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The golden strip of correlated top quark, gaugino, and vector-like mass in no-scale, no-parameter \mathcal{F} -SU(5)

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ABSTRACT

We systematically establish the hyper-surface within the tan β , top quark mass m_t , universal gaugino mass $M_{1/2}$, and vector-like mass M_V parameter volume which is compatible with the application of the no-scale supergravity boundary conditions, particularly the vanishing of the Higgs bilinear soft term B_{μ} , near to the Planck mass at the point $M_{\mathcal{F}}$ of ultimate \mathcal{F} -lipped SU(5) unification. $M_{\mathcal{F}}$ is elevated from the penultimate partial unification near the traditional GUT scale at a mass M_{32} by the inclusion of extra \mathcal{F} -theory derived heavy vector-like multiplets. We demonstrate that simultaneous adherence to all current experimental constraints, most importantly contributions to the muon anomalous magnetic moment $(g - 2)_{\mu}$, the branching ratio limit on $(b \rightarrow s\gamma)$, and the 7-year WMAP relic density measurement, dramatically reduces the allowed solutions to a highly non-trivial "golden strip" with tan $\beta \simeq 15$, $m_t = 173.0-174.4$ GeV, $M_{1/2} = 455-481$ GeV, and $M_V = 691-1020$ GeV, effectively eliminating all extraneously tunable model parameters. We emphasize that the consonance of the theoretically viable m_t range with the experimentally established value is an independently correlated "postdiction". The predicted range of M_V is testable at the Large Hadron Collider (LHC). The partial lifetime for proton decay in the leading $(e|\mu)^+\pi^0$ channels falls around 4.6×10^{34} Y, testable at the future DUSEL and Hyper-Kamiokande facilities.

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1. Introduction

We have recently demonstrated [1] the existence of a model dubbed no-scale \mathcal{F} -SU(5), resting essentially and in equal measure on the tripodal foundations of the flipped SU(5) × U(1)_X Grand Unified Theory (GUT) [2–4], extra \mathcal{F} -theory derived TeV scale vector-like multiplets [5–9], and the high scale boundary conditions of no-scale supergravity [10–14], which simultaneously satisfies all current experimental constraints, while eliminating all extraneously tunable free parameters. The hybridization of these three distinct conceptual progenitors was shown to uniquely define a "golden point" of phenomenological intersection, at which is *dynamically* established the universal gaugino boundary mass $M_{1/2}$, the ratio of Higgs vacuum expectation values $\tan \beta$, the dual \mathcal{F} -lipped unification scales M_{32} and $M_{\mathcal{F}}$, and also consequently the

electroweak symmetry breaking (EWSB) scale, the full contingent of supersymmetric particle masses, the proton lifetime, and all interrelated experimental observables.

This presentation, however, bore the caveat that the mass of the vector-like particles M_V was taken to be a constituent definition of the \mathcal{F} -theory context. While values around 1 TeV are considered reasonable a priori selections for natural proximity to the point of EWSB, consistent decoupling of the traditional GUT and Planck scales, and even potential testability at the LHC, it was not obvious that similar constructions with different masses – it has sometimes been suggested that M_V could be as large as 10^{15} GeV – might not also exist, thereby smearing the "golden point" into a "golden string". We have therefore undertaken a comprehensive scan of the viable range of this input.

Concurrently, we set out to establish the effect which variations within the quoted error margins of the key electroweak (EW) reference data (α_s , M_Z) and m_t would have on implementation of the no-scale \mathcal{F} -SU(5) scenario. The induced change in $|B_{\mu}(M_{\mathcal{F}})|$ with respect to shifts in α_s and M_Z were mild, on the order of 1 GeV, which we thus adopted as our definition of acceptable deviation from the strict $B_{\mu}(M_{\mathcal{F}}) = 0$ condition, this about the

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size of the EW radiative corrections. The variation with respect to m_t was found to be more severe by an order of magnitude, and this we opted instead to recognize by effectively treating m_t as an additional input, scanning also over discrete values for this parameter, and selecting the appropriate m_t to restore compliance with $|B_{\mu}(M_{\mathcal{F}})| \leq 1$ at each point in the $(\tan \beta, M_{1/2}, M_V)$ volume.

