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QCD evolutions of twist-3 chirality-odd operators

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ABSTRACT

We study the scale dependence of twist-3 distributions defined with chirality-odd quark–gluon operators. To derive the scale dependence we explicitly calculate these distributions of multi-parton states instead of a hadron. Taking one-loop corrections into account we obtain the leading evolution kernel in the most general case. In some special cases the evolutions are simplified. We observe that the obtained kernel in general does not get simplified in the large-N_c limit in contrast to the case of those twist-3 distributions defined only with chirality-odd quark operators. In the later, the simplification is significant.

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Predictions can be made from QCD with the concept of factorizations for processes with large momentum transfers. A typical example is DIS. For unpolarized DIS, the differential cross section at the leading power of the momentum transfer Q is predicted as a convolution of perturbative coefficient functions with parton distribution functions (PDF's). PDF's are defined with twist-2 QCD operators and describe nonperturbative effects of hadrons. Although PDF's cannot be predicted with perturbative QCD, but their scale dependence, hence the Q^2-dependence of the differential cross section, can be determined by perturbation theory. The dependence is governed by the famous DGLAP equation. In the past, the predicted scale-dependence or DGLAP equation has played an indispensable role for testing QCD as the correct theory of strong interaction.

In general, factorized differential cross sections also contain contributions involving hadronic matrix elements of higher-twist operators. Although these contributions are suppressed by inverse powers of Q, they contain more information about inner structure of hadrons. Among them, the most interesting are those involving twist-3 operators. These contributions are responsible for certain asymmetries in differential cross sections. These asymmetries can be measured in experiment and hence provide information about hadronic matrix elements of twist-3 operators. A well-known example is the study of Single transverse-Spin Asymmetries (SSA). The asymmetries can be factorized with the ETQS matrix elements defined with chirality-even quark–gluon operators at twist-3 [1,2]. The scale dependence of these twist-3 operators have been studied in [3–8]. Besides them, there are chirality-odd quark–gluon operators at twist-3. In this work, we study the scale dependence of these operators.

We consider a spin-1/2 hadron moving in the z-direction with the momentum $P^\mu = (P^+, P^-, 0, 0)$. We will use the light-cone coordinate system, in which a vector a^μ is expressed as $a^\mu = (a^+, a^-, \vec{a}_\perp) = ((a^0 + a^3)/\sqrt{2}, (a^0 - a^3)/\sqrt{2}, a^1, a^2)$ and $a_\perp^2 = (a^1)^2 + (a^2)^2$. In this coordinate system we introduce two light-cone vectors: $n^\mu = (0, 1, 0, 0)$ and $l^\mu = (1, 0, 0, 0)$. There are two distributions which can be defined with chirality-odd quark–gluon operators at twist-3. They are:

$$\begin{aligned}
&T_F^{(\sigma)}(x_1, x_2) \\
&\quad = g_s \int \frac{dy_1\, dy_2}{4\pi} e^{-iy_1 x_1 P^+ + iy_2 x_2 P^+} \\
&\qquad \times \langle P|\bar{\psi}(y_1 n)\big(i\gamma_{\perp\mu}\gamma^+\big)G^{+\mu}(0)\psi(y_2 n)|P\rangle, \\
&\lambda \tilde{T}_F^{(\sigma)}(x_1, x_2) \\
&\quad = g_s \int \frac{dy_1\, dy_2}{4\pi} e^{-iy_1 x_1 P^+ + iy_2 x_2 P^+} \\
&\qquad \times \langle P, \lambda|\bar{\psi}(y_1 n)\big(i\gamma_{\perp\mu}\gamma^+\gamma_5\big)G^{+\mu}(0)\psi(y_2 n)|P, \lambda\rangle, \qquad (1)
\end{aligned}$$

where the matrix element in the first equation is spin-averaged and that in the second equation is of a longitudinally polarized hadron with the helicity $\lambda = \pm 1/2$. $x_{1,2}$ are momentum fractions. The definitions are given in the light-cone gauge $n \cdot G = 0$. In other gauges gauge links along the direction n should be supplemented to make the definitions gauge invariant. From general principle one can show that the two distributions are real and:

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$$T_F^{(\sigma)}(x_1,x_2)=T_F^{(\sigma)}(x_2,x_1),$$
$$\tilde{T}_F^{(\sigma)}(x_1,x_2)=-\tilde{T}_F^{(\sigma)}(x_2,x_1). \tag{2}$$

Replacing the gluon field strength tensor $G^{+\mu}(0)$ in Eq. (1) with the covariant derivative $D^\mu=\partial^\mu+ig_sG^\mu(0)$, one can obtain another two twist-3 distributions. With equation of motion one can relate those two distributions to the two distributions defined in Eq. (1), respectively (see, e.g., [9,10,14]).

