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Physics Letters B 665 (2008) 284–293

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Unfolded description of AdS_4 Kerr black hole

V.E. Didenko, A.S. Matveev, M.A. Vasiliev *

I.E. Tamm Department of Theoretical Physics, Lebedev Physical Institute, Leninsky prospect 53, 119991 Moscow, Russia

ARTICLE INFO

Article history:
Received 15 April 2008
Accepted 30 May 2008
Available online 6 June 2008
Editor: M. Cvetič

ABSTRACT

It is shown that AdS_4 Kerr black hole is a solution of simple unfolded differential equations that form a deformation of the zero-curvature description of empty AdS_4 space–time. Our construction uses the Killing symmetries of the Kerr solution. All known and some new algebraic properties of the Kerr–Schild solution result from the obtained black hole unfolded system in the coordinate-independent way. Kerr–Schild type solutions of free equations in AdS_4 for massless fields of any spin associated with the proposed black hole unfolded system are found.

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1. Introduction

Since its discovery in 1963, the celebrated Kerr metric [1] is still a hot research topic.[1] Being of great physical significance, it exhibits deep mathematical beauty. In particular, the Kerr–Schild form [3] of the Kerr metric illustrates the remarkable fact that Kerr black hole (BH), being an exact solution of Einstein equations, also verifies its first order fluctuational part, i.e., the second order terms vanish independently. In other words, Kerr BH solves free spin-two equations. This property suggests the relevance of perturbative study of corrections to Kerr-like solutions in such field-theoretical extensions of gravity as string theory or higher-spin (HS) gauge theory (see [4–8] for reviews). The Kerr–Schild form allowed Myers and Perry to generalize the four-dimensional Kerr metric to any dimension [9].

BHs with non-zero cosmological constant have also been extensively studied in the context of AdS/CFT correspondence [10–13] where the (anti-)de Sitter geometry plays a distinguished role. Although the generalization of the Kerr solution that describes a rotating BH in AdS_4 was discovered by Carter [14] soon after Kerr's paper [1], its higher-dimensional generalization has been found only recently. A generalization of the Myers–Perry solution with non-zero cosmological term in five dimensions has been given by Hawking, Hunter and Taylor-Robinson in [12] and then extended to any dimension by Gibbons, Lu, Page and Pope in [15]. These higher-dimensional BHs are shown [16] to possess the hidden symmetry associated with the Yano–Killing tensor [17] which is the characteristic feature of Petrov type D metrics.

Despite considerable progress in the construction of different BH metrics, including the charged rotating Kerr–Newman solution [18], the generalization of AdS_4 Taub-NUT to any dimension [19] and discovery of various supergravity BHs in the series of works by Chong, Cvetic, Gibbons, Lu, Pope [20,21], a number of problems remain open. In particular, a generalization of the well-known four-dimensional Kerr–Newman solution to charged rotating BH solution in any dimension is available neither in flat nor in $(A)dS$ case. This suggests that some more general approaches can be useful.

Usually an analysis in the BH background uses a particular coordinate system. The aim of this Letter is to reformulate the AdS_4 Kerr BH in a coordinate-independent way using the formalism of the unfolded dynamics [22,23] that operates with first-order covariant field equations. We show how the AdS_4 Kerr BH arises as a solution of the BH unfolded system (BHUS) associated with a certain BH Killing vector.

In general, unfolded equations generalize the Cartan–Maurer equations for Lie algebras by incorporating p-form gauge potentials. They provide a powerful tool for the study of partial differential equations. More precisely, consider, following [22], a set of differential p-forms $W^A(x)$ with $p \geqslant 0$ and generalized curvatures R^A defined as

$$R^A = dW^A + F^A(W), \tag{1.1}$$

* Corresponding author.
E-mail addresses: didenko@lpi.ru (V.E. Didenko), matveev@lpi.ru (A.S. Matveev), vasiliev@lpi.ru (M.A. Vasiliev).

[1] For the history of question we refer to the recent publication by Roy Patric Kerr [2].

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doi:10.1016/j.physletb.2008.05.067

where $d = dx^n \frac{\partial}{\partial x^n}$ is the space–time exterior differential and some functions F^A are built of wedge products of the differential forms W^B. The functions $F^A(W)$ subjected to the generalized Jacobi identity

$$F^B \frac{\delta F^A}{\delta W^B} = 0 \tag{1.2}$$

guarantee the generalized Bianchi identity $dR^A = R^B \frac{\delta F^A}{\delta W^B}$ which shows, in particular, that the differential equations for $W^A(x)$

$$R^A = 0 \tag{1.3}$$

are consistent. In this case Eqs. (1.3) are called unfolded. Note that every unfolded system is associated with some solution of the generalized Jacobi identity (1.2) on the functions $F^A(W)$ built of exterior products of the differential forms W^A which, in turn, defines a free differential algebra [24,25] (more precisely, its generalization to the case with zero-forms included).

The reformulation of partial differential equations in the unfolded form has a number of advantages. In particular, the first-order equations formulated in terms of exterior algebra are manifestly coordinate-independent and reconstruct a local solution in terms of its values at any given point of space–time modulo gauge invariance for p-forms with $p \geqslant 1$. (For the recent summary of the properties of unfolded dynamics we refer the reader to [26].)