As might be expected, we found that the four degrees of freedom may conspire by intra-compensatory variation to define a large hyper-surface of acceptable solutions for the no-scale boundary condition. However, simultaneous compatibility with experimental results for the branching ratio of $(b \rightarrow s\gamma)$, the non-Standard Model (SM) contribution to $(g-2)_{\mu}$, and the WMAP cold dark matter (CDM) relic density measurement constitutes a much more stringent condition. The mutually consistent intersection is a non-trivial "golden strip" – $\tan \beta \simeq 15$, $M_{1/2} = 455-481$ GeV, and $M_{\rm V} = 691-1020$ GeV – narrowly encompassing our original golden point.

We find the emergent restriction of M_V to just a rather light range which may be probed by the LHC to be quite noteworthy. The tightly bound GUT couplings and scale imply a well resolved dimension six partial lifetime for proton decay in the leading $(e|\mu)^+\pi^0$ channels around 4.6×10^{34} Y, within reach of the future Hyper-Kamiokande [15] and Deep Underground Science and Engineering Laboratory (DUSEL) [16] experiments. A posteriori, we recognize the fortune which smiled on our early efforts, wherein by some chance we may have struck gold on a first swing of the pick, not realizing that substantial deviation from the region of $M_V \simeq 1$ TeV would destroy the model.

Our greatest surprise and delight however, is reserved for the new findings regarding m_t . It seems that under no circumstance is a satisfactory realization of the present scenario possible unless m_t , considered again here as independent input, lies within the range 173.0–174.4 GeV. The top mass being now well known [17], we firmly resist the temptation to refer to this result as a "prediction", although we consider the pressing impact of the raw correlation between theoretical and experimental numbers to be undiminished. The remarkable sonority of this "postdiction", which appears only after exhaustion to the last of all freely tunable model parameters, suggests to us that deeper currents may be in motion below the surface of no-scale \mathcal{F} -SU(5).

2. \mathcal{F} -SU(5) no-scale models

Gauge coupling unification strongly suggests the existence of a GUT. In minimal supersymmetric SU(5) models there are problems with doublet-triplet splitting and dimension five proton decay by colored Higgsino exchange. These difficulties can be elegantly overcome in flipped SU(5) GUT models via the missing partner mechanism [4]. Written in full, the gauge group of flipped SU(5) is $SU(5) \times U(1)_X$, which can be embedded into SO(10). A most notable intrinsic feature of the flipped SU(5) GUT is the presence of dual unification scales, with the ultimate merger of $SU(5) \times U(1)_X$, at a scale referred to here as $M_{\mathcal{F}}$, occurring subsequent in energy to the penultimate $SU(3)_c$ and $SU(2)_L$ mixing at M_{32} .

No-scale supergravity was proposed [10–14] to address the cosmological flatness problem. For the simple Kähler potential given in [1], we automatically obtain the no-scale boundary condition $M_0 = A = B_{\mu} = 0$ on the universal boundary scalar mass and tri/bi-linear soft terms, while $M_{1/2} > 0$ is allowed, and indeed required for supersymmetry (SUSY) breaking. This appealing reductionist perspective has however, historically been undermined by a basic inconsistency of the $M_0 = 0$ condition as applied at a GUT scale of order 10^{16} GeV with precision phenomenology.