The operators in Eq. (1) contain the operator of the gluon field strength tensor. There are two chirality-odd twist-3 operators which only consist of quark field operators. Correspondingly, one can also define two distributions. One is $e(x)$ for unpolarized hadrons, another is $h_L(x)$ for longitudinally polarized hadrons. Again, these two distributions are not independent. One can use operators identities in [11] or equation of motion to show that these two are related with the above two and plus some contributions with local operators. The evolution of $e(x)$ and $h_L(x)$ have been studied in [12–14] and the evolution equations have been solved in moment space. The evolution of twist-3 quark–gluon operators has been studied in [15] with the emphasis on the solutions of evolution equations. In this work we derive the evolution kernel for the two twist-3 distributions defined in Eq. (1) in momentum fraction space with a different method.

Under renormalization there is no mixing between the two operators in Eq. (1). The evolution kernels can be conveniently expressed with one function by introducing the combinations:

$$\begin{aligned}\tilde{T}_\pm(x_1,x_2,\mu)&=\langle P,\pm|\mathcal{O}(x_1,x_2)|P,\pm\rangle\\&=T_F^{(\sigma)}(x_1,x_2,\mu)\pm\tilde{T}_F^{(\sigma)}(x_1,x_2,\mu),\\ \mathcal{O}(x_1,x_2)&=g_s\int\frac{dy_1\,dy_2}{4\pi}e^{-ix_1y_1P^++ix_2y_2P^+}\bar{\psi}(y_1n)i\gamma_{\perp\mu}\\&\quad\times\gamma^+(1+\gamma_5)G^{+\mu}(0)\psi(y_2n).\end{aligned} \tag{3}$$

The functions $\tilde{T}_\pm(x_1,x_2)$ are nonzero in the region of $|x_{1,2}|\leqslant 1$ and $|x_1-x_2|\leqslant 1$. The scale-dependence can be written in the form:

$$\begin{aligned}&\frac{\partial}{\partial\ln\mu}\tilde{T}_\pm(x_1,x_2,\mu)\\&\quad=\frac{\alpha_s}{\pi}\int d\xi_1\,d\xi_2\,\mathcal{F}_\pm(x_1,x_2,\xi_1,\xi_2)\tilde{T}_\pm(\xi_1,\xi_2,\mu).\end{aligned} \tag{4}$$

The integration region of $\xi_{1,2}$ is fixed by the support of $\tilde{T}_\pm$. It is easy to find that $\mathcal{F}_+$ is related to $\mathcal{F}_-$. Here we will take $\mathcal{F}_-$ to give our results.

The distributions are defined for a given hadron, but the kernel does not depend on the hadron. It is completely determined by the dynamics of QCD. For large μ it can be calculated with perturbative QCD. Because of this one can use various parton states to calculate the distribution $\tilde{T}_-$ to find its μ-dependence, hence the kernel $\mathcal{F}_-$. For the case with operators of twist-2 one can use single-parton state for the purpose. But for the two distributions defined here, one simply finds null results with single-parton states because the chirality is flipped by the operators. Therefore, one has to use multi-parton states to calculate the distribution. By using multi-parton states factorizations of SSA with twist-3 operators have been studied in [16–18]. In [7] such multi-parton states have been employed to study the scale dependence of twist-3 operators relevant for SSA.

We introduce the following state $|n(\lambda)\rangle$ as a superposition of single- and multi-parton states:

$$\big|n(\lambda)\big\rangle=|q\rangle+c_1|qG\rangle+c_2|qGG\rangle+c_3|q\bar{q}q\rangle+\cdots. \tag{5}$$

The state $|n\rangle$ is with the momentum $(P^+,0,0,0)$. All partons in the partonic states moving in the z-direction and the sum of their momenta are that of $|n\rangle$. The sum of helicity of partons is λ. In the above we have suppressed quantum numbers of partons, which will be specified later. In principle one can introduce wave functions depending on momenta of partons. For simplicity, we take these wave-functions as δ-functions and hence c_i $(i=1,2,3,\ldots)$ are real constants. If we calculate $\tilde{T}_-$ of the state $|n(\lambda)\rangle$ instead of a hadron, we obtain nonzero contributions from interference between different partonic states. At tree-level, the contributions can be schematically written as:

$$\begin{aligned}\tilde{T}_-(x_1,x_2)=\;&\mathcal{C}_1\big\langle q(-)\big|\mathcal{O}(x_1,x_2)\big|q(+)g(-)\big\rangle\\&+\mathcal{C}_2\big\langle q(-)g(+)\big|\mathcal{O}(x_1,x_2)\big|q(+)\big\rangle\\&+\mathcal{C}_3\big\langle q(-)\bar{q}(-)\big|\mathcal{O}(x_1,x_2)\big|g(-)\big\rangle\\&+\mathcal{C}_4\big\langle g(+)\big|\mathcal{O}(x_1,x_2)\big|q(+)\bar{q}(+)\big\rangle,\end{aligned} \tag{6}$$

where $\pm$ in brackets indicate the helicity of partons. It should be noted that there are possible spectators. E.g., in the first term, the spectators must carry the total helicity $\lambda_s=0$ which can be a quark pair or gluon pair. In the second term the spectators must have $\lambda_s=-1$ because we have here $\lambda=-1/2$. For the last two terms we have $\lambda_s=\pm 1/2$, respectively. The contributions from spectators give overall factors as products of δ-functions for each term. These overall factors are contained in the constants $\mathcal{C}_i$ which also depend on c_i as in Eq. (5). Because of existence of spectator-partons, the parton states in the above are not necessarily with the total momentum P.

Beyond tree-level, the four matrix elements of partons in Eq. (6) receive corrections. These corrections make $\tilde{T}_-$ μ-dependent. We define the four matrix elements as:

$$\begin{aligned}T_{-qg}(x_1,x_2,y_0,z_0)&=\big\langle q(p,-)\big|\mathcal{O}(x_1,x_2)\big|q(p_1,+)g(p_2,-)\big\rangle,\\T_{+qg}(x_1,x_2,y_0,z_0)&=\big\langle q(p_1,-)g(p_2,+)\big|\mathcal{O}(x_1,x_2)\big|q(p,+)\big\rangle,\\T_{-q\bar{q}}(x_1,x_2,y_0,z_0)&=\big\langle \bar{q}(p_2,-)q(p_1,-)\big|\mathcal{O}(x_1,x_2)\big|g(p,-)\big\rangle,\\T_{+q\bar{q}}(x_1,x_2,y_0,z_0)&=\big\langle g(p,+)\big|\mathcal{O}(x_1,x_2)\big|q(p_1,+)\bar{q}(p_2,+)\big\rangle,\end{aligned} \tag{7}$$

with the momenta:

$$p_1^+=y_0P^+,\qquad p_2^+=(z_0-y_0)P^+,\qquad p^+=z_0P^+. \tag{8}$$

The color of the state $|qg\rangle$ is the same as the single-quark state. Details can be found in [16,18]. The four matrix elements are not independent. They are pair-wise related:

$$\begin{aligned}T_{-qg}(x_1,x_2,y_0,z_0)&=T_{+qg}(x_2,x_1,y_0,z_0),\\T_{-q\bar{q}}(x_1,x_2,y_0,z_0)&=T_{+q\bar{q}}(x_2,x_1,y_0,z_0).\end{aligned} \tag{9}$$

As we will see, the μ-dependence of the four matrix elements determine the kernel $\mathcal{F}_-$ in different regions of $\xi_{1,2}$. Because of this, the determined kernel will not depend on the states, i.e., the coefficients c_i and $\mathcal{C}_i$. To determine the kernel in the full region of $\xi_{1,2}$, one also needs to calculate $\tilde{T}_-$ of the state $|\bar{n}\rangle$ – the charge-conjugated state of $|n\rangle$. However, $\tilde{T}_-$ of $|\bar{n}\rangle$ can be obtained from $\tilde{T}_-$ of $|n\rangle$ through charge conjugation.

We take T_{-qg} as an example to show how the corresponding contribution to $\mathcal{F}_-$ is determined. At tree-level, we have:

$$\begin{aligned}T_{-qg}^{(0)}(x_1,x_2,y_0,z_0)&=2\pi g_s\sqrt{2z_0y_0}\big(N_c^2-1\big)\\&\quad\times(z_0-y_0)\delta(z_0-x_1)\delta(x_2-y_0).\end{aligned} \tag{10}$$

It should be noted $z_0>y_0$ because of $p_2^+>0$. If we have the one-loop result $T_{-qg}^{(1)}$, we can substitute the results of $T_{-qg}^{(0,1)}$ into Eq. (4)