As an example of the unfolded system consider the Killing equation in the AdS_4 space written in the local frame

$$DK_a = \kappa_{ba} h^b, \qquad D\kappa_{ab} = 2\lambda^2 K_{[a} h_{b]}, \qquad D^2 = R_{ab} = \lambda^2 h_a \wedge h_b, \tag{1.4}$$

where K_a is some Killing vector, $\kappa_{ab} = -\kappa_{ba}$, D is the Lorentz covariant differential, h_a is the AdS_4 vierbein one-form, $-\lambda^2$ is the cosmological constant, and $a, b = 0, \ldots, 3$. The AdS_4 geometry is described by the zero curvature equations

$$d\Omega_{ab} + \Omega_a{}^c \wedge \Omega_{cb} - \lambda^2 h_a \wedge h_b = 0, \tag{1.5}$$

$$dh_a + \Omega_a{}^c \wedge h_c = 0, \tag{1.6}$$

where Ω_{ab} is the AdS_4 Lorentz connection one-form. The first of Eqs. (1.4) is obviously equivalent to the AdS_4 Killing equation as it implies $D_a K_b + D_b K_a = 0$, while the second one follows as a consequence of Bianchi identities for the space of constant curvature. Eqs. (1.4)–(1.6) represent the unfolded system for the set of forms $W_A = (h_a, \Omega_{ab}, K_a, \kappa_{ab})$.

In this Letter we show that there exists a simple deformation of Eqs. (1.4)–(1.6) with the same set of forms that describes the AdS_4 Kerr BH in four dimensions with the BH mass being a free deformation parameter. Since the proposed BH unfolded system (BHUS) expresses all derivatives of fields in terms of the fields themselves, all invariant relationships on the derivatives of BH geometric quantities become direct consequences of BHUS. In particular, it manifestly expresses the Petrov type D BH Weyl tensor in terms of the Killing two-form.

The AdS_4 Killing two-form that results from the exterior differential of the one-form dual to the Killing vector plays the central role in our consideration. The integrability properties of the AdS_4 BHUS imply, in particular, the existence of the sourceless Maxwell tensor and Yano–Killing tensor, both being proportional to the Killing two-form. Let us stress that, even though the Killing two-form in AdS_4 has non-vanishing divergence, it remains related to the BH Weyl tensor and to the sourceless Maxwell field.

Although the obtained results may have different applications, the original motivation for this study was due to the search of the Kerr BH solution in the nonlinear HS gauge theory which itself is formulated in the unfolded form and AdS space (see, e.g., reviews [5,8] and references therein). The proposed formulation looks particularly promising in that respect. We show how the proposed BHUS makes it possible to generate Kerr–Schild like solutions of the free bosonic HS field equations for all spins $s = 0, 1, \ldots$ in AdS_4. Remarkably, for $s = 1$ and $s = 2$ the obtained solutions satisfy in addition the minimally coupled to gravity equations. For other spins the covariantization of Fronsdal equations [27] in the Kerr–Schild metric leads to certain nonlinear terms which are explicitly calculated.

The rest of the Letter is organized as follows. First, in Section 2 we recall the basic facts about the AdS_4 Kerr BH solution, its Kerr–Schild metric decomposition and its Killing symmetry. In Section 3 we reconsider the Cartan formalism most appropriate for our purpose. In Section 4 we present the BH unfolded system, build the null geodesic congruence using Killing projectors, find relations between the Killing two-form, Maxwell tensor and Yano–Killing tensor. In Section 5 we reproduce the Kerr solution from AdS_4 space via a Kerr–Schild type algebraic shift. Finally, in Section 6 we show how Kerr–Schild type solutions for free HS equations can be obtained. Section 7 contains summary and conclusions.

2. *AdS*$_4$ Kerr black hole

The matric of a rotating BH of mass M in AdS_4 admits the Kerr–Schild form[2] [14]

$$g_{mn}(X) = \eta_{mn}(X) + \frac{2M}{U(X)} k_m(X) k_n(X), \qquad g^{mn}(X) = \eta^{mn}(X) - \frac{2M}{U(X)} k^m(X) k^n(X), \tag{2.1}$$

where $\eta_{mn}(X)$ $(m, n = 0, \ldots, 3)$ is the background AdS_4 metric with negative cosmological constant $-\lambda^2$ and $k^m(X)$ defines the null geodesic congruence with respect to the both full metric $g_{mn}(X)$ and background one $\eta_{mn}(X)$

$$k^m k_m = 0, \qquad k^m \mathcal{D}_m k_n = k^m D_m k_n = 0. \tag{2.2}$$

Here $\mathcal{D}$ and D are the full and background covariant differentials, respectively.

[2] Throughout this Letter we use units with $8\pi G = 1$.

A useful coordinate system has the background metric of the form [15]

$$\eta^{mn} = \begin{pmatrix} \frac{1-a^2\lambda^2}{(1+r^2\lambda^2)(1-\lambda^2\frac{a^2z^2}{r^2})} & 0 & 0 & 0 \\ 0 & -1-\lambda^2(x^2-a^2) & -\lambda^2 xy & -\lambda^2 xz \\ 0 & -\lambda^2 xy & -1-\lambda^2(y^2-a^2) & -\lambda^2 yz \\ 0 & -\lambda^2 xz & -\lambda^2 yz & -1-\lambda^2 z^2 \end{pmatrix}, \tag{2.3}$$

where the radial coordinate $r(X)$ is defined through the ellipsoid of revolution equation

$$\frac{x^2+y^2}{r^2+a^2} + \frac{z^2}{r^2} = 1 \tag{2.4}$$

and a is a rotational parameter. The BH has angular momentum $J = Ma$.