In the more traditional flipped SU(5) formulations, the scale $M_{\mathcal{F}}$ occurs only slightly above M_{32} , larger by a factor of perhaps

only two or three [18]. Our interest however, is in scenarios where the ratio $M_{\mathcal{F}}/M_{32}$ is considerably larger, on the order of 10 to 100. Key motivations for this picture include the desire to address the monopole problem via hybrid inflation, and the opportunity for realizing true string scale gauge coupling unification in the free fermionic model building context [5,19], or the decoupling scenario in F-theory models [6,7]. We have previously also considered the favorable effect of such considerations on the decay rate of the proton [8,9].

The greatest present benefit however, is the effortless manner in which the lifting of the $SU(5) \times U(1)_X$ scale salvages the dynamically established boundary conditions of no-scale supergravity. Being highly predictive, these conditions are thus also intrinsically highly constrained, and notoriously difficult to realize generically. Our continuing study, succinctly dubbed no-scale \mathcal{F} -SU(5) [5–9], of the \mathcal{F} -lipped SU(5) GUT [19] supplemented by \mathcal{F} -theory derived vector-like multiplets at the TeV scale, provides the essential rationale; The accompanying modification to the gauge coupling renormalization group equations (RGEs) naturally separates $M_{32} \simeq 1.0 \times 10^{16}$ GeV, near the traditional GUT scale, from $M_{\mathcal{F}} \simeq 7.5 \times 10^{17}$ GeV, approaching the reduced Planck mass [5–7], at which point the no-scale boundary conditions fit like hand to glove.

In this scenario, we introduce the following two pairs of vectorlike flipped $SU(5) \times U(1)_X$ multiplets near the TeV scale [5]

$$(XF_{(10,1)} \equiv (XQ, XD^c, XN^c), XF_{(\overline{10},-1)}), (Xl_{(1,-5)}, \overline{Xl}_{(1,5)} \equiv XE^c),$$

$$(1)$$

where XQ, XD^c , XE^c , XN^c have the same quantum numbers as the quark doublet, the right-handed down-type quark, charged lepton, and neutrino, respectively. We thus suggest the name *flippons* for these hypothetical particles.

We emphasize that the specific representations of vector-like fields which we currently employ have been explicitly constructed within the F-theory model building context [6]. However, the mass of these fields, and even the fact of their existence, is not mandated by the F-theory, wherein it is also possible to realize models with only the traditional flipped (or standard) SU(5) field content. We claim only an inherent consistency of their conceptual origin out of the F-theoretic construction, and take the manifest phenomenological benefits which accompany the elevation of $M_{\mathcal{F}}$ as justification for the greater esteem which we hold for this particular model above other alternatives. In our present point of view, the uniqueness of the theory is imposed by the experimental constraints, punctuated by the relative efficiency with which no-scale \mathcal{F} -SU(5) achieves such agreement, in contrast to its competitors.

There are, however, delicate questions of compatibility between the F-theoretic model building origins of \mathcal{F} -SU(5) with vector-like fields, and the purely field-theoretic RGE running which we employ up to the high scale. As one approaches the Planck scale $M_{\rm Pl}$, consideration must be given to the role which will be played by Kaluza–Klein (KK) and string mode excitations, and if we indeed posit a substantial increase in the string scale $M_{\rm S}$, also to α' corrections associated with the corresponding reduction in the global volume of the six-dimensional internal space via the scaling $R_{\rm Global} \propto (M_{\rm Pl}/M_{\rm S})^{1/3}$.

For local F-theory models with large volume compactifications, we acknowledge that the string scale determined by direct calculation cannot be large [20]. However, to describe Nature, the local F-theory models must be embedded into a globally consistent framework. In such global constructions, the string and KK mass scales can indeed be comfortably positioned around 4×10^{17} GeV, as is likewise the case with the usual heterotic string constructions. It should be remarked in any event, that since the running of the gauge couplings is logarithmically dependent upon the mass scale, the contributions to the RGEs from the string and KK mode excitations are quite small. Moreover, there may further exist contributions to the RGE running of the gauge couplings from the heavy threshold corrections of heavy fields, as studied for example in Ref. [7], where the Type IA1 Model of Table X is associated with a unification about 2×10^{17} GeV, somewhat below the usual string scale.

