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Thermal Radiation Effect on Mixed Convection Flow over a Vertical Stretching Sheet Embedded in a Porous Medium with Suction (Injection)

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Abstract

The problem of steady laminar mixed convection flow of an incompressible viscous fluid over a vertical stretching sheet with variable wall temperature and concentration in the presence of porous medium and thermal radiation is examined. The governing boundary layer equations are transformed into a non-dimensional form by a group of non-similar transformations. The resulting coupled non-linear partial differential equations have been solved numerically by using implicit finite difference scheme in combination with the quasi-linearization technique. The effects of various parameters on the velocity and temperature profiles as well as skin friction and heat transfer coefficients are presented. The results are found to be in good agreement with the existing solutions in literature.

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Keywords: Mixed convection; Porous medium; Suction or Injection; Thermal radiation

1. Introduction

The study of flow and heat transfer over a stretching surface has gained considerable attention of researchers due to its applications, such as extraction of polymer sheet, wire drawing, paper production, glass-fiber production, hot rolling, solidification of liquid crystals, petroleum production, continuous cooling and fibers spinning and exotic lubricants. Meanwhile, since the pioneering work of Sakiadis [1], related to stretching sheet have increased greatly in recent due to its wide applications in industrial and engineering systems as shown by [2–4].

The problem of mixed convection flow past a stretching sheet embedded in porous medium arises in some metallurgical processes which involve the cooling of continuous strips or filaments by drawing them through quiescent fluid and the rate of cooling can be better controlled and final product of desired characteristics can be achieved if the strips are drawn through porous media as investigated by Khader and Megahed [5] and Pal and Mondal [6]. The study of radiative heat transfer flow is useful in manufacturing industries for the design of reliable equipments, nuclear plans, gas turbines and various propulsion devices for aircraft, missiles, satellites and space vehicles. Also, the aspect of

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thermal radiation on convective flows has very important processes in high temperatures. Based on these applications, Pal and Mondal [7] and Shateyi and Motsa [8] determined the thermal radiation of a gray fluid which is emitting and absorbing radiation in non-scattering medium.

Aforementioned studies were primarily concerned without suction or injection. Pal and Mondal [9] and Cortell [10] reported the influence of uniform suction or injection in the presence of chemical reaction, thermal radiation, heat generation or absorption, Soret and Dufour effects over a stretching sheet. [11–13] obtained a non-similar solution into a mixed convection flow over a stretching sheet and cone.

However, so far no attention has been given to the effects of thermal radiation and porous medium over a stretching sheet in the presence of suction or injection. The governing boundary layer equations along with the boundary conditions are first cast into a dimensionless form, and the resulting equations are then solved by an implicit finite difference method of the quasi-linearization technique.

2. Mathematical formulation

Consider a steady two dimensional laminar mixed convection flow of a viscous incompressible and optically thick radiating fluid over a vertical stretching sheet embedded in a porous medium placed in the plane $y = 0$. The stretching surface has linear velocity U_w while two equal and opposite forces are introduced along the x-axis so that the sheet is stretched with a speed proportional to the distance from the fixed origin $x = 0$. The fluid is considered to be gray, absorbing and emitting radiation but not-scattering medium and using Rosseland approximation to describe the radiative heat flux in the energy equation. The radiative heat flux in the x-direction is considered negligible in comparison to the y-direction. The temperature and the species concentration vary with the distance from the origin. The flow is governed by the following equations:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = U_e\frac{\partial U_e}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2} - \frac{\nu}{K}(u - U_e) + g[\beta(T - T_\infty) + \beta^*(C - C_\infty)], \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p}\frac{\partial q_r}{\partial y}, \tag{3}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2}. \tag{4}$$