Components of the Kerr–Schild vector $k^m(X)$ are

$$k^0 = \frac{1}{1+r^2\lambda^2}, \qquad k^1 = -\frac{xr-ay}{r^2+a^2}, \qquad k^2 = -\frac{yr+ax}{r^2+a^2}, \qquad k^3 = -\frac{z}{r}, \tag{2.5}$$

and

$$U(X) = r + \frac{a^2z^2}{r^3}. \tag{2.6}$$

Direct calculation gives

$$\frac{2}{U} = \frac{1}{Q} + \frac{1}{\bar{Q}} = -D_m k^m = -\mathcal{D}_m k^m, \tag{2.7}$$

where

$$Q = r - \frac{iaz}{r}, \qquad \bar{Q} = r + \frac{iaz}{r}. \tag{2.8}$$

Note that the metric (2.1) still provides a Kerr solution after the transformation

$$\tau(a, x, y, z, t) = (-a, -x, -y, -z, t). \tag{2.9}$$

In other words, the Kerr–Schild Ansatz also works for the vector $n^i = \tau(k^i)$.

One can check that the Maxwell tensor

$$F = dA^{(1)} = dA^{(2)}, \qquad A^{(1)}_m = \frac{k_m}{U}, \qquad A^{(2)}_m = \frac{n_m}{U}, \tag{2.10}$$

where $n_m = \eta_{mp} n^p$, verifies the sourceless Maxwell equations both in the AdS_4 and in the BH geometry

$$D_m F^m{}_n = \mathcal{D}_m F^m{}_n = 0. \tag{2.11}$$

The Kerr BH has two Killing vectors, namely the time translation $\mathcal{V}^m_t$ and rotation around z-axis $\mathcal{V}^m_\phi$,

$$\mathcal{V}^m_t = \frac{\partial}{\partial t} = (1,0,0,0), \qquad \mathcal{V}^m_\varphi = \frac{\partial}{\partial\varphi} = (0, y, -x, 0). \tag{2.12}$$

Let us introduce a Killing two-form $\kappa = dK$ as the exterior derivative of the one-form $dx^m K_m$ dual to some Killing vector K^m. This two-form will also be called Papapetrou field. It was originally introduced in [28], where it was shown to give rise to the sourceless Maxwell tensor in Ricci flat manifolds with isometries.

A particular linear combination $\mathcal{V}^m$ of the Killing vectors (2.12)

$$\mathcal{V}^m = \left(1, a\lambda^2 y, -a\lambda^2 x, 0\right) \tag{2.13}$$

that satisfies the condition $k_m\mathcal{V}^m = 1$, will be used later on for the definition of the Killing two-form associated with a Kerr BH in AdS_4.

3. Cartan formalism

Let $dx^m\,\mathbf{\Omega}_m{}^{ab}$ be an antisymmetric Lorentz connection one-form and $dx^m\,\mathbf{h}_m{}^a$ be a vierbein one-form. These can be identified with the gauge fields of the AdS_4 symmetry algebra $o(3,2)$. The corresponding AdS_4 curvatures have the form

$$\mathbf{R}^{ab} = d\mathbf{\Omega}^{ab} + \mathbf{\Omega}^{ac}\wedge\mathbf{\Omega}_c{}^b - \lambda^2\mathbf{h}^a\wedge\mathbf{h}^b, \tag{3.1}$$

$$\mathbf{R}^a = d\mathbf{h}^a + \mathbf{\Omega}^{ac}\wedge\mathbf{h}_c, \tag{3.2}$$

where $a, b, c = 0, \ldots, 3$ are Lorentz indices.

The zero-torsion condition $\mathbf{R}^a = 0$ expresses algebraically the Lorentz connection $\mathbf{\Omega}$ via derivatives of $\mathbf{h}$. Then the λ-independent part of the curvature two-form (3.1) identifies with the Riemann tensor. Einstein equations imply that the Ricci tensor vanishes up to a constant trace part proportional to the cosmological constant. In other words, only those components of the tensor (3.1) may remain non-vanishing on-shell that belong to the Weyl tensor

$$\mathbf{R}_{ab} = \frac{1}{2}\mathbf{h}^c\wedge\mathbf{h}^d C_{cdab}, \tag{3.3}$$

where C_{abcd} is the Weyl tensor in the local frame, $C_{abcd} = -C_{bacd} = -C_{abdc} = C_{cdab}$.

The analysis in four dimensions considerably simplifies in spinor notation. Vector notation is translated to the spinor one and vice versa with the help of Pauli σ-matrices. For example, for a Lorentz vector U_a we have

$$U_{\alpha\dot{\alpha}} = (\sigma^a)_{\alpha\dot{\alpha}} U_a, \qquad U_a = \frac{1}{2}(\sigma_a)^{\alpha\dot{\alpha}} U_{\alpha\dot{\alpha}}, \tag{3.4}$$

where $\alpha, \dot{\alpha} = 1, 2$. Spinor indices are raised and lowered by the $sp(2)$ antisymmetric tensors $\varepsilon_{\alpha\beta}$ and $\varepsilon_{\dot{\alpha}\dot{\beta}}$

$$\xi_\alpha = \xi^\beta \varepsilon_{\beta\alpha}, \qquad \xi^\alpha = \varepsilon^{\alpha\beta}\xi_\beta, \qquad \bar{\xi}_{\dot{\alpha}} = \bar{\xi}^{\dot{\beta}}\varepsilon_{\dot{\beta}\dot{\alpha}}, \qquad \bar{\xi}^{\dot{\alpha}} = \varepsilon^{\dot{\alpha}\dot{\beta}}\bar{\xi}_{\dot{\beta}}, \tag{3.5}$$

where $\varepsilon_{12} = \varepsilon^{12} = 1$, $\varepsilon_{\alpha\beta} = -\varepsilon_{\beta\alpha}$, $\varepsilon^{\alpha\beta} = -\varepsilon^{\beta\alpha}$.

