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ABSTRACT

We test the validity of the QCD sum rules applied to the meson $Z^+(4430)$, by considering a diquark–antidiquark type of current with $J^P = 0^-$ and with $J^P = 1^-$. We find that, with the studied currents, it is possible to find an acceptable Borel window. In such a Borel window we have simultaneously a good OPE convergence and a pole contribution which is bigger than the continuum contribution. We get $m_Z = (4.52 \pm 0.09)$ GeV and $m_Z = (4.84 \pm 0.14)$ GeV for the currents with $J^P = 0^-$ and $J^P = 1^-$, respectively. We conclude that the QCD sum rules results favors $J^P = 0^-$ quantum numbers for the $Z^+(4430)$ meson.

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During the past years, a series of exotic charmonium-like mesons, called X , Y and Z , have been discovered in B mesons decays. Among them, the charged resonance state $Z^+(4430)$, observed by Belle Collaboration [1] in the $Z^+ \rightarrow \psi' \pi^+$ decay mode, is the most intriguing one since it cannot be described as ordinary $c\bar{c}$ meson.

The nature of the $Z^+(4430)$ meson is completely open and there are already many theoretical interpretations about its structure [2–19]. However, an intriguing possibility is the interpretation as tetraquark or molecular state. In Ref. [5], the closeness of the $Z^+(4430)$ mass to the threshold of $D^{*+}(2010)\bar{D}_1(2420)$ lead the authors to consider the $Z^+(4430)$ as a $D^*\bar{D}_1$ molecule. This hypothesis was tested in Ref. [9] by using the QCD sum rules approach, with a good agreement with the experimental data. The interpretation of $Z^+(4430)$ as tetraquark state was done in Refs. [3,4,7].

Since $Z^+(4430)$ was observed in the $\psi' \pi^+$ channel, it is an isovector state with positive G -parity: $I^G = 1^+$. However, nothing is known about its spin and parity quantum numbers. For a $D^*\bar{D}_1$ molecular state in s -wave, the allowed J^P are $0^-, 1^-$ or 2^- , although the 2^- assignment is probably suppressed in the $B \rightarrow Z(4430)K$ decay, by the small phase space. In this work we use QCD sum rules (QCDSR) [20–22], to study the two-point function of the state $Z^+(4430)$ considered as a tetraquark state with $J^P = 0^-$ and $J^P = 1^-$.

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In previous calculations, the QCDSR approach was used to study the $X(3872)$ by using a diquark–antidiquark current [23], the $Z^+(4430)$ meson, by using a D^*D_1 molecular current [9] and the Y mesons [24] by using molecular and diquark–antidiquark type of currents. In all cases a very good agreement with the experimental mass was obtained.

Let us consider first the $Z^+(4430)$ by using a diquark–antidiquark current with $J^P = 0^-$ and positive G parity. One can invoke simple arguments using constituent quark model to show why the tetraquark with the suggested quantum number could be stable. In the constituent quark model, a multi-quark exotic is expected to have some scalar diquark component in the color anti-triplet configuration, as this is the most attractive quark–quark channel. However, when a tetraquark has $J^P = 0^+$ quantum number, it would energetically be more favorable to decay in s -wave into two pseudo-scalar mesons. In terms of the spin–spin interaction, one can say that the attraction in the quark–antiquark configuration in the two pseudo-scalar mesons is phenomenologically more than a factor 3 larger than that in the two scalar quark–quark channel in the tetraquark [25]. However, when the tetraquark configuration has $J^P = 0^-$ quantum number, at least one of the diquark could be in the attractive channel, while the remaining diquark is in the pseudo-scalar channel. On the other hand, it cannot decay into final states containing a pseudo-scalar meson in s -wave; hence the tetraquark could be quasi-stable. To test such configuration in a non-perturbative way, we are implementing the QCD sum rule method.