The stretching surface has a uniform temperature T_w and the free stream temperature is T_∞ with $T_w > T_\infty$. Also, it has a uniform concentration C_w and the free stream concentration is C_∞ with $C_w > C_\infty$. Diffusion approximation model is used for an optically thick medium for equating radiative transfer in which an approximation form of the radiative heat flux gradient q_r, in the y-direction is called Rosseland or diffusion approximation which has the following form:

$$q_r = -\frac{4\sigma^*}{a_R}\frac{\partial T^4}{\partial y}, \tag{5}$$

where σ is the Stefan-Boltzmann constant and a_R is the mean absorption coefficient. The temperature differences within the flow are assumed to be sufficiently small so that T^4 may be expressed as a linear function of temperature T using a truncated Taylor series about the free stream temperature T_∞, i.e.,

$$T^4 \approx 4T_\infty^3 T - 3T_\infty^4.$$

The boundary conditions are given by:

$$\left.\begin{aligned} u = U_w(x) = U_{w0}x^m, \quad v = v_w(x), \quad T = T_w(x) = T_\infty + Bx^n,\\ C = C_w(x) = C_\infty + B^*x^n \ \text{ at } y = 0,\\ u \to U_\infty(x) = U_{\infty 0}x^m, \quad T \to T_\infty, \quad C \to C_\infty \ \text{ as } y \to \infty.\end{aligned}\right\} \tag{6}$$

Where B and B^* are constants, $U_w(x)$ is the stretching sheet velocity of the surface represent the mass transfer velocity at the surface of the sheet, v_w is suction/injection parameter with $v_w > 0$ for injection (blowing), $v_w < 0$ for suction and $v_w = 0$ corresponds to an impermeable sheet.
The composite reference velocity is defined as $U(x) = U_w(x) + U_\infty(x)$, $U_0 = U_{w0} + U_{\infty 0} (\neq 0)$. To facilitate numerical solutions, we introduce the following dimensionless variables:

$$\xi = \left(\frac{U(x)x}{\nu}\right)^{\frac{1}{2}}, \qquad \eta = y\left(\frac{U(x)}{\nu x}\right)^{\frac{1}{2}},$$

$$\psi(x,y) = (\nu x U(x))^{\frac{1}{2}} f(\xi,\eta), \quad f_\eta(\xi,\eta) = F(\xi,\eta),$$
$$T - T_\infty = (T_w(x) - T_\infty)\ G(\xi,\eta), \quad C - C_\infty = (C_w(x) - C_\infty)\ H(\xi,\eta), \tag{7}$$

where $0 \leq \xi \leq 1$ and ψ is the stream function which is defined in the usual form as

$$u = \frac{\partial\psi}{\partial y}, \quad v = -\frac{\partial\psi}{\partial x}. \tag{8}$$

So the continuity Eq. (1) is automatically satisfied. Thus substituting equation (7) into (8), we obtain u, v as follows

$$u = U_0 x^m F(\xi,\eta), \qquad v = -\frac{1}{2}(\nu U_0)^{\frac{1}{2}} x^{\frac{m-1}{2}}\Big[(m+1)(f + \xi f_\xi) + (m-1)\eta F\Big]. \tag{9}$$

To transform Eqs. (2)-(4) into a non-dimensional equations, we introduce the following dimensionless parameters and variables:

$$\lambda = \frac{Gr}{Re_x^2}, \quad Gr = \frac{g\beta(T_w - T_\infty)x^3}{\nu^2}, \quad Pr = \frac{\mu c_p}{k},$$

$$\lambda^* = \frac{Gr^*}{Re_x^2}, \quad Gr^* = \frac{g\beta^*(C_w - C_\infty)x^3}{\nu^2}, \quad Sc = \frac{\nu}{D},$$

$$Re_x = \frac{U_0 x^{m+1}}{\nu}, \quad N = \frac{\lambda^*}{\lambda}, \quad k_1 = \frac{1}{Da_x Re_x} = \frac{\nu x}{KU}, \quad R = \frac{16\sigma^* T_\infty^3}{3ka_R}.$$