Lorentz irreducible spinor decompositions of the Maxwell and Weyl tensors F_{ab} and C_{abcd} read, respectively, as

$$F_{\alpha\dot{\alpha}\beta\dot{\beta}} = \varepsilon_{\alpha\beta}\bar{F}_{\dot{\alpha}\dot{\beta}} + \varepsilon_{\dot{\alpha}\dot{\beta}}F_{\alpha\beta}, \qquad C_{\alpha\dot{\alpha}\beta\dot{\beta}\gamma\dot{\gamma}\delta\dot{\delta}} = \varepsilon_{\alpha\beta}\varepsilon_{\gamma\delta}\bar{C}_{\dot{\alpha}\dot{\beta}\dot{\gamma}\dot{\delta}} + \varepsilon_{\dot{\alpha}\dot{\beta}}\varepsilon_{\dot{\gamma}\dot{\delta}}C_{\alpha\beta\gamma\delta}, \tag{3.6}$$

where[3] $F_{\alpha\beta}$, $C_{\alpha\beta\gamma\delta}$ and their conjugates are totally symmetric multispinors.

To describe Killing symmetries let us rewrite the Cartan equations (3.1), (3.2) in the spinor notation. Lorentz connection one-forms $\mathbf{\Omega}_{\alpha\alpha}, \bar{\mathbf{\Omega}}_{\dot{\alpha}\dot{\alpha}}$ and vierbein one-form $\mathbf{h}_{\alpha\dot{\alpha}}$ can be identified with the gauge fields of $sp(4) \sim o(3,2)$. Vacuum Einstein equations with cosmological constant acquire the form

$$\mathcal{R}_{\alpha\alpha} = d\mathbf{\Omega}_{\alpha\alpha} + \mathbf{\Omega}_\alpha{}^\gamma \wedge \mathbf{\Omega}_{\gamma\alpha} = \frac{\lambda^2}{2}\mathbf{H}_{\alpha\alpha} + \frac{1}{8}\mathbf{H}^{\gamma\gamma}C_{\gamma\gamma\alpha\alpha}, \tag{3.7}$$

$$\bar{\mathcal{R}}_{\dot{\alpha}\dot{\alpha}} = d\bar{\mathbf{\Omega}}_{\dot{\alpha}\dot{\alpha}} + \bar{\mathbf{\Omega}}_{\dot{\alpha}}{}^{\dot{\gamma}} \wedge \bar{\mathbf{\Omega}}_{\dot{\gamma}\dot{\alpha}} = \frac{\lambda^2}{2}\bar{\mathbf{H}}_{\dot{\alpha}\dot{\alpha}} + \frac{1}{8}\bar{\mathbf{H}}^{\dot{\gamma}\dot{\gamma}}\bar{C}_{\dot{\gamma}\dot{\gamma}\dot{\alpha}\dot{\alpha}}, \tag{3.8}$$

$$\mathcal{R}_{\alpha\dot{\alpha}} = d\mathbf{h}_{\alpha\dot{\alpha}} + \frac{1}{2}\mathbf{\Omega}_\alpha{}^\gamma \wedge \mathbf{h}_{\gamma\dot{\alpha}} + \frac{1}{2}\bar{\mathbf{\Omega}}_{\dot{\alpha}}{}^{\dot{\gamma}} \wedge \mathbf{h}_{\alpha\dot{\gamma}} = 0, \tag{3.9}$$

where $\mathcal{R}_{\alpha\beta}$, $\bar{\mathcal{R}}_{\dot{\alpha}\dot{\beta}}$ are the components of the Lorentz curvature two-form

$$\mathcal{D}^2\xi_{\alpha\dot{\alpha}} = \frac{1}{2}\mathcal{R}_\alpha{}^\beta\xi_{\beta\dot{\alpha}} + \frac{1}{2}\bar{\mathcal{R}}_{\dot{\alpha}}{}^{\dot{\beta}}\xi_{\alpha\dot{\beta}} \tag{3.10}$$

and

$$\mathbf{H}^{\alpha\alpha} = \mathbf{h}^\alpha{}_{\dot{\alpha}} \wedge \mathbf{h}^{\alpha\dot{\alpha}}, \qquad \bar{\mathbf{H}}^{\dot{\alpha}\dot{\alpha}} = \mathbf{h}_\alpha{}^{\dot{\alpha}} \wedge \mathbf{h}^{\alpha\dot{\alpha}}. \tag{3.11}$$

The Killing equation is

$$\mathcal{D}\mathcal{V}_{\alpha\dot{\alpha}} = \frac{1}{2}\mathbf{h}^\gamma{}_{\dot{\alpha}}\kappa_{\gamma\alpha} + \frac{1}{2}\mathbf{h}_\alpha{}^{\dot{\gamma}}\bar{\kappa}_{\dot{\gamma}\dot{\alpha}}, \tag{3.12}$$

where $\kappa_{\alpha\alpha}$ and $\bar{\kappa}_{\dot{\alpha}\dot{\alpha}}$ just represent non-zero components of first derivatives of the Killing vector $\mathcal{V}_{\alpha\dot{\alpha}}$. In vector notation this gives $\mathcal{D}_m\mathcal{V}_n = \kappa_{mn} = -\kappa_{nm}$, hence leading to the Killing equation $\mathcal{D}_{(m}\mathcal{V}_{n)} = 0$.