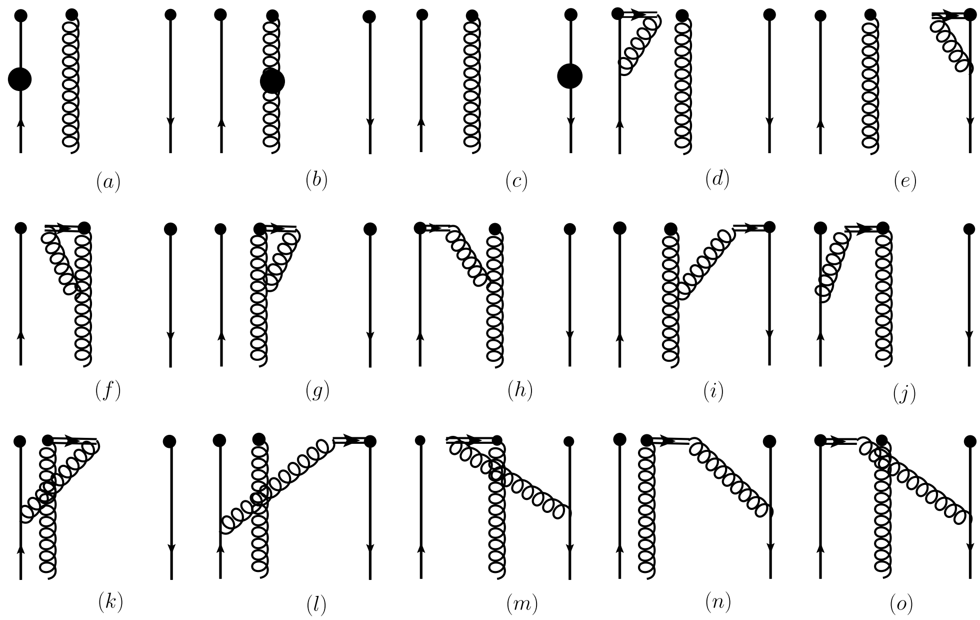

Fig. 1. A set of diagrams of one-loop corrections to the defined twist-3 matrix elements. This set only contains the self-energy corrections and corrections from the gluon emission from a gauge link represented by a double line.

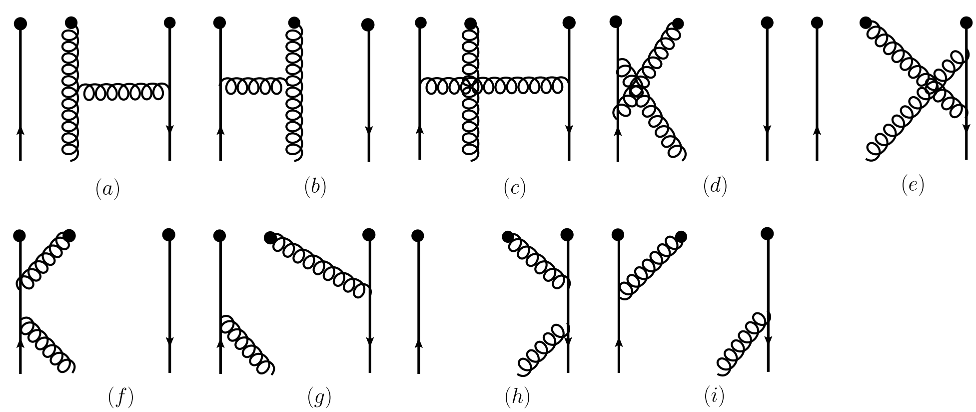

Fig. 2. Another set of diagrams for one-loop corrections of the defined twist-3 matrix elements.

through Eq. (6). Then we can find for $\xi_2 > 0$:

$$g_s\alpha_s\mathcal{F}_-(x_1,x_2,\xi_1,\xi_2)\theta(\xi_1-\xi_2) = \frac{1}{2(N_c^2-1)(\xi_1-\xi_2)\sqrt{2\xi_1\xi_2}}\frac{\partial}{\partial\ln\mu}T^{(1)}_{-qg}(x_1,x_2,\xi_2,\xi_1). \quad (11)$$

The θ-function $\theta(\xi_1-\xi_2)$ appears because of $z_0-y_0>0$. Therefore, $\mathcal{F}_-$ in the region of $\xi_1>\xi_2$ and $\xi_2>0$ is determined by $T^{(1)}_{-qg}$. Similarly, one can find that $\mathcal{F}_-$ in the region of $\xi_2>\xi_1$ and $\xi_1>0$ is determined by $T^{(1)}_{+qg}$. Combining the two contributions one has the kernel in the region of positive $\xi_{1,2}$.