Using the non-similar transformation in Eq. (8), the following non-linear partial differential equations obtained from Eqs. (2)-(4) are given by:

$$F_{\eta\eta} + \left(\frac{m+1}{2}\right) f F_\eta + m(\varepsilon^2 - F^2) + \lambda(G + NH) + k_1(F - \varepsilon) = \left(\frac{m+1}{2}\right)\xi(F F_\xi - f_\xi F_\eta), \tag{10}$$

$$G_{\eta\eta} + \left(\frac{Pr}{1+R}\right)\left(\frac{m+1}{2}\right) f G_\eta - \left(\frac{Pr}{1+R}\right) F G n = \left(\frac{Pr}{1+R}\right)\left(\frac{m+1}{2}\right)\xi(F G_\xi - f_\xi G_\eta), \tag{11}$$

$$H_{\eta\eta} + Sc\left(\frac{m+1}{2}\right) f H_\eta - Sc F H n = Sc\left(\frac{m+1}{2}\right)\xi(F H_\xi - f_\xi H_\eta), \tag{12}$$

where

$$f = \int_0^\eta F d\eta + f_w \text{ and } f_w \text{ is given by } \quad f_w = \frac{2}{(m-1)} A\xi^{\frac{1-m}{1+m}}.$$

The non-dimensional boundary conditions of the problem

$$\begin{aligned} &F(\xi,\eta) = 1 - \varepsilon, \quad G(\xi,\eta) = 1, \quad H(\xi,\eta) = 1 \quad \text{at} \quad \eta = 0, \\ &F(\xi,\eta) = \varepsilon, \quad G(\xi,\eta) = 0, \quad H(\xi,\eta) = 0 \quad \text{at} \quad \eta = \eta_\infty, \end{aligned} \tag{13}$$

where

$$\varepsilon = \frac{U_\infty(x)}{U(x)} = \frac{U_{\infty 0}}{U_{w0} + U_{\infty 0}}$$

corresponds to the ratio of free stream velocity to the composite reference velocity.

The physical quantities of interest are the local skin friction coefficient C_{fx}, the local Nusselt number Nu_x and the local Sherwood number Sh_x are expressed in dimensionless form are as follows:

$$C_{fx} = \frac{2\left[\mu\left(\frac{\partial u}{\partial y}\right)\right]_{y=0}}{\rho U^2} = 2F_\eta(\xi,0)(Re_x)^{-1/2},$$

$$Nu_x = -\frac{\left[x\left(\frac{\partial T}{\partial y}\right)\right]_{y=0}}{T_w(x) - T_\infty} = -G_\eta(\xi,0)(Re_x)^{1/2},$$

$$Sh_x = -\frac{\left[x\left(\frac{\partial C}{\partial y}\right)\right]_{y=0}}{C_w(x) - C_\infty} = -H_\eta(\xi,0)(Re_x)^{1/2}.$$

3. Method of Solution

The set of non-linear coupled partial differential equations (10)-(12) along with the boundary condition (13) is solved by an implicit finite difference scheme in combination with the quasi-linearization technique. With the help of quasi-linearization technique, the non-linear coupled partial differential equations (10)-(12) are replaced by the following sequence of linear partial differential equations:

$$F^{i+1}_{\eta\eta} + X^i_1 F^{i+1}_\eta + X^i_2 F^{i+1} + X^i_3 G^{i+1} + X^i_4 F^{i+1}_\xi + X^i_5 H^{i+1} = X^i_6, \quad (14)$$

$$G^{i+1}_{\eta\eta} + Y^i_1 G^{i+1}_\eta + Y^i_2 G^{i+1} + Y^i_3 F^{i+1} + Y^i_4 G^{i+1}_\xi = Y^i_5, \quad (15)$$

$$H^{i+1}_{\eta\eta} + Z^i_1 H^{i+1}_\eta + Z^i_2 H^{i+1} + Z^i_3 F^{i+1} + Z^i_4 H^{i+1}_\xi = Z^i_5. \quad (16)$$

The coefficient functions with iterative index (i) are known and the functions with iterative index ($i+1$) are to be determined.

The corresponding boundary conditions of Eqs. (14)-(16) are

$$\begin{aligned} &F^{i+1} = 1-\varepsilon,\ \ G^{i+1} = 1,\ \ H^{i+1} = 1 \quad \text{at} \quad \eta = 0,\\ &F^{i+1} = \varepsilon,\ \ G^{i+1} = 0,\ \ H^{i+1} = 0 \quad \text{at} \quad \eta = \eta_\infty. \end{aligned} \quad (17)$$