A possible current describing such state is given by

$$j = \frac{i\epsilon_{abc}\epsilon_{dec}}{\sqrt{2}} [(u_a^T C \gamma_5 c_b)(\bar{d}_d C \bar{c}_e^T) - (u_a^T C c_b)(\bar{d}_d \gamma_5 C \bar{c}_e^T)], \quad (1)$$

where the index T means matrix transposition, a, b, \dots are color indices and C is the charge conjugation matrix.

The QCD sum rules for the meson mass are constructed from the two-point correlation function:

$$\Pi(q) = i \int d^4x e^{iq \cdot x} \langle 0 | T [j(x) j^\dagger(0)] | 0 \rangle. \quad (2)$$

Phenomenologically, the correlator can be expressed as a dispersion integral

$$\Pi^{\text{phen}}(q^2) = \int ds \frac{\rho^{\text{phen}}(s)}{s - q^2} + \dots, \quad (3)$$

where $\rho^{\text{phen}}(s)$ is the spectral density and the dots represent subtraction terms. The spectral density is described, as usual, as a single sharp pole representing the lowest resonance plus a smooth continuum representing higher mass states:

$$\rho^{\text{phen}}(s) = \lambda^2 \delta(s - m_Z^2) + \rho^{\text{cont}}(s), \quad (4)$$

where λ is proportional to the meson decay constant, f_Z , which parametrizes the coupling of the current to the meson Z^+ :

$$\langle 0 | j | Z^+ \rangle = f_Z m_Z^4 = \lambda. \quad (5)$$

It is important to notice that there is no one to one correspondence between the current and the state, since the current in Eq. (1) can be rewritten in terms of sum a over molecular type currents, by the use of the Fierz transformation. However, the parameter λ , appearing in Eq. (5), gives a measure of the strength of the coupling between the current and the state.

We follow the prescription that the continuum contribution to the spectral density, $\rho^{\text{cont}}(s)$ in Eq. (4), vanishes below a certain continuum threshold s_0 . Above this threshold, it is given by the result obtained with the OPE [26]:

$$\rho^{\text{cont}}(s) = \rho^{\text{OPE}}(s) \Theta(s - s_0). \quad (6)$$

On the OPE side, we work at leading order in α_s and consider the contributions of condensates up to dimension eight. To keep the charm quark mass finite, we use the momentum-space expression for the charm quark propagator. The light quark part of the correlation function is calculated in the coordinate-space. Then, the resulting light-quark part is Fourier transformed to the momentum space in D dimensions and it is dimensionally regularized at $D = 4$. The correlation function in the OPE side can be written as

$$\Pi^{\text{OPE}}(q^2) = \int_{4m_c^2}^{\infty} ds \frac{\rho^{\text{OPE}}(s)}{s - q^2} + \Pi^{\text{mix}(\bar{q}q)}(q^2), \quad (7)$$

where $\rho^{\text{OPE}}(s)$ is given by the imaginary part of the correlation function: $\pi \rho^{\text{OPE}}(s) = \text{Im}[\Pi^{\text{OPE}}(s)]$.

After equating the two representations of the correlation function, assuming quark-hadron duality, making a Borel transform to both sides, and transferring the continuum contribution to the OPE side, the sum rule for the pseudo-scalar meson Z^+ , up to dimension-eight condensates, is given by

$$\lambda^2 e^{-m_Z^2/M^2} = \int_{4m_c^2}^{s_0} ds e^{-s/M^2} \rho^{\text{OPE}}(s) + \Pi^{\text{mix}(\bar{q}q)}(M^2), \quad (8)$$

where

$$\rho^{\text{OPE}}(s) = \rho^{\text{pert}}(s) + \rho^{(\bar{q}q)}(s) + \rho^{(G^2)}(s) + \rho^{\text{mix}}(s) + \rho^{(\bar{q}q)^2}(s), \quad (9)$$