Differentiation of (3.12) with the help of (3.7), (3.8) and (3.10) yields

$$\mathcal{D}\kappa_{\alpha\alpha} = \lambda^2\mathbf{h}_\alpha{}^{\dot{\gamma}}\mathcal{V}_{\alpha\dot{\gamma}} + \frac{1}{4}\mathbf{h}^{\beta\dot{\beta}}\mathcal{V}^\beta{}_{\dot{\beta}}C_{\beta\beta\alpha\alpha}, \tag{3.13}$$

$$\mathcal{D}\bar{\kappa}_{\dot{\alpha}\dot{\alpha}} = \lambda^2\mathbf{h}^\gamma{}_{\dot{\alpha}}\mathcal{V}_{\gamma\dot{\alpha}} + \frac{1}{4}\mathbf{h}^{\beta\dot{\beta}}\mathcal{V}_\beta{}^{\dot{\beta}}\bar{C}_{\dot{\beta}\dot{\beta}\dot{\alpha}\dot{\alpha}} \tag{3.14}$$

along with compatibility conditions $\mathcal{D}^{\beta\dot{\alpha}}C_{\beta\alpha\alpha\alpha} = 0$, $\mathcal{D}^{\alpha\dot{\beta}}\bar{C}_{\dot{\beta}\dot{\alpha}\dot{\alpha}\dot{\alpha}} = 0$. In case $C_{\alpha\alpha\alpha\alpha} = 0$, $\bar{C}_{\dot{\alpha}\dot{\alpha}\dot{\alpha}\dot{\alpha}} = 0$ Eqs. (3.12)–(3.14) describe an isometry of AdS_4.

4. Black hole unfolded equations

4.1. Consistency condition for Killing unfolded system

Let us investigate solutions of Einstein equations with negative cosmological constant and the Weyl tensor of the form

$$C_{\alpha\alpha\alpha\alpha} = f(X)\kappa_{\alpha\alpha}\kappa_{\alpha\alpha}, \tag{4.1}$$

where $f(X)$ is some function of space–time coordinates X, and $\kappa_{\alpha\alpha}$ is a Killing two-form corresponding to some Killing vector $\mathcal{V}_{\alpha\dot{\alpha}}$. So, we assume at least one isometry. The Weyl tensor (4.1) is of the type D by Petrov classification [29].

From the Killing equations (3.12)–(3.14) we derive the following unfolded equations

[3] The symmetrization over denoted by the same letter spinor indices is implied.

$$\mathcal{D}\mathcal{V}_{\alpha\dot\alpha} = \frac{1}{2}\mathbf{h}^{\gamma}{}_{\dot\alpha}\kappa_{\gamma\alpha} + \frac{1}{2}\mathbf{h}_{\alpha}{}^{\dot\gamma}\bar\kappa_{\dot\gamma\dot\alpha}, \tag{4.2}$$

$$\mathcal{D}\kappa_{\alpha\alpha} = \lambda^2\mathbf{h}_{\alpha}{}^{\dot\gamma}\mathcal{V}_{\alpha\dot\gamma} + \frac{f}{4}\mathbf{h}^{\beta\dot\beta}\mathcal{V}^{\beta}{}_{\dot\beta}\kappa_{\beta\beta\alpha\alpha}, \tag{4.3}$$

$$\mathcal{D}\bar\kappa_{\dot\alpha\dot\alpha} = \lambda^2\mathbf{h}^{\gamma}{}_{\dot\alpha}\mathcal{V}_{\gamma\dot\alpha} + \frac{\bar f}{4}\mathbf{h}^{\beta\dot\beta}\mathcal{V}_{\beta}{}^{\dot\beta}\bar\kappa_{\dot\beta\dot\beta\dot\alpha\dot\alpha}, \tag{4.4}$$

$$\mathcal{R}_{\alpha\alpha} = \frac{\lambda^2}{2}\mathbf{H}_{\alpha\alpha} + \frac{f}{8}\mathbf{H}^{\beta\beta}\kappa_{\beta\beta\alpha\alpha}, \tag{4.5}$$

$$\bar{\mathcal{R}}_{\dot\alpha\dot\alpha} = \frac{\lambda^2}{2}\bar{\mathbf{H}}_{\dot\alpha\dot\alpha} + \frac{\bar f}{8}\bar{\mathbf{H}}^{\dot\beta\dot\beta}\bar\kappa_{\dot\beta\dot\beta\dot\alpha\dot\alpha}, \tag{4.6}$$

$$\mathcal{D}\mathbf{h}_{\alpha\dot\alpha} = 0, \tag{4.7}$$

$$df = -\left(\frac{1}{12}f^2 + \frac{5f\lambda^2}{2\kappa^2}\right)\mathbf{h}^{\alpha\dot\gamma}\mathcal{V}^{\alpha}{}_{\dot\gamma}\kappa_{\alpha\alpha}, \tag{4.8}$$