The most important question is whether our model can in fact be embedded into a globally consistent framework. It seems to us that a field-theoretic application of the no-scale boundary conditions may prove to be valid in this case. This is a point which we continue to study, and on which we have not yet reached a definitive conclusion. Therefore, the above potential stringy modifications duly noted, we stipulate here by choice to consider the minimal possible scenario, wherein their substantive onset is deferred to $M_{\mathcal{F}}$, the naturally elevated secondary unification point of no-scale \mathcal{F} -SU(5), and the true GUT scale of this model. The consequent phenomenology is in our view of sufficient merit and interest to justify the investigation of the model on its own terms, postponing for now the consideration of what may ultimately be deemed to constitute effects of higher order.

3. The golden strip

In the no-scale formulation, one imposes $M_0 = A = B_\mu = 0$ at the unification scale M_F , and allows distinct inputs for the single parameter $M_{1/2}(M_F)$ to translate under RGEs to distinct low scale outputs. Equivalently, we instead allow $M_{1/2}$ and $\tan \beta$ to float freely and implement a precision self-consistency assessment [1], customized from the codebase of [21] and [22], to isolate solutions for $B_\mu(M_F) = 0$. We adhere to the following experimental constraints: 1) WMAP 7-year measurements of the CDM density [23], 0.1088 $\leq \Omega_{\chi} \leq 0.1158$. 2) Experimental limits on the FCNC process, $b \rightarrow s\gamma$, using the limits 2.86 × 10⁻⁴ $\leq Br(b \rightarrow$ $s\gamma) \leq 4.18 \times 10^{-4}$ [24,25]. 3) Anomalous magnetic moment of the muon, $g_\mu - 2$, with a lower bound of $a_\mu > 11 \times 10^{-10}$ [26]. 4) Process $B_s^0 \rightarrow \mu^+\mu^-$, using an upper bound of $Br(B_s^0 \rightarrow \mu^+\mu^-) <$ 5.8 × 10⁻⁸ [27]. 5) LEP limit on the lightest CP-even Higgs boson mass, $m_b \geq 114$ GeV [28,29].

Only a small portion of viable parameter space is consistent with the $B_{\mu}(M_{\mathcal{F}}) = 0$ condition, which thus constitutes a strong constraint. Since the boundary value of the universal gaugino mass $M_{1/2}$, and even the unification scale $M_{\mathcal{F}} \simeq 7.5 \times 10^{17}$ GeV itself, are established by the low energy experiments via RGE running, we are not left with any surviving scale parameters in the present model. The floor of the "valley gorge" in Fig. 1 represents accord with the $B_{\mu} = 0$ target for variations in $(M_{1/2}, M_V)$. We fix tan $\beta = 15$, as appears to be rather generically required in no-scale \mathcal{F} -SU(5) to realize radiative EWSB and match the observed CDM density.

We have allowed for uncertainty in the most sensitive experimental input, the top quark mass, by effectively redefining m_t as an independent free parameter. Lesser sensitivities to uncertainty in (α_s, M_Z) are included in the ± 1 GeV deviation from strict adherence to $B_{\mu} = 0$. We have established that there is a two-dimensional sheet (of some marginal thickness to recognize the mentioned uncertainty) defining $|B_{\mu}(M_{\mathcal{F}})| \leq 1$ for each point in the three-dimensional $(M_{1/2}, M_V, m_t)$ volume, as shown in Fig. 2. This sheet is inclined in the region of interest at the very shallow angle of 0.2° to the $(M_{1/2}, M_V)$ plane, such that m_t is largely decoupled from variation in the plane.

A particularly interesting facet of this model is the surprisingly strong correlation between the vector-like mass M_V and the



Fig. 1. The $B_{\mu} = 0$ target for variations in $(M_{1/2}, M_V)$, with $\tan \beta = 15$. The specific m_t which is required to minimize $|B_{\mu}(M_{\mathcal{F}})|$ is annotated along the solution string.