with

$$\begin{aligned} \rho^{\text{pert}}(s) &= \frac{1}{2^9 \pi^6} \int_{\alpha_{\min}}^{\alpha_{\max}} \frac{d\alpha}{\alpha^3} \int_{\beta_{\min}}^{1-\alpha} \frac{d\beta}{\beta^3} (1 - \alpha - \beta) [(\alpha + \beta)m_c^2 - \alpha\beta s]^4, \\ \rho^{(\bar{q}q)}(s) &= 0, \\ \rho^{(G^2)}(s) &= \frac{\langle g^2 G^2 \rangle}{2^8 \pi^6} \int_{\alpha_{\min}}^{\alpha_{\max}} \frac{d\alpha}{\alpha^2} \int_{\beta_{\min}}^{1-\alpha} d\beta [(\alpha + \beta)m_c^2 - \alpha\beta s] \\ &\quad \times \left(m_c^2 \frac{1 - \alpha - \beta}{3\alpha} + \frac{(\alpha + \beta)m_c^2 - \alpha\beta s}{4\beta} \right), \\ \rho^{\text{mix}}(s) &= 0, \\ \rho^{(\bar{q}q)^2}(s) &= -\frac{m_c^2 \langle \bar{q}q \rangle^2}{12\pi^2} \sqrt{1 - 4m_c^2/s}, \\ \Pi^{\text{mix}(\bar{q}q)}(M^2) &= \frac{m_c^2 \langle \bar{q}g\sigma \cdot Gq \rangle \langle \bar{q}q \rangle}{24\pi^2} \int_0^1 d\alpha \frac{e^{-\frac{m_c^2}{\alpha(1-\alpha)M^2}}}{1 - \alpha} \left[\frac{m_c^2}{\alpha M^2} - \alpha \right]. \end{aligned} \quad (10)$$

The integration limits are given by $\alpha_{\min} = (1 - \sqrt{1 - 4m_c^2/s})/2$, $\alpha_{\max} = (1 + \sqrt{1 - 4m_c^2/s})/2$ and $\beta_{\min} = \alpha m_c^2 / (s\alpha - m_c^2)$.

One should note that a evaluation of the higher-dimension condensate contributions is technically difficult and non-trivial, which cannot be obtained by a simple routine iteration of the quark propagator in an external field. Violation of the factorization hypothesis become increasingly important in higher dimensions and so the results become increasingly model dependent, as more condensates will have to be introduced if factorization is not valid [27].

Similarly to the results in Ref. [9], the current in Eq. (1) does not get contribution from the quark and mixed condensates. This is very different from the OPE behavior obtained for the diquark-antidiquark current used for the $X(3872)$ and $Y(4660)$ mesons in Refs. [23,24], but very similar to the OPE behavior obtained for the axial double-charmed meson T_{cc} , also described by a diquark-antidiquark current [28].

In the numerical analysis, the input values are taken as [22, 29]: $m_c(m_c) = (1.23 \pm 0.05) \text{ GeV}$, $\langle \bar{q}q \rangle = -(0.23 \pm 0.03)^3 \text{ GeV}^3$, $\langle \bar{q}g\sigma \cdot Gq \rangle = m_0^2 \langle \bar{q}q \rangle$ with $m_0^2 = 0.8 \text{ GeV}^2$, $\langle g^2 G^2 \rangle = 0.88 \text{ GeV}^4$.

We evaluate the sum rule in the Borel range $2.2 \leq M^2 \leq 3.5 \text{ GeV}^2$. To determine the allowed Borel window, we analyse the OPE convergence and the pole contribution: the minimum value of the Borel mass is fixed by considering the convergence of the OPE, and the maximum value of the Borel mass is determined by imposing that the pole contribution must be bigger than the continuum contribution. To fix the continuum threshold range we extract the mass from the sum rule, for a given s_0 , and accept such value of s_0 if the obtained mass is around 0.5 GeV smaller than $\sqrt{s_0}$. However, in this case, to be able to compare our results with the results obtained by using a molecular type current in Ref. [9], we use the same continuum range as in Ref. [9]: $4.8 \leq \sqrt{s_0} \leq 5.0 \text{ GeV}$.