$$d\bar f = -\left(\frac{1}{12}\bar f^2 + \frac{5\bar f\lambda^2}{2\bar\kappa^2}\right)\mathbf{h}^{\gamma\dot\alpha}\mathcal{V}_{\gamma}{}^{\dot\alpha}\bar\kappa_{\dot\alpha\dot\alpha}, \tag{4.9}$$

where $\mathbf{H}^{\alpha\alpha}$ and $\bar{\mathbf{H}}^{\dot\alpha\dot\alpha}$ are defined in (3.11), $\mathcal{R}_{\alpha\alpha}$ and $\bar{\mathcal{R}}_{\dot\alpha\dot\alpha}$ are the curvatures (3.7) and (3.8), and we use notations $\kappa_{\alpha\beta}\kappa^{\beta}{}_{\gamma} = \kappa^2\varepsilon_{\alpha\gamma}$, $\kappa_{\alpha\alpha\alpha\alpha} = \kappa_{\alpha\alpha}\kappa_{\alpha\alpha}$.

The system (4.2)–(4.9) has the unfolded form (1.3) and is formally consistent. We call it BH unfolded system (BHUS). It is invariant under the following transformation

$$\tau_\mu\colon\quad (\mathcal{V}_{\alpha\dot\alpha}, \kappa_{\alpha\alpha}, \bar\kappa_{\dot\alpha\dot\alpha}, f, \bar f, \mathbf{h}_{\alpha\dot\alpha}, \boldsymbol{\Omega}_{\alpha\alpha}, \boldsymbol{\Omega}_{\dot\alpha\dot\alpha}) \to \big(\mu\mathcal{V}_{\alpha\dot\alpha}, \mu\kappa_{\alpha\alpha}, \mu\bar\kappa_{\dot\alpha\dot\alpha}, \mu^{-2}f, \mu^{-2}\bar f, \mathbf{h}_{\alpha\dot\alpha}, \boldsymbol{\Omega}_{\alpha\alpha}, \boldsymbol{\Omega}_{\dot\alpha\dot\alpha}\big), \tag{4.10}$$

where μ is a real parameter. Another symmetry of the system (4.2)–(4.9) is the parity transform

$$\rho\colon\quad (\mathcal{V}_{\alpha\dot\alpha}, \kappa_{\alpha\alpha}, \bar\kappa_{\dot\alpha\dot\alpha}, f, \bar f, \mathbf{h}_{\alpha\dot\alpha}, \boldsymbol{\Omega}_{\alpha\alpha}, \boldsymbol{\Omega}_{\dot\alpha\dot\alpha}) \to (-\mathcal{V}_{\alpha\dot\alpha}, \kappa_{\alpha\alpha}, \bar\kappa_{\dot\alpha\dot\alpha}, f, \bar f, -\mathbf{h}_{\alpha\dot\alpha}, \boldsymbol{\Omega}_{\alpha\alpha}, \boldsymbol{\Omega}_{\dot\alpha\dot\alpha}). \tag{4.11}$$

As we show below, the τ-symmetry transformation (2.9) is a composition of (4.10) and (4.11), $\tau = \tau_{-1}\circ\rho$.

Eqs. (4.8) and (4.9) result from the Bianchi identities for the curvatures (4.5) and (4.6). It can be shown that $f(X) = f(\kappa^2)$ as a consequence of (4.3). Indeed, we have

$$d\kappa^2 = \left(\lambda^2 + \frac{1}{3}f\kappa^2\right)\mathbf{h}^{\alpha\dot\alpha}\mathcal{V}^{\alpha}{}_{\dot\alpha}\kappa_{\alpha\alpha}. \tag{4.12}$$

Comparing (4.8) and (4.12), we obtain

$$\frac{d\kappa^2}{2\lambda^2 + \frac{2}{3}f\kappa^2} + \frac{df}{\frac{1}{6}f^2 + 5\lambda^2\frac{f}{\kappa^2}} = 0. \tag{4.13}$$

Its general solution is

$$f = 6\mathcal{M}\frac{\mathcal{G}^3}{\kappa^2}, \tag{4.14}$$

where the complex parameter $\mathcal{M}$ appears as an integration constant and $\mathcal{G}$ is defined implicitly by

$$\mathcal{M}\mathcal{G}^3 - \mathcal{G}\sqrt{-\kappa^2} = \lambda^2 \tag{4.15}$$

and satisfies

$$d\mathcal{G} = -\frac{\mathcal{G}^2}{2\sqrt{-\kappa^2}}\mathbf{h}^{\alpha\dot\alpha}\mathcal{V}^{\alpha}{}_{\dot\alpha}\kappa_{\alpha\alpha}. \tag{4.16}$$

Analogously,

$$\bar f = 6\bar{\mathcal{M}}\frac{\bar{\mathcal{G}}^3}{\bar\kappa^2}, \qquad \bar{\mathcal{M}}\bar{\mathcal{G}}^3 - \bar{\mathcal{G}}\sqrt{-\bar\kappa^2} = \lambda^2. \tag{4.17}$$