The one-loop correction to T_{-qg} is from the diagrams given in Fig. 1 and Fig. 2. The calculation of these diagrams is rather standard. Therefore we give the result directly. We introduce the following function which is just the kernel in the region of $\xi_{1,2}>0$:

$$\begin{aligned}\mathcal{F}_1(x_1,x_2,\xi_1,\xi_2) = \theta(x_2)\big[&\delta(\xi_1-x_1)f_1(\xi_2,x_1,x_2)\\ &+\delta(\xi_1-\xi_2-x_1+x_2)f_0(\xi_2,x_1,x_2)\\ &+\delta(\xi_2-x_2)f_2(\xi_1,x_1,x_2)\big]\\ &+(x_1\leftrightarrow x_2,\xi_1\leftrightarrow\xi_2),\end{aligned} \quad (12)$$

with

$$f_0(\xi,x_1,x_2) = \frac{1}{2N_c}\frac{x_2\theta(\xi-x_2)}{\xi(x_2-\xi)_+},$$

$$\begin{aligned}f_1(\xi,x_1,x_2) = \frac{N_c}{4}&\big(\theta(x_1-x_2)\theta(x_2-\xi)-\theta(x_2-x_1)\theta(\xi-x_2)\big)\\ &\times\left(\frac{2}{(x_2-\xi)_+}-\frac{x_1-x_2}{(x_1-\xi)^2}-\frac{2}{x_1-\xi}\right)\\ &+\frac{N_c}{2}\theta(\xi-x_2)\frac{x_2}{\xi}\left(\frac{1}{(\xi-x_2)_+}-\frac{1}{\xi-x_1}\right)\\ &+\frac{1}{2}\delta(\xi-x_2)\left[\frac{3(N_c^2-1)}{4N_c}\right.\\ &\left.-\frac{N_c^2-1}{N_c}\ln x_2-N_c\ln|x_1-x_2|\right],\end{aligned}$$

$$\begin{aligned}&f_2(\xi,x_1,x_2)\\ &\quad=-\frac{N_c}{4(\xi-x_2)}\theta(x_1)\left[\theta(x_1-\xi)\theta(x_2-x_1)\frac{x_1-x_2}{x_2(\xi-x_2)}\right.\\ &\quad\times(3x_2-2x_1-2\xi)\end{aligned}$$

$$+2\theta(\xi-x_1)\theta(x_2-x_1)\frac{x_1}{\xi x_2}(x_2-x_1-\xi)$$
$$\left.-\theta(\xi-x_1)\theta(x_1-x_2)\frac{2x_2x_1-3\xi x_2-2x_1^2+x_1\xi+2\xi^2}{\xi(\xi-x_2)}\right]$$
$$+\frac{N_c^2-1}{2N_c}\theta(x_2-x_1)\theta(x_1)\frac{x_1(x_2-x_1)}{x_2^2(\xi-x_2)}$$
$$+\frac{\theta(x_2-x_1)}{2N_c(\xi-x_2)}(x_2-x_1-\xi)\left[-\frac{\xi-x_2+x_1}{\xi(\xi-x_2)}\right.$$
$$\times\theta(-x_1)\theta(\xi-x_2+x_1)$$
$$-\theta(x_1)\left(-\frac{x_1\theta(x_2-\xi-x_1)}{x_2(\xi-x_2)}\right.$$
$$\left.\left.+\frac{x_2-x_1}{\xi x_2}\theta(\xi-x_2+x_1)\right)\right]. \tag{13}$$

The $+$-distributions here are defined as:

$$\int_0^x dz\,\frac{f(z)}{(x-z)_+}=\int_0^x dz\,\frac{f(z)-f(x)}{x-z}+f(x)\ln x,$$
$$\int_x^1 dz\,\frac{f(z)}{(z-x)_+}=\int_x^1 dz\,\frac{f(z)-f(x)}{z-x}+f(x)\ln(1-x). \tag{14}$$

It should be noted that our $+$-distribution is not the standard $+$-distribution defined later. The function f_0 and f_1 are determined by the contributions from Fig. 1. In general for each diagram with a gluon emitted from the gauge link there is a light-cone singularity. But, the sum is free from the singularity.

Now we turn to the contributions from $T_{-q\bar q}$. At tree-level we have

$$T^{(0)}_{-q\bar q}(x_1,x_2,y_0,z_0)=-2\pi g_s\big(N_c^2-1\big)\sqrt{2y_0(z_0-y_0)}$$
$$\times z_0\delta(x_1-y_0)\delta(x_2+z_0-y_0). \tag{15}$$

With the one-loop result of $T_{-q\bar q}$ one can determine the kernel

$$g_s\alpha_s\mathcal{F}_-(x_1,x_2,\xi_1,\xi_2)\theta(\xi_1)\theta(-\xi_2)$$
$$=-\frac{1}{2(N_c^2-1)(\xi_1-\xi_2)\sqrt{-2\xi_1\xi_2}}\frac{\partial}{\partial\ln\mu}$$
$$\times T^{(1)}_{-q\bar q}(x_1,x_2,\xi_1,\xi_1-\xi_2). \tag{16}$$