From Fig. 1 we see that we obtain a quite good OPE convergence for $M^2 \geq 2.3 \text{ GeV}^2$. Therefore, we fix the lower value of M^2 in the Borel window as $M_{\min}^2 = 2.3 \text{ GeV}^2$. This figure also shows that the dimension-eight condensate contribution is very small as compared with the four-quark condensate contribution.

The comparison between pole and continuum contributions for $\sqrt{s_0} = 4.9 \text{ GeV}$ is shown in Fig. 2, from where we see that the pole contribution is bigger than the continuum for $M^2 \leq 3.1 \text{ GeV}^2$. The same analysis for the other values of the continuum threshold gives $M^2 \leq 2.9 \text{ GeV}^2$ for $\sqrt{s_0} = 4.8 \text{ GeV}$ and $M^2 \leq 3.3 \text{ GeV}^2$ for $\sqrt{s_0} = 5.0 \text{ GeV}$.

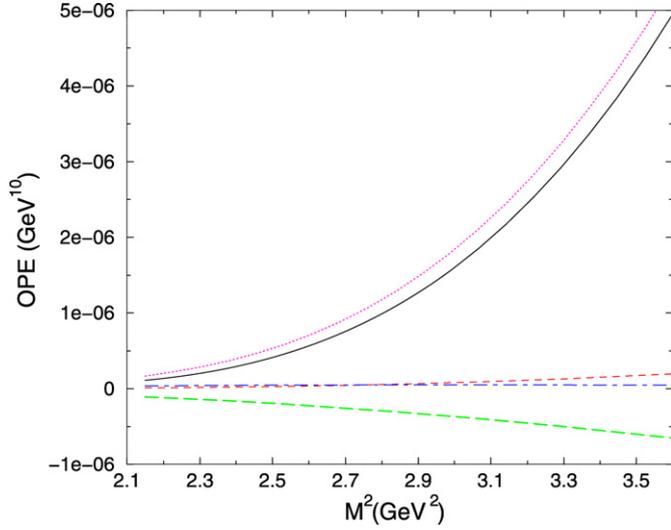


Fig. 1. The OPE convergence in the region $2.2 \leq M^2 \leq 3.5 \text{ GeV}^2$ for $\sqrt{s_0} = 4.9 \text{ GeV}$. Perturbative contribution (dotted line), $\langle g^2 G^2 \rangle$ contribution (dashed line), $\langle \bar{q}q \rangle^2$ contribution (long-dashed line), $\langle \bar{q}g\sigma.Gq \rangle \langle \bar{q}q \rangle$ (dot-dashed line) and the total contribution (solid line).

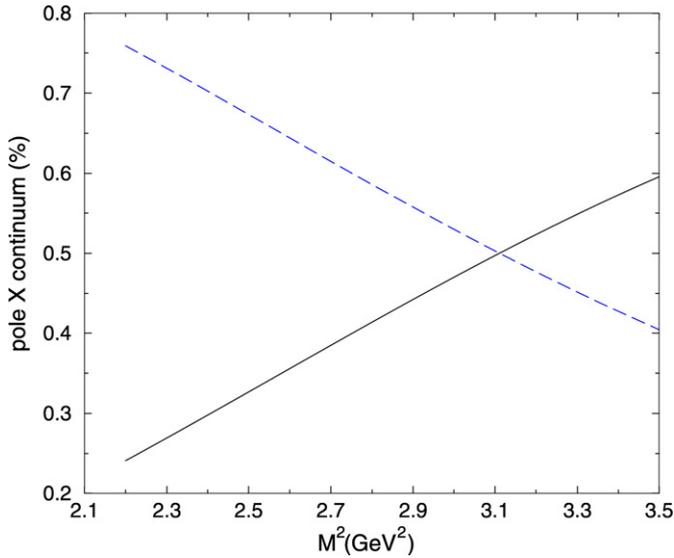


Fig. 2. The dashed line shows the relative pole contribution (the pole contribution divided by the total, pole plus continuum, contribution) and the solid line shows the relative continuum contribution for $\sqrt{s_0} = 4.9 \text{ GeV}$.