The coefficients in Eqs. (14)-(16) are as follows:

$$X^i_1 = f\left(\frac{m+1}{2}\right) + \left(\frac{m+1}{2}\right)\xi f_\xi,$$

$$X^i_2 = -2mF + k_1 - \left(\frac{m+1}{2}\right)\xi F_\xi,$$

$$X^i_3 = \lambda,$$

$$X^i_4 = -\left(\frac{m+1}{2}\right)\xi F,$$

$$X^i_5 = N\lambda,$$

$$X^i_6 = -\left(\frac{m+1}{2}\right)\xi F F_\xi - m(\varepsilon^2 + F^2) + \varepsilon k_1;$$

$$Y^i_1 = \left(\frac{Pr}{1+R}\right)\left(\frac{m+1}{2}\right)\left[f + \xi f_\xi\right],$$

$$Y^i_2 = -nF\left(\frac{Pr}{1+R}\right),$$

$$Y_3^i = -\left(\frac{Pr}{1+R}\right)\left[\left(\frac{m+1}{2}\right)\xi G_\xi + nG\right],$$

$$Y_4^i = -\left(\frac{Pr}{1+R}\right)\left(\frac{m+1}{2}\right)\xi F,$$

$$Y_5^i = -\left(\frac{Pr}{1+R}\right)\left[\left(\frac{m+1}{2}\right)\xi F G_\xi + nFG\right];$$

$$Z_1^i = Sc\left[\left(\frac{m+1}{2}\right)f + \left(\frac{m+1}{2}\right)\xi f_\xi\right],$$

$$Z_2^i = -Sc\, n\, F,$$

$$Z_3^i = -Sc\left[\left(\frac{m+1}{2}\right)\xi H_\xi + nH\right],$$

$$Z_4^i = -Sc\left(\frac{m+1}{2}\right)\xi F,$$

$$Z_5^i = -Sc\left[\left(\frac{m+1}{2}\right)\xi F H_\xi + nFH\right].$$

At each iteration step, the set of linear partial differential equations (14)-(16) were expressed in difference form by using central difference in η-direction and backward difference in ξ-direction. Then the system of linear algebraic equations with a block tri-diagonal matrix is solved by using Varga's algorithm [14]. To ensure the convergence of the numerical solution to the exact solution, the step sizes $\Delta\eta$ and $\Delta\xi$ are optimized and taken as 0.01 and 0.005, respectively. The solution is assumed to have converged when the difference reaches less than 10^{-5}.

4. Results and Discussion

The effect of various physical parameters on the flow field are examined and discussed in this section. To check the accuracy of the result, the present solution is compared with the available particular solution in the literature for heat transfer parameter $-G_\eta(0)$ in Table 1.

Table 1. Comparison of heat transfer parameter $-G_\eta(0)$ for various values of Pr

Pr	0.7	1.0	2.0	7.0	10.0	100.0
Tsou et al. [2]	0.3492	0.4438	–	–	1.6804	5.545
Ali [3]	0.3476	0.4416	–	–	1.6713	–
Soundalgekar and Murty [4]	0.3508	–	0.6831	–	1.6808	–
Patil et al. [11]	0.35004	0.44401	0.68314	1.38625	1.68011	5.54610
Patil [12]	0.352215	0.444428	0.683204	1.386861	1.680150	5.547512
Present work	0.3520	0.4441	0.6831	1.3870	1.6803	5.5476

The effects of permeability parameter k_1 and temperature exponent parameter n on velocity profiles (F) in the boundary layer for $\xi = 1.0$, $Pr = 0.7$, $\lambda = 1.0$, $m = 0.0$, $A = 1.0$, $Sc = 0.60$, $N = 0.5$, $\varepsilon = 1.0$ and $R = 1.0$ are shown in Fig. 1. It is clearly observed that the velocity increases as the permeability parameter k_1 increases along the sheet. This is due to the fact that the porous medium produces a resistive type of force which leads to reduction and increase in the fluid velocity. Also, the momentum boundary layer thickness increases for linear stretched surface temperature ($n = 1$) while they decrease for uniform surface temperature ($n = 0$).