WMAP dark matter relic density. The underlying mechanism may be traced to the more directly visible effect which M_V has on the secondary unification scale M_F . A larger (or smaller) value of M_V will reduce (or increase) M_F , thus suppressing (or enhancing) the role of the RGEs across the contracted (or extended) energy gap. A key consequence will be a heavier (or lighter) low energy Bino mass. In our model, which features 99.8% Bino dark matter, the dark matter density, being proportional to the Bino mass squared, will thus also sharply rise (or fall). For a fixed $M_{1/2}$, we may thus place upper and lower bounds on M_V via the corresponding upper and lower limits of the WMAP dark matter density.

The $(g-2)_{\mu}$ and $b \rightarrow s\gamma$ constraints vary most strongly with $M_{1/2}$. The two considered effects are each at their lower limits at the boundary, but they exert pressure in opposing directions on $M_{1/2}$ due to the fact that the leading gaugino and squark contributions to $Br(b \rightarrow s\gamma)$ enter with an opposing sign to the SM term and Higgs contribution. For the non-SM contribution to Δa_{μ} , the effect is additive, and establishes an upper mass limit on $M_{1/2}$. Incidentally, the same experiment forms the central rationale for the adoption of $sign(\mu) > 0$, such that appropriate interference terms between SM and SUSY contributions are realized. Conversely, the requirement that SUSY contributions to $Br(b \rightarrow s\gamma)$ not be overly large, undoing the SM effect, requires a sufficiently large, i.e. lower bounded, $M_{1/2}$. The WMAP-7 CDM measurement, by contrast, exhibits a fairly strong correlation with both $M_{1/2}$ and M_V (as elaborated prior), cross-cutting the $M_{1/2}$ bound, and confining the vector-like mass to 691-1020 GeV. We note that the mixing of the SM fermions and vector particles may give additional contributions to $Br(b \rightarrow s\gamma)$ and Δa_{μ} , but we do not consider them here.

The intersection of these three key constraints with the $|B_{\mu}(M_{\mathcal{F}})| \leq 1$ surface, as depicted in Fig. 2, defines the "golden strip" of no-scale \mathcal{F} -*SU*(5). All of the prior is accomplished with no reference to the experimental top quark mass, redefined here as a *free* input. However, the extremely shallow angle of inclination (0.2°) of the $|B_{\mu}(M_{\mathcal{F}})| \leq 1$ sheet above the $(M_{1/2}, M_V)$ plane and into the m_t axis causes the golden strip to *imply* an exceedingly narrow range of compatibility for m_t , between 173.0 and 174.4 GeV, in perfect alignment with the physically observed value of $m_t = 173.1 \pm 1.3$ GeV [17].

Within the golden strip, we select the benchmark point of Table 1. The golden strip is further consistent with the CDMSII [30] and Xenon100 [31] upper limits, with the spin-independent cross section extending from $\sigma_{SI} = 1.3-1.9 \times 10^{-10}$ pb. Likewise,



Fig. 2. With tan $\beta \simeq 15$ fixed by WMAP-7, the residual parameter volume is three-dimensional in (M_{12}, M_V, m_t) , with the $|B_\mu(M_F)| \le 1$ (slightly thickened) surface forming a shallow (0.2°) incline above the (M_{12}, M_V) plane. The overlayed blue contours on the flattened diagram mark the $p \to (e|\mu|^+\pi^0$ proton lifetime prediction, in units of 10^{34} years. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this Letter.)