To extract the mass m_Z we take the derivative of Eq. (8) with respect to $1/M^2$, and divide the result by Eq. (8). In Fig. 3, we show the Z^+ meson mass, for different values of $\sqrt{s_0}$, in the relevant sum rule window, with the upper and lower validity limits indicated. From this figure we see that the results are very stable as a function of M^2 .

To check the dependence of our results with the value of the charm quark mass, we fix $\sqrt{s_0} = 4.9 \text{ GeV}$ and vary the charm quark mass in the range $m_c = (1.23 \pm 0.05) \text{ GeV}$. Using $2.5 \leq M^2 \leq 3.1 \text{ GeV}^2$ we get: $m_Z = (4.51 \pm 0.06) \text{ GeV}$. Including the uncertainty due to the value of the continuum threshold and the value of the Borel parameter we arrive at

$$m_{Z(0^-)} = (4.52 \pm 0.09) \text{ GeV}, \quad (12)$$

which is a little bigger than the experimental value [1], but still consistent with it, considering the uncertainties. Comparing our result with the result obtained in Ref. [9]: $m_{D^*D_1} = (4.40 \pm 0.10) \text{ GeV}$, where the $Z^+(4430)$ was considered by using a D^*D_1

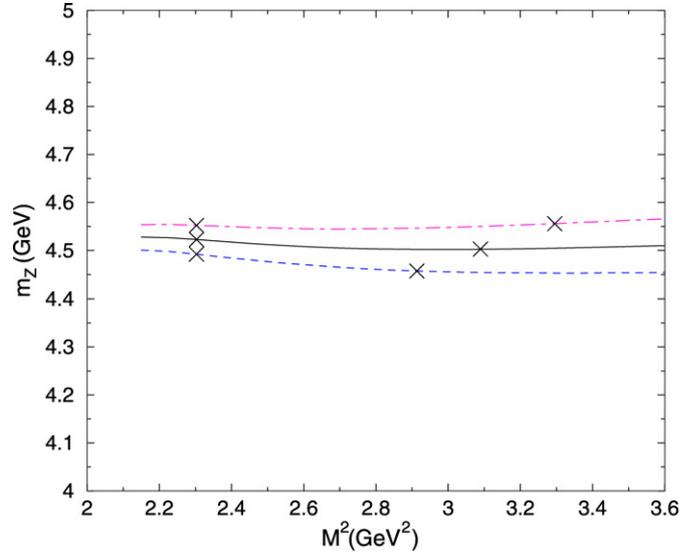


Fig. 3. The Z^+ with $J^P = 0^-$ meson mass as a function of the sum rule parameter (M^2) for different values of $\sqrt{s_0}$: $\sqrt{s_0} = 4.8 \text{ GeV}$ dashed line, $\sqrt{s_0} = 4.9 \text{ GeV}$ solid line and $\sqrt{s_0} = 5.0 \text{ GeV}$ dot-dashed line. The crosses indicate the region allowed for the sum rules.

molecular current with $J^P = 0^-$, we see that the result in Ref. [9] is in a better agreement with the experimental value. However, as mentioned above, since there is no one to one correspondence between the structure of the current and the state, we cannot use this result to conclude that the $Z^+(4430)$ is better explained as a molecular state than as a diquark–antidiquark state. To get a measure of the coupling between the state and the current, we use Eq. (8) to evaluate the parameter λ , defined in Eq. (5). We get:

$$\lambda_{Z(0^-)} = (3.75 \pm 0.48) \times 10^{-2} \text{ GeV}^5, \quad (13)$$

while for the current used in Ref. [9] we get:

$$\lambda_{D^*D_1} = (5.66 \pm 1.26) \times 10^{-2} \text{ GeV}^5. \quad (14)$$

Therefore, it is possible to conclude that the physical particle with $J^P = 0^-$ and quark content $c\bar{c}u\bar{d}$ couples with a larger strength with the molecular D^*D_1 type current than with the current in Eq. (1).