The kernel determined from $T_{-q\bar q}$ is in the region of $\xi_1>0$ and $\xi_2<0$. From $T_{+q\bar q}$ the kernel in the region $\xi_1<0$ and $\xi_2>0$ can be obtained. The one-loop correction to $T_{-q\bar q}$ is still given by the diagrams in Fig. 1 and Fig. 2, where the quark line in the left side should be taken as an outgoing anti-quark. The calculation of these diagrams are again straightforward. The result can be summarized by the function

$$\mathcal{F}_2(x_1,x_2,\xi_1,\xi_2)$$
$$=\delta(\xi_1-x_1)\theta(x_1)\big(h_1(-\xi_2,x_1,x_2)+h_2(-\xi_2,-x_2,-x_1)\big)$$
$$+\delta(\xi_2-x_2)\theta(-x_2)\big(h_1(\xi_1,-x_2,-x_1)+h_2(\xi_1,x_1,x_2)\big)$$
$$+\delta(\xi_1-\xi_2-x_1+x_2)\big(\theta(-x_2)h_0(-\xi_2,x_1,x_2)$$
$$+\theta(x_1)h_0(\xi_1,-x_2,-x_1)\big), \tag{17}$$

with the functions

$$h_0(\xi,x_1,x_2)=\frac{1}{2N_c}\theta(x_2+\xi)\frac{x_2}{\xi(\xi+x_2)_+},$$
$$h_1(\xi,x_1,x_2)$$
$$=\frac{N_c}{4}\left[\theta(-x_2)\theta(x_2+\xi)\left(\frac{\xi+2x_1-x_2}{(\xi+x_1)^2}-\frac{2x_2}{\xi(\xi+x_1)}\right)\right.$$
$$\left.\times\frac{x_1-x_2}{(\xi+x_2)_+}+\theta(x_1-x_2)\theta(x_2)\frac{\xi+2x_1-x_2}{(\xi+x_1)^2}\frac{x_1-x_2}{\xi+x_2}\right]$$
$$+\frac{1}{2}\theta(-x_2)\delta(x_2+\xi)\left[\frac{3}{4}\frac{N_c^2-1}{N_c}-\frac{N_c^2-1}{N_c}\ln|x_2|\right.$$
$$\left.-N_c\ln|x_1-x_2|\right],$$
$$h_2(\xi,x_1,x_2)=-\frac{N_c}{4}\frac{1}{(\xi-x_2)^2}\left[\theta(x_1)\theta(\xi-x_1)\right.$$
$$\times\frac{-2\xi^2-x_1\xi+2x_1^2+3\xi x_2-2x_1x_2}{\xi}$$
$$\left.+\theta(-x_1)\theta(x_1-x_2)\frac{x_1-x_2}{x_2}(2\xi+2x_1-3x_2)\right]$$
$$+\frac{1}{2N_c}\frac{\xi+x_1-x_2}{(\xi-x_2)^2}\theta(-x_1)\theta(\xi-x_2+x_1)$$
$$\times\left[\theta(x_1-x_2)\frac{x_1}{x_2}+\theta(x_2-x_1)\frac{\xi-x_2+x_1}{\xi}\right]$$
$$+\frac{N_c^2-1}{2N_c}\theta(-x_1)\theta(x_1-x_2)\frac{x_1(x_1-x_2)}{x_2^2(\xi-x_2)}. \tag{18}$$

The function $\mathcal{F}_2$ is just the kernel in the region of $\xi_1>0$ and $\xi_2<0$. The kernel in the region of $\xi_1<0$ and $\xi_2>0$ can be obtained from $T_{+q\bar q}$ through the relation in Eq. (9).

Finally, we have the kernel in the full region of $\xi_{1,2}$ as:

$$\mathcal{F}_-(x_1,x_2,\xi_1,\xi_2)$$
$$=\theta(\xi_1)\theta(-\xi_2)\mathcal{F}_2(x_1,x_2,\xi_1,\xi_2)$$
$$+\theta(\xi_2)\theta(-\xi_1)\mathcal{F}_2(x_2,x_1,\xi_2,\xi_1)$$
$$+\theta(\xi_1)\theta(\xi_2)\mathcal{F}_1(x_1,x_2,\xi_1,\xi_2)$$
$$+\theta(-\xi_1)\theta(-\xi_2)\mathcal{F}_1(-x_2,-x_1,-\xi_2,-\xi_1). \tag{19}$$

The kernel in the region with $\xi_{1,2}<0$ can be obtained from that in the region with $\xi_{1,2}>0$ through charge-conjugation. Eq. (19) is our main result. Since $T_F^{(\sigma)}$ and $\tilde T_F^{(\sigma)}$ are not mixed under renormalization, the correct kernel should satisfy:

$$\mathcal{F}_-(x_1,x_2,\xi_1,\xi_2)=\mathcal{F}_-(x_2,x_1,\xi_2,\xi_1). \tag{20}$$

Our result has the property. Therefore, taking the part of $\mathcal{F}_-$ symmetric between x_1 and x_2, one can obtain the evolution of $T_F^{(\sigma)}$ as a convolution with $T_F^{(\sigma)}$. The anti-symmetric part of $\mathcal{F}_-$ gives the kernel of the evolution of $\tilde T_F^{(\sigma)}$.