Table 1

Spectrum (in GeV) for the benchmark point. Here, $M_{1/2} = 464$ GeV, $M_V = 850$ GeV, $m_t = 173.6$ GeV, $\Omega_{\chi} = 0.112$, $\sigma_{SI} = 1.7 \times 10^{-10}$ pb, and $\langle \sigma v \rangle_{\gamma \gamma} = 1.7 \times 10^{-28}$ cm³/s. The central prediction for the $p \rightarrow (e|\mu)^+ \pi^0$ proton lifetime is 4.6×10^{34} years. The lightest neutralino is 99.8% Bino.

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$\widetilde{\chi}_1^0$	96	$\widetilde{\chi}_1^{\pm}$ 187	\widetilde{e}_R 153	\widetilde{t}_1 499	\widetilde{u}_R	975	m _h	120.6
$\widetilde{\chi}_2^0$	187	$\widetilde{\chi}_2^{\pm}$ 849	\widetilde{e}_L 519	\widetilde{t}_2 929	\widetilde{u}_L	1062	$m_{A,H}$	946
$\widetilde{\chi}_3^0$	845	$\widetilde{\nu}_{e/\mu}$ 513	$\widetilde{ au}_1$ 105	\widetilde{b}_1 880	\widetilde{d}_R	1018	$m_{H^{\pm}}$	948
$\widetilde{\chi}_4^0$	848	$\widetilde{ u}_{ au}$ 506	$\widetilde{\tau}_2$ 514	<i>b</i> ₂ 992	\widetilde{d}_L	1065	පි	629

the allowed region satisfies the Fermi–LAT space telescope constraints [32], with the photon–photon annihilation cross section $\langle \sigma v \rangle_{\gamma\gamma}$ ranging from $\langle \sigma v \rangle_{\gamma\gamma} = 1.5-1.7 \times 10^{-28} \text{ cm}^3/\text{s}.$

4. Experimental signature

We remark in closing on a distinctive evidentiary "smoking gun" for the no-scale \mathcal{F} -SU(5) scenario. This is direct detection near the TeV scale of the components of the extra vector-like flippon multiplets of Eq. (1). In particular, these vector particles mirror the quark and lepton quantum numbers, and crucially also the "flipped" charge assignment. Since the flippons consist of a pair of ten-plets (XF, \overline{XF}), and a pair of *charged* SU(5) singlets (XI, \overline{XI}), but no five-plets, the grouping is unambiguous. The discovery of flippons would firmly establish the flipped group structure.

5. Discussion and conclusion

We have continued and extended our prior study [1] of no-scale supergravity in the context of a \mathcal{F} -lipped $SU(5) \times U(1)_X$ GUT supplemented with \mathcal{F} -theory derived TeV-scale vector-like particles. There did not have to be an experimentally viable $B_{\mu}(M_{\mathcal{F}}) = 0$ solution at all, and indeed successful implementation of this boundary has eluded a myriad of prior attempts. Because the universal gaugino mass, and even the final unification scale $M_{\mathcal{F}}$ itself are determined by the low energy known experiments via selfconsistent RGE running, there are no surviving arbitrary mass scales or extraneously tunable inputs. We stress again that the union of top-down model based constraints with bottom-up experimental data exhausts the available freedom of parameterization in a uniquely consistent and predictive manner, prior to invocation of the m_t value. Retaining no residual malleability, the model is forced to live or die by the success of its extraordinarily finely attenuated postdiction of the top quark mass - a trial which it surmounts with colors flying, phenomenologically defining a "golden strip" of correlated top quark, gaugino, and vectorlike mass, with $m_t = 173.0-174.4$ GeV, $M_{1/2} = 455-481$ GeV, and $M_{\rm V} = 691-1020$ GeV. A narrowly defined yet broadly applicable prediction has been made for $\tan \beta \simeq 15$. The required TeV scale vector multiplets and dimension six $(e|\mu)^+\pi^0$ proton decay, both bearing the distinctive signature of their flipped origin, are each poised to play a potentially prominent role in certain of the most exciting particle physics experiments of the coming decade. This luxury of portent and paucity of accommodation is the power of no-scale \mathcal{F} -SU(5).

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