We now consider the $Z^+(4430)$ by using a diquark–antidiquark current with $J^P = 1^-$ and positive G parity. The lowest-dimension interpolating operator describing such current is given by

$$j_\mu = \frac{\epsilon_{abc}\epsilon_{dec}}{\sqrt{2}} [(u_a^T C \gamma_5 c_b)(\bar{d}_d \gamma_\mu \gamma_5 C \bar{c}_e^T) + (u_a^T C \gamma_5 \gamma_\mu c_b)(\bar{d}_d \gamma_5 C \bar{c}_e^T)]. \quad (15)$$

The two-point correlation function is now given by

$$\begin{aligned} \Pi_{\mu\nu}(q) &= i \int d^4x e^{iq \cdot x} \langle 0 | T [j_\mu(x) j_\nu^\dagger(0)] | 0 \rangle \\ &= -\Pi(q^2)(g_{\mu\nu}q^2 - q_\mu q_\nu), \end{aligned} \quad (16)$$

from where we get

$$\Pi_\mu^\mu(q) = -3q^2 \Pi(q^2), \quad (17)$$

and, therefore, we can write a sum rule for $\Pi(q^2)$ as before. The spectral density is now given by

$$\begin{aligned} \rho^{\text{pert}}(s) &= -\frac{1}{2^8 3 \pi^6 s} \int_{\alpha_{\min}}^{\alpha_{\max}} \frac{d\alpha}{\alpha^3} \int_{\beta_{\min}}^{1-\alpha} \frac{d\beta}{\beta^3} (1 - \alpha - \beta) \\ &\quad \times [(\alpha + \beta)m_c^2 - \alpha\beta s]^3 [m_c^2 - 2m_c^2(\alpha + \beta) + \alpha\beta s], \end{aligned}$$

$$\begin{aligned}
\rho^{(\bar{q}q)}(s) &= 0, \\
\rho^{(G^2)}(s) &= \frac{m_c^2 \langle g^2 G^2 \rangle}{3^2 2^9 \pi^6 s} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{1-\alpha} \frac{d\beta}{\beta^3} (1-\alpha-\beta) \\
&\quad \times \left[4(2\alpha+2\beta-1)m_c^2 - \frac{3m_c^2\beta}{\alpha} - \beta s(7\alpha-3) \right], \\
\rho^{\text{mix}}(s) &= \frac{m_c \langle \bar{q}g\sigma \cdot Gq \rangle}{2^6 3\pi^4 s} \int_{\alpha_{\min}}^{\alpha_{\max}} \frac{d\alpha}{\alpha} \int_{\beta_{\min}}^{1-\alpha} \frac{d\beta}{\beta^2} (2\alpha+\beta) [(\alpha+\beta)m_c^2 - \alpha\beta s], \\
\rho^{(\bar{q}q)^2}(s) &= -\frac{\langle \bar{q}q \rangle^2}{36\pi^2} \left(\frac{5m_c^2}{s} - \frac{1}{2} \right) \sqrt{1-4m_c^2/s}, \\
\rho^{\text{mix}(\bar{q}q)^2}(s) &= -\frac{\langle \bar{q}q \rangle \langle \bar{q}g\sigma \cdot Gq \rangle}{3^2 2^4 \pi^2 s} (1+4m_c^2/s) \sqrt{1-4m_c^2/s}, \\
\Pi^{\text{mix}(\bar{q}q)}(M^2) &= -\frac{\langle \bar{q}q \rangle \langle \bar{q}g\sigma \cdot Gq \rangle}{3^2 2^4 \pi^2} \\
&\quad \times \left(\frac{2}{3} - 3 \int_0^1 d\alpha \exp\left[-\frac{m_c^2}{\alpha(1-\alpha)M^2}\right] \left[\alpha - 2\alpha^2 + \frac{2m_c^2}{M^2} \right] \right). \quad (19)
\end{aligned}$$