4.2. Killing projectors

Let two pairs of mutually conjugated projectors $\Pi^{\pm}_{\alpha\beta}$ and $\bar\Pi^{\pm}_{\dot\alpha\dot\beta}$ have the form

$$\Pi^{\pm}_{\alpha\beta} = \frac{1}{2}\left(\epsilon_{\alpha\beta} \pm \frac{1}{\sqrt{-\kappa^2}}\kappa_{\alpha\beta}\right), \qquad \bar\Pi^{\pm}_{\dot\alpha\dot\beta} = \frac{1}{2}\left(\epsilon_{\dot\alpha\dot\beta} \pm \frac{1}{\sqrt{-\bar\kappa^2}}\bar\kappa_{\dot\alpha\dot\beta}\right). \tag{4.18}$$

They satisfy

$$\Pi^{\pm}_{\alpha}{}^{\beta}\Pi^{\pm}_{\beta\gamma} = \Pi^{\pm}_{\alpha\gamma}, \qquad \Pi^{\pm}_{\alpha}{}^{\beta}\Pi^{\mp}_{\beta\gamma} = 0, \qquad \bar\Pi^{\pm}_{\dot\alpha}{}^{\dot\beta}\bar\Pi^{\pm}_{\dot\beta\dot\gamma} = \bar\Pi^{\pm}_{\dot\alpha\dot\gamma}, \qquad \bar\Pi^{\pm}_{\dot\alpha}{}^{\dot\beta}\bar\Pi^{\mp}_{\dot\beta\dot\gamma} = 0. \tag{4.19}$$

From the definition (4.18) it follows that

$$\Pi^{\pm}_{\alpha\beta} = -\Pi^{\mp}_{\beta\alpha}, \qquad \bar\Pi^{\pm}_{\dot\alpha\dot\beta} = -\bar\Pi^{\mp}_{\dot\beta\dot\alpha}. \tag{4.20}$$

Hereinafter we will focus on the holomorphic (i.e., undotted) sector of the BHUS. All relations in the antiholomorphic sector result by conjugation.

From (4.18), (4.2) and (4.3) it follows that

$$\mathcal{D}\Pi^{\pm}_{\alpha\beta} = \pm\frac{\mathcal{G}}{2}\left(\Pi^{+}_{\alpha\gamma}\Pi^{+}_{\beta\gamma} + \Pi^{-}_{\alpha\gamma}\Pi^{-}_{\beta\gamma}\right)\mathcal{V}^{\gamma}{}_{\dot\gamma}\mathbf{h}^{\gamma\dot\gamma}. \tag{4.21}$$

The projectors (4.18) split the two-dimensional (anti)holomorphic spinor space into the direct sum of two one-dimensional subspaces. For any ξ_α we set

$$\xi^{\pm}_{\alpha} = \Pi^{\pm}_{\alpha}{}^{\beta}\xi_\beta, \qquad \xi^{+}_{\alpha} + \xi^{-}_{\alpha} = \xi_\alpha, \tag{4.22}$$

so that $\Pi^{\mp}_{\alpha}{}^{\beta}\xi^{\pm}_{\beta} = 0$. This allows us to build light-like vectors with the aid of projectors. Indeed, consider an arbitrary vector $U_{\alpha\dot\alpha}$. Using (4.18) define $U^{\pm}_{\alpha\dot\alpha}$ and $U^{\pm\mp}_{\alpha\dot\alpha}$ as

$$U^{\pm}_{\alpha\dot\alpha} = \Pi^{\pm}_{\alpha}{}^{\beta}\bar\Pi^{\pm}_{\dot\alpha}{}^{\dot\beta}U_{\beta\dot\beta}, \qquad U^{+-}_{\alpha\dot\alpha} = \Pi^{+}_{\alpha}{}^{\beta}\bar\Pi^{-}_{\dot\alpha}{}^{\dot\beta}U_{\beta\dot\beta}, \qquad U^{-+}_{\alpha\dot\alpha} = \Pi^{-}_{\alpha}{}^{\beta}\bar\Pi^{+}_{\dot\alpha}{}^{\dot\beta}U_{\beta\dot\beta}. \tag{4.23}$$

Obviously, $U^{\pm}_{\alpha\dot\beta}U^{\pm\alpha\dot\gamma} = 0$ and $U^{\pm}_{\alpha\dot\alpha}U^{\pm\beta\dot\alpha} = 0$. Then $U^{-}_{\alpha\dot\alpha}$ can be cast into the form

$$U^{-}_{\alpha\dot\alpha} = \psi_\alpha\bar\zeta_{\dot\alpha}, \tag{4.24}$$

from where it follows that

$$U^{\pm}_{\alpha\dot\beta}U^{\pm}_{\beta\dot\alpha} = U^{\pm}_{\alpha\dot\alpha}U^{\pm}_{\beta\dot\beta}, \qquad U^{-+}_{\alpha\dot\beta}U^{+-}_{\beta\dot\alpha} = -\frac{(U^{-+}U^{+-})}{(U^{-}U^{+})}U^{-}_{\alpha\dot\alpha}U^{+}_{\beta\dot\beta}, \tag{4.25}$$

where $(AB) = A_{\alpha\dot\alpha}B^{\alpha\dot\alpha}$.