The evolution kernel for arbitrary $x_{1,2}$ is rather lengthy. But the convolution in Eq. (4) is a one-dimensional integral at one-loop. In some special cases, the kernel is simplified. E.g., for $x_1=x_2=x>0$, i.e., the case where the gluon carries zero momentum fraction entering a hard scattering, we have:

$$\frac{\partial}{\partial\ln\mu}T_F^{(\sigma)}(x,x,\mu)$$
$$=\frac{\alpha_s}{\pi}\left\{-\frac{N_c^2+3}{4N_c}T_F^{(\sigma)}(x,x,\mu)\right.$$

$$+\int_x^1 \frac{dz}{z}\frac{1}{(1-z)_+}\left(N_c T_F^{(\sigma)}(x,\xi)-\frac{z}{N_c}T_F^{(\sigma)}(\xi,\xi)\right)$$
$$+\frac{1}{N_c}\int_0^{1-x} d\xi\,\frac{\xi}{(\xi+x)^2}T_F^{(\sigma)}(x,-\xi)\bigg\},$$
$$\frac{\partial}{\partial\ln\mu}\tilde{T}_F^{(\sigma)}(x,x,\mu)=0 \tag{21}$$

with $z=x/\xi$. Here the $+$-distribution is the standard one defined as:

$$\int_0^1 dz\,\frac{\theta(z-x)}{(1-z)_+}t(z)=\int_x^1 dz\,\frac{t(z)-t(1)}{1-z}+t(1)\ln(1-x). \tag{22}$$

The result in Eq. (21) agrees with that in [8]. Part of the evolution has been also derived in [19]. It agrees with our corresponding result. There are also cases in which a quark carries zero momentum fraction entering a hard scattering. In these cases, one has either $x_1=0$ or $x_2=0$. The evolutions in these cases are:

$$\frac{\partial}{\partial\ln\mu}\tilde{T}_-(0,x,\mu)$$
$$=\frac{\alpha_s}{\pi}\left\{\int_x^1\frac{dz}{z}\left[\left(-\frac{N_c}{2}(1+z+z^2)\tilde{T}_-(0,\xi)\right.\right.\right.$$
$$\left.+\frac{1}{2N_c}\tilde{T}_-(\xi,x)+\frac{1}{2N_c}\tilde{T}_-(\xi-x,\xi)\right)$$
$$+\frac{1}{(1-z)_+}\left(N_c\tilde{T}_-(0,\xi)+\frac{N_c}{2}\tilde{T}_-(x-\xi,x)\right.$$
$$\left.\left.-\frac{1}{2N_c}\tilde{T}_-(\xi-x,\xi)\right)\right]$$
$$\left.+\frac{(1-z)^2}{2N_c}\tilde{T}_-(0,-\xi)+\frac{3(N_c^2-1)}{4N_c}\tilde{T}_-(0,x)\right\}. \tag{23}$$

Changing every $\tilde{T}_-(x_1,x_2)$ to $\tilde{T}_-(x_2,x_1)$ in the above equation, we obtain the evolution of $\tilde{T}_-(x,0)$.

The evolutions in these special cases of Eqs. (21), (23) are not closed, i.e., they depend on $T_F^{(\sigma)}(x_1,x_2)$ and $\tilde{T}_-(x_1,x_2)$. However, the twist-3 matrix elements with $x_1=x_2$ or $x_{1,2}=0$ can play a special role in collinear factorization. They can be directly related to physical observables. E.g., in the collinear factorization of an SSA studied in [20] there is a contribution proportional to $T_F^{(\sigma)}(x,x)$, in addition to the contribution involving chirality-even twist-3 operators. In the study of collinear factorizations of SSA [2,18,21], one has found that the twist-3 matrix elements of chirality-even operators in the special cases corresponding to the special cases here are directly related to physical observables. It is also in general expected that $\tilde{T}_-(x,0)$ or $\tilde{T}_-(0,x)$ will be related to some physical observables. To show this further studies are needed. Therefore, the evolutions in these special cases of Eqs. (21), (23) are particularly interesting.