Although with this current we still do not get contribution from the quark condensate, we do get contribution from the mixed condensate. As can be seen by Fig. 4, the mixed condensate contribution is of the same order as the four-quark condensate contribution, but with opposite signal. The contribution of the dimension-eight condensate is now of the same order as the four-quark condensate contribution, for small values of M^2 .

In this case we find that the continuum threshold is in the range $\sqrt{s_0} = (5.3 \pm 0.1)$ GeV and, from Fig. 4, we see that there is a good OPE convergence for $M^2 \geq 3.9$ GeV².

The upper limits for M^2 for each value of $\sqrt{s_0}$ are given in Table 1, from where we see that the Borel window in this case has higher values of the Borel parameter, as compared with the case for Z^+ with $J^P = 0^-$.

In the case of Z^+ with $J^P = 1^-$ we get a worse Borel stability than for the Z^+ with $J^P = 0^-$, in the allowed sum rule window, as a function of M^2 , as can be seen by Fig. 5. We also observe that the results are, in this case, more sensitive to the values of m_c .

Using the Borel window, for each value of s_0 , to evaluate the mass, and then varying the value of the continuum threshold in the range $5.2 \leq \sqrt{s_0} \leq 5.4$ GeV, we get $m_{Z(1^-)} = (4.80 \pm 0.08)$ GeV.

Because of the complex spectrum of the exotic states, some times lower continuum threshold values are favorable in order to completely eliminate the continuum above the resonance state. Therefore, in Fig. 5 we also include the result for $\sqrt{s_0} = 5.1$ GeV. We see that we get a very narrow Borel window, and for values of the continuum threshold smaller than 5.1 GeV there is no allowed Borel window. Taking into account the variations on M^2 , s_0 and m_c in the regions indicated above we get:

$$m_{Z(1^-)} = (4.84 \pm 0.14) \text{ GeV}, \quad (20)$$

which is much bigger than the experimental value and bigger than the result obtained using the current with $J^P = 0^-$ in Eq. (12).

For the value of the parameter λ defined in Eq. (5) we get:

$$\lambda_{Z(1^-)} = (8.36 \pm 0.85) \times 10^{-5} \text{ GeV}^5. \quad (21)$$

In conclusion, we have presented a QCDSR analysis of the two-point function of the recently observed $Z^+(4430)$ meson, considered as a tetraquark state, with a diquark-antidiquark configuration. Since the spin-parity quantum numbers of the $Z^+(4430)$ meson are not known, we have considered two different possibilities:

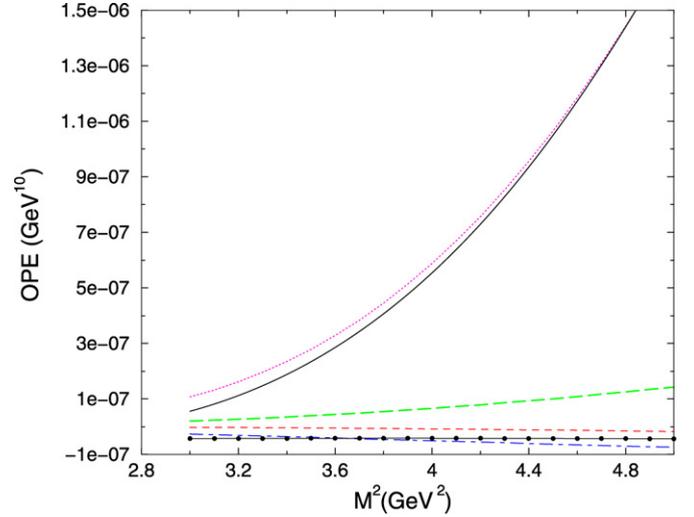


Fig. 4. The OPE convergence for the sum rule for Z^+ with $J^P = 1^-$, using $\sqrt{s_0} = 5.3$ GeV. The dotted, dashed, long-dashed, dot-dashed, solid with dots and solid lines give, respectively, the perturbative, gluon condensate, mixed condensate, four-quark condensate, dimension-eight condensate and total contributions.

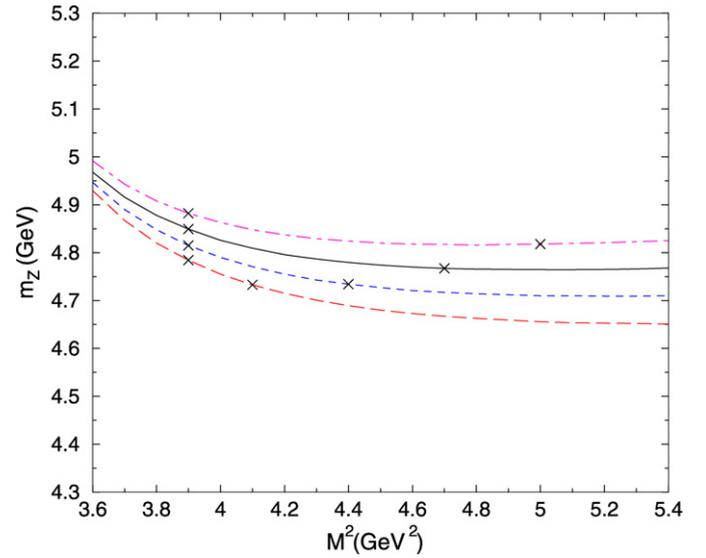


Fig. 5. The Z^+ with $J^P = 1^-$ meson mass as a function of the sum rule parameter for different values of $\sqrt{s_0}$: $\sqrt{s_0} = 5.1$ GeV long-dashed line, $\sqrt{s_0} = 5.2$ GeV dashed line, $\sqrt{s_0} = 5.3$ GeV solid line and $\sqrt{s_0} = 5.4$ GeV dot-dashed line. The crosses indicate the region allowed for the sum rules.

Table 1
Upper limits in the Borel window for Z^+ with $J^P = 1^-$.

| $\sqrt{s_0}$ (GeV) | M_{\max}^2 (GeV ²) |
|--------------------|----------------------------------|
| 5.2 | 4.4 |
| 5.3 | 4.7 |
| 5.4 | 5.0 |

$J^P = 0^-$ and $J^P = 1^-$. We have found a very good OPE convergence for these two cases, although this is not in general the case for tetraquark states [30]. We got a Z^+ mass in some agreement with the experimental result in the case with $J^P = 0^-$. However, in the case $J^P = 1^-$, we got a much higher value for the mass. This is consistent with the expectation from the constituent quark model, since in this model the scalar diquark component in the color anti-triplet configuration is the most attractive quark-quark channel.

Comparing our result, for the case $J^P = 0^-$, with the case where the $Z^+(4430)$ meson was considered by using a D^*D_1 molecular current, also with $J^P = 0^-$ [9], the differences are also not really big. Since there is no one-to-one correspondence between the structure of the current and the state, we cannot conclude that the $Z^+(4430)$ is better explained as a molecular state than as a diquark–antidiquark state. However, comparing the results obtained for the quantum numbers $J^P = 0^-$ and 1^- , from our calculations we conclude that the $Z^+(4430)$ is probably a $J^P = 0^-$ state.

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