid for different values of thermophoresis Parameter Nt . It is seen that the temperature distribution increases by increasing Nt . Here also the same effect as Brownian parameter on the temperature field.

Fig.10 illustrates the nanoparticles concentration profiles for various values of Nb . The nanoparticles concentration is found to decrease when the Brownian motion effect intensifies. It is interesting to see that the smaller values of Nb strongly affect the concentration function. It is noticed that magnitude of concentration function is significantly increase with an increase in Nt . The influence of the thermophoresis parameter on nanoparticles concentration is graphically displayed in Fig. 11. It is observed that the nanoparticles concentration decreases when there is an increase in the values of thermophoresis parameter.

The trapping bolus phenomenon reads variation of travelling of circulating bolus covered by streamlines as the flow progresses. Figs.12 & 13 illustrates the of Hartmann number M on the streamlines. It reveals that the volume of the trapped bolus decreases with increase of M .

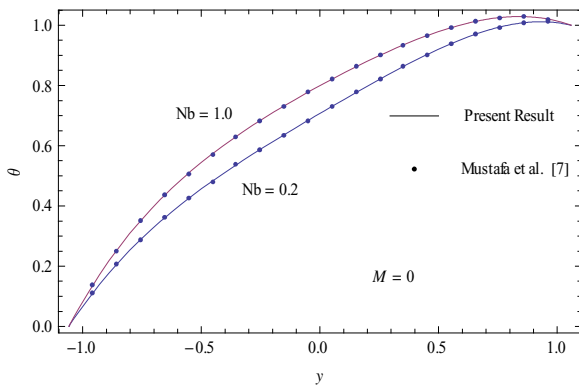


Fig.2. Comparison of temperature profiles.

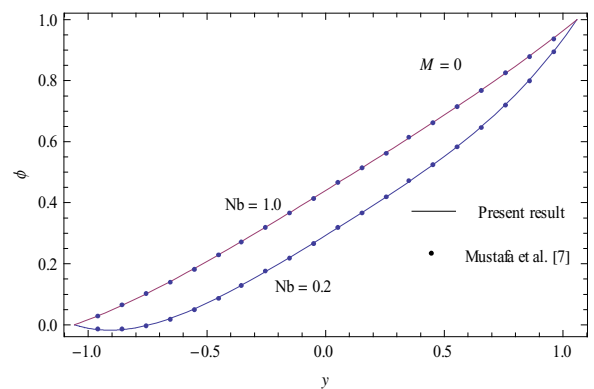


Fig.3. Comparison nanoparticles concentration.

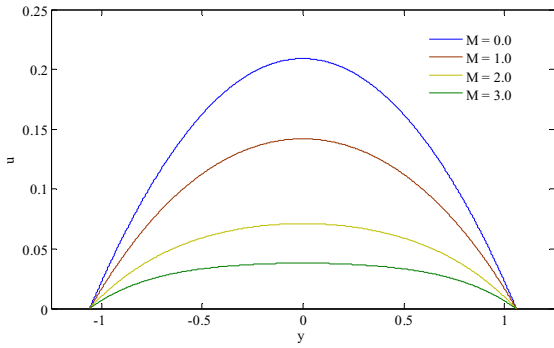


Fig.4. Effect of M on velocity.

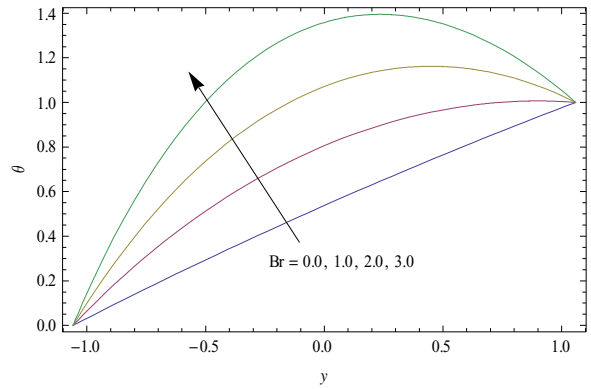


Fig.7. Effects of Br on temperature.

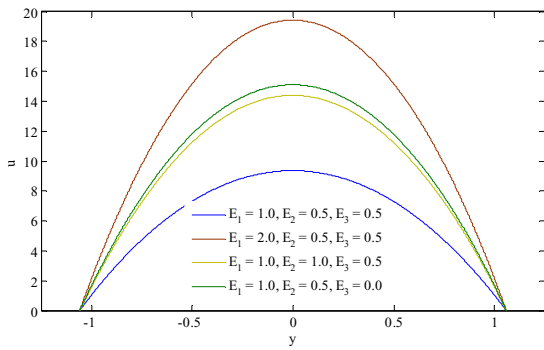


Fig.5. Effects of E_1, E_2 and E_3 on velocity.

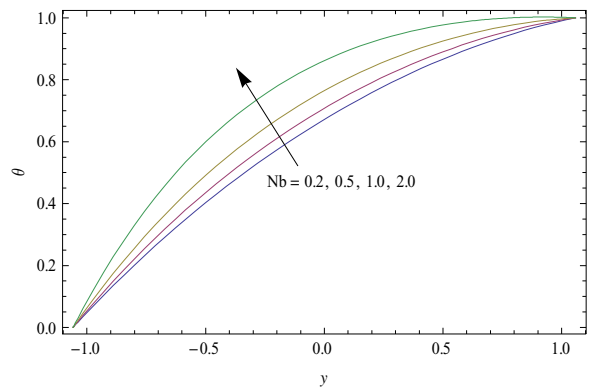


Fig.8. Effects of Nb on temperature.