The two studied twist-3 distributions do not mix in the evolution with other twist-3 distributions mentioned in the beginning, e.g., with $e(x)$ and $h_L(x)$. But the evolution of the distribution $e(x)$ or $h_L(x)$ does mix with $T_F^{(\sigma)}$ and $\tilde{T}_F^{(\sigma)}$ [12–14]. One may derive the evolution of $e(x)$ and $h_L(x)$ through their relations to $T_F^{(\sigma)}$ and $\tilde{T}_F^{(\sigma)}$, respectively. But the derivation is very tedious. It has been shown in [12] that in the large-N_c limit the evolution of $e(x)$ or $h_L(x)$ is simplified and obeys DGLAP-type equations. The result for this in momentum fraction space can be found in [14]. These evolutions in the large-N_c limit can also be derived with the approach used here. In the below we take $e(x)$ as an example.

For an unpolarized hadron there is one twist-3 distribution $e(x)$ defined only with bilinear operator of quark fields in the light-cone gauge:

$$e(x)=P^+\int\frac{dy}{4\pi}\,e^{-iyxP^+}\langle P|\bar{\psi}(yn)\psi(0)|P\rangle. \tag{24}$$

With equation of motion it can be shown that $e(x)$ for $x\neq 0$ is related to $T_F^{(\sigma)}$. The relation is [9,10,14]:

$$xe(x)=\frac{1}{4\pi}\int dx_1\,dx_2\,\frac{T_F^{(\sigma)}(x_1,x_2)}{x_2-x_1}\left(\frac{\delta(x-x_1)}{x_1}-\frac{\delta(x-x_2)}{x_2}\right). \tag{25}$$

Using our results of $T_F^{(\sigma)}$ at tree-level and one-loop, one can also derive the tree-level- and one-loop result for $e(x)$. From these results and by taking the large-N_c limit, we find the following evolution for $x>0$:

$$\frac{\partial e(x)}{\partial\ln\mu}=\frac{\alpha_s N_c}{2\pi}\left[\frac{1}{2}e(x)+\int_x^1\frac{dz}{z}\,e(\xi)\left(\frac{2}{(1-z)_+}+1\right)\right]$$
$$+\mathcal{O}\left(N_c^{-1}\right), \tag{26}$$

with $z=x/\xi$. This result agrees with that in [14]. Although the evolution of $e(x)$ and $h_L(x)$ gets simplified in the large-N_c limit, the evolutions of $T_F^{(\sigma)}$ and $\tilde{T}_F^{(\sigma)}$ are not much simplified in the limit. This can be observed from our results. This supports the observation made in [22] that simplifications of evolutions of higher-twist distributions can be accidental.

In this Letter we have mainly concentrated on the evolutions of $T_F^{(\sigma)}$ and $\tilde{T}_F^{(\sigma)}$. We have not studied the evolutions of other twist-3 distributions in detail, e.g., $h_L(x)$, because they are all related to $T_F^{(\sigma)}$ or $\tilde{T}_F^{(\sigma)}$. For h_L its relation to $\tilde{T}_F^{(\sigma)}$ is more complicated than that in Eq. (25), there is an additional twist-3 operator entering the relation [9,10]. For physical predictions like differential cross sections, the most useful evolutions should be those $T_F^{(\sigma)}$ or $\tilde{T}_F^{(\sigma)}$. The reason is the following: The studied twist-3 distributions are the most general distributions, e.g., $T_F^{(\sigma)}(x_1,x_2)$ contains more information than that of $e(x)$. In collinear factorization of physical observables one should take $T_F^{(\sigma)}$ or $\tilde{T}_F^{(\sigma)}$ instead of other twist-3 operators to perform the factorization. Otherwise, inconsistencies can appear. A well-known example is the factorization of the structure function $g_T=g_1+g_2$ of DIS [23]. At tree-level the function can be factorized with a twist-3 operator defined only with bilinear quark field operator, similar to $e(x)$. But beyond the tree-level and for consistent factorization one should factorize the function with a twist-3 operator consisting of bilinear quark field operator and a gluon field strength tensor operator, similar to $T_F^{(\sigma)}$ or $\tilde{T}_F^{(\sigma)}$.

To summarize: We have calculated the twist-3 distributions of multi-parton states. The distributions are defined with chirality-odd quark–gluon operators. The evolution kernel of the distributions has been obtained by the calculation at one-loop. The evolution at one-loop is a one-dimensional convolution. In some special cases the evolution takes a short form. We have also derived the evolution of $e(x)$ in the large-N_c limit and found an agreement with existing results.

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