4.3. Kerr–Schild vector in BH unfolded system

Let us identify $\kappa_{\alpha\alpha}$ in (4.18) with $\kappa_{\alpha\alpha}$ in the BHUS and introduce two null vectors

$$k_{\alpha\dot\alpha} = \frac{2}{(\mathcal{V}^{-}\mathcal{V}^{+})}\mathcal{V}^{-}_{\alpha\dot\alpha}, \qquad n_{\alpha\dot\alpha} = \frac{2}{(\mathcal{V}^{-}\mathcal{V}^{+})}\mathcal{V}^{+}_{\alpha\dot\alpha} \tag{4.26}$$

with the evident property $\frac{1}{2}k_{\alpha\dot\alpha}\mathcal{V}^{\alpha\dot\alpha} = \frac{1}{2}n_{\alpha\dot\alpha}\mathcal{V}^{\alpha\dot\alpha} = 1$. It is a matter of definition which of two vectors $k_{\alpha\dot\alpha}$ or $n_{\alpha\dot\alpha}$ to identify with the Kerr–Schild vector. Indeed, Eqs. (4.2)–(4.9) are invariant under the symmetry τ_{-1} (4.10) which acts on $k_{\alpha\dot\alpha}$ as $\tau_{-1}(k_{\alpha\dot\alpha}) = -n_{\alpha\dot\alpha}$. Let us choose $k_{\alpha\dot\alpha}$ as the Kerr–Schild vector.

Obviously,

$$k_{\alpha\dot\alpha}k^{\beta\dot\alpha} = 0. \tag{4.27}$$

From BHUS and the projector properties, the geodesity condition follows by straightforward calculation

$$k^{\alpha\dot\alpha}\mathcal{D}_{\alpha\dot\alpha}k_{\beta\dot\beta} = 0. \tag{4.28}$$

In addition, $k_{\alpha\dot\alpha}$ is the eigenvector of the Papapetrou field $\kappa_{\alpha\alpha}$

$$\kappa_{\alpha\beta}k^{\beta}{}_{\dot\alpha} = \sqrt{-\kappa^2}\,k_{\alpha\dot\alpha} \tag{4.29}$$

and has the following properties as a consequence of (4.2), (4.3) and (4.16)

$$\mathcal{D}_{\alpha\dot\alpha}k^{\alpha\dot\alpha} = -2(\mathcal{G}+\bar{\mathcal{G}}), \qquad k^{\alpha\dot\alpha}\mathcal{D}_{\alpha\dot\alpha}\mathcal{G} = 2\mathcal{G}^2, \tag{4.30}$$

$$\mathcal{D}_{\alpha\dot\alpha}\left((\mathcal{G}+\bar{\mathcal{G}})k^{\alpha\dot\alpha}\right) = -4\mathcal{G}\bar{\mathcal{G}}, \qquad \mathcal{D}_{\alpha\dot\alpha}\left(\mathcal{G}\bar{\mathcal{G}}k^{\alpha\dot\alpha}\right) = 0, \tag{4.31}$$

$$k_{\alpha}{}^{\dot\alpha}\mathcal{D}_{\alpha\dot\alpha}k_{\gamma\dot\gamma} = \mathcal{G}k_{\alpha}{}^{\dot\alpha}\mathcal{V}_{\gamma\dot\alpha}k_{\alpha\dot\gamma}. \tag{4.32}$$

The BHUS (4.2)–(4.9) admits a sourceless Maxwell tensor. Indeed, consider $F_{\alpha\alpha}$ of the form

$$F_{\alpha\alpha} = \frac{\mathcal{G}^2}{\sqrt{-\kappa^2}}\kappa_{\alpha\alpha}. \tag{4.33}$$

That (4.33) solves Maxwell equations can be easily verified as follows. Using (4.15), (4.26) and (4.2), (4.3) one can make sure that

$$F_{\alpha\alpha} = \frac{1}{2}\mathcal{D}_{\alpha\dot\alpha}\left((\mathcal{G}+\bar{\mathcal{G}})k_{\alpha}{}^{\dot\alpha}\right) \tag{4.34}$$

and $F_{\alpha\alpha}$ is τ_{-1}-invariant. In other words, the vector-potential $A_m = \frac{1}{2}(\mathcal{G}+\bar{\mathcal{G}})k_m$ gives the Maxwell tensor field $F = dA$.

Due to (4.33), the Weyl tensor (4.1) can be rewritten in the form

$$C_{\alpha\alpha\alpha\alpha} = -\frac{6\mathcal{M}}{\mathcal{G}}F_{\alpha\alpha}F_{\alpha\alpha}. \tag{4.35}$$

The differentiation of (4.33) with the aid of (4.3) yields

$$\mathcal{M}\mathcal{D}F_{\alpha\alpha} = \frac{1}{4}\mathbf{h}^{\beta\dot\beta}\mathcal{V}^{\beta}{}_{\dot\beta}C_{\beta\beta\alpha\alpha}. \tag{4.36}$$

The Maxwell equations

$$\mathcal{D}_{\gamma\dot\alpha}F_\alpha{}^\gamma=0,\qquad \mathcal{D}_{\alpha\dot\gamma}\bar F_{\dot\alpha}{}^{\dot\gamma}=0 \tag{4.37}$$

are now simple consequences of (4.36).

The Maxwell tensor (4.33) is related to Killing–Yano tensor via

$$Y_{\alpha\alpha}=\frac{i}{\mathcal{G}^3}F_{\alpha\alpha}. \tag{4.38}$$

Indeed, (4.38) satisfies

$$\mathcal{D}_{\alpha\dot\alpha}Y_{\alpha\alpha}=0,\qquad \mathcal{D}_{\beta\dot\alpha}Y^\beta{}_\alpha+\mathcal{D}_{\alpha\dot\beta}\bar Y^{\dot\beta}{}_{\dot\alpha}=0, \tag{4.39}$$

which implies the Yano–Killing equation