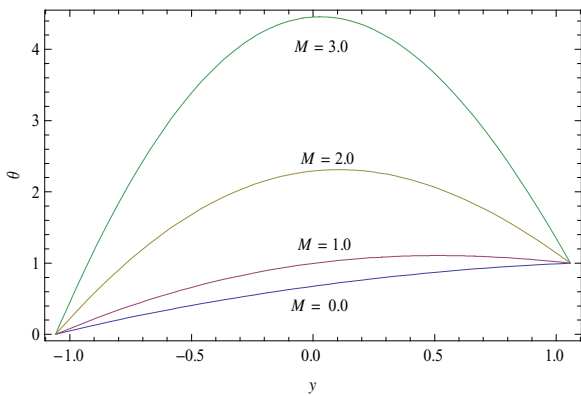


Fig.6. Effects of M on temperature.

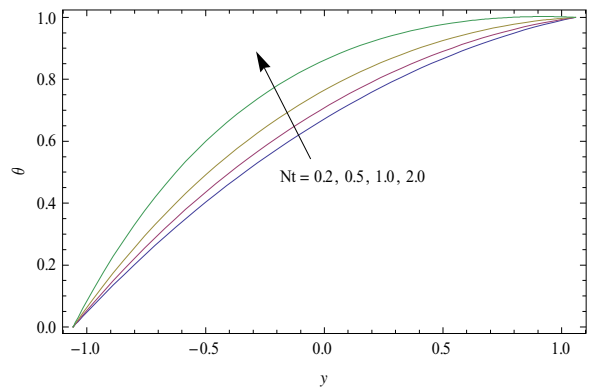


Fig.9. Effects of Nt on temperature.

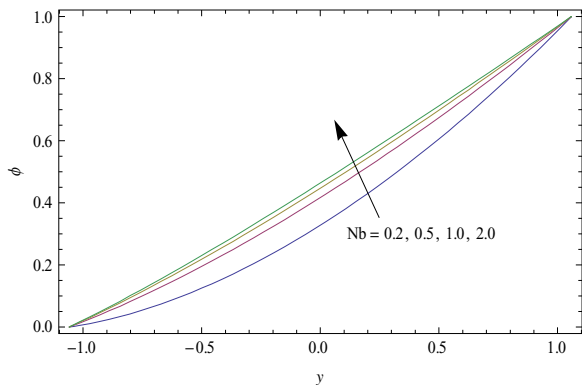


Fig.10. Effects of Nb on nanoparticles concentration.

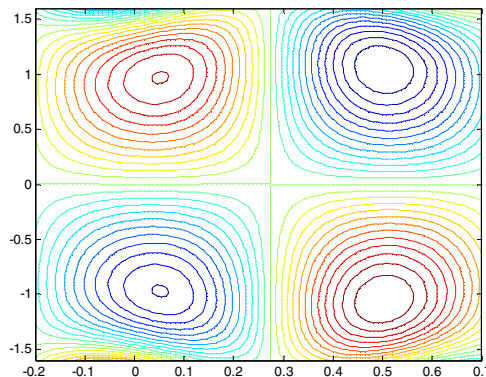


Fig.12. Stream lines for $M = 2.0$.

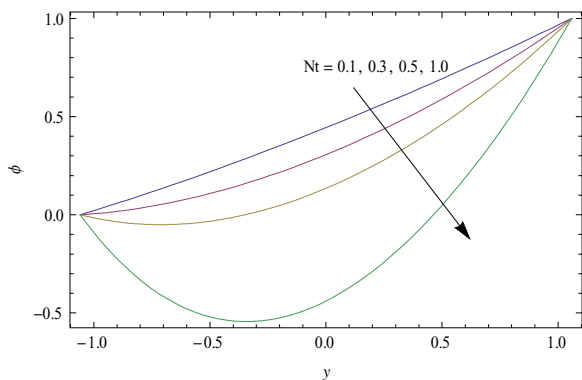


Fig.11. Effects of Nt on nanoparticles concentration.

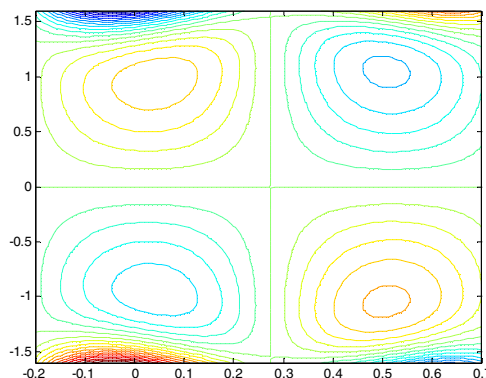


Fig.13. Stream lines for $M = 4.0$.

5. Conclusions

The main key features of the present study are as follows.

1. Axial velocity decreases with an increase in Hartmann number while it increases with increasing wall elasticity parameters
2. The temperature increases in presence of Joule heating effect. Also, an increase in temperature becomes more prominent for $M > 2$.
3. An appreciable increase in the temperature and nanoparticles concentration with the increase in the strength of Brownian motion effects.
4. The size of trapped bolus decreases with increasing Hartmann number.

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