The effect of radiation parameter R on velocity and temperature profiles (F, G) for $k_1 = 0.5$, $Pr = 0.7$, $\lambda = 1.0$, $m = 0.0$, $n = 1.0$, $Sc = 0.60$, $N = 0.5$, $\varepsilon = 1.0$, $A = 1.0$ and $\xi = 1.0$ are shown in Figs. 2(a) and 2(b). It is observed that the velocity profile increases with increasing of radiation parameter (R). That is because the presence of the radiation yields an increase in acceleration of the fluid motion and hence an increase in the liquid velocity (see Fig. 2(a)). It can also be observed from Fig. 2(b) that in each point with the increase of the radiation parameter (R), the temperature rises. The physical reason is that due to the fact that a higher radiation parameter implies a larger surface heat flux which leads to increase the temperature of the fluid.

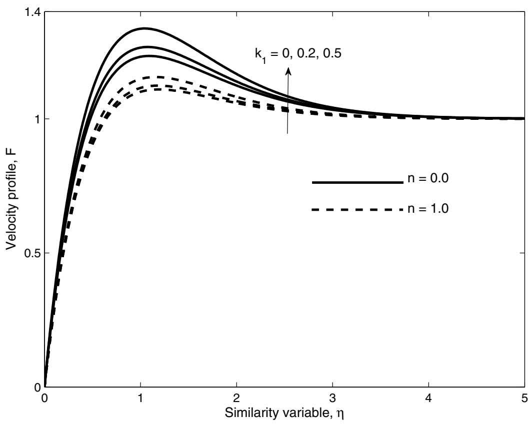

Fig. 1. Effects of permeability parameter k_1 and temperature exponent parameter n on velocity profiles

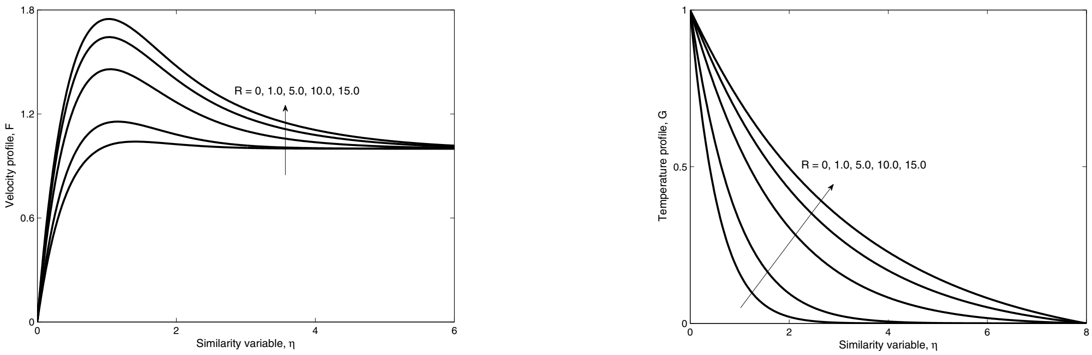

Fig. 2. Effect of radiation parameter R on (a) velocity and (b) temperature profiles

Figures 3(a) and 3(b) display the effects of streamwise co-ordinate ξ and surface mass transfer A on velocity and temperature profiles (F, G) for $k_1 = 0.1$, $Pr = 0.7$, $\lambda = 1.0$, $m = 0.0$, $n = 0.0$, $Sc = 0.22$, $N = 0.5$, $\varepsilon = 0.5$ and $R = 1.0$. It can be seen that increasing the suction parameter $(A > 0)$ will reduce the momentum and thermal boundary layer thickness for both the velocity and temperature profiles inside the boundary layer. It is noticed in Fig. 3(a), the velocity inside the boundary layer reaches a fixed value which is zero. However, increasing the injection parameter will increase both the velocity and temperature boundary layer thickness. This is because the buoyancy forces are not capable to heat the injected fluid any more. The velocity overshoot decreases with the increase of streamwise co-ordinate ξ. Due to increase in the streamwise co-ordinate, the velocity and thermal boundary layer thicknesses reduce near the wall of the sheet within the boundary layer.

The effect of radiation parameter R on skin friction and heat transfer coefficients ($C_{fx}(Re_x)^{1/2}$ and $Nu_x(Re_x)^{-1/2}$) are shown in Figs. 4(a) and 4(b), when $k_1 = 0.1$, $Pr = 0.7$, $\lambda = 1.0$, $m = 0.0$, $n = 1.0$, $N = 1.0$, $Sc = 0.60$, $\varepsilon = 1.0$ and $A = 1.0$. It can be seen that the skin friction coefficient increases with the increase of thermal radiation parameter (R) while it decreases for the heat transfer coefficient. It should be noted from Fig. 4(b), increase in the values of (R) have the tendency to increase the conduction effect and to increase the thermal boundary layer. This, in turn, causes the temperature to increase at every point away from the sheet surface. Since the wall slope of the temperature profile

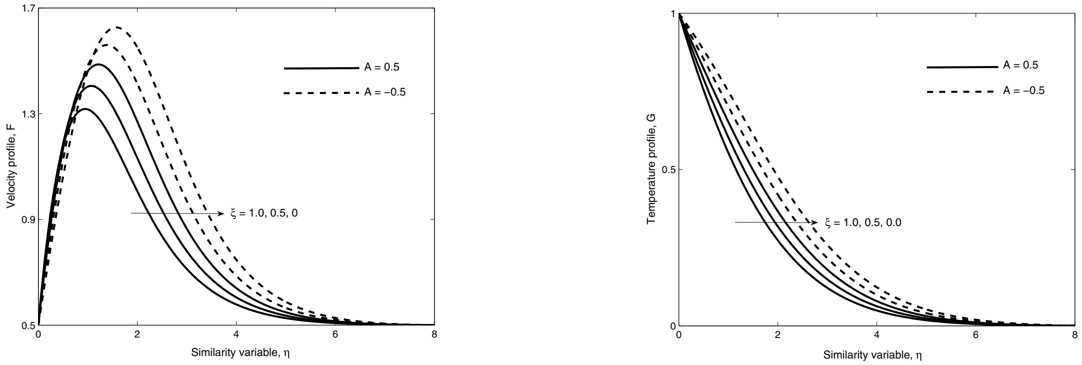

Fig. 3. Effects of surface mass transfer parameter A and streamwise co-ordinate ξ on (a) velocity and (b) temperature profiles

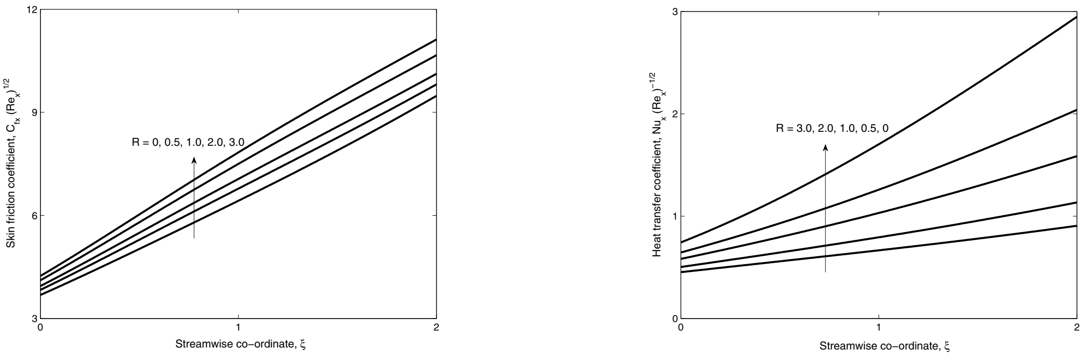

Fig. 4. Effect of radiation parameter R on (a) skin friction and (b) heat transfer coefficients

increases as (R) increases and the heat transfer coefficient decreases because it is proportional to the negative value of the wall slope of the temperature profile.

5. Conclusion

A numerical study is performed for the problem of thermal radiation embedded in porous medium over a stretching sheet in the presence of suction or injection. The present study indicates that, an increase in thermal radiation parameter increases the skin friction coefficient while it decreases heat transfer coefficient. The injection parameter ($A < 0$) tends to increase the magnitude of the velocity overshoot but suction parameter ($A > 0$) reduces the magnitude of the velocity overshoot. The influence of temperature exponent parameter (n) increases thermal boundary layer thickness for linear stretched surface ($n = 1$) while it decreases for uniform surface temperature ($n = 0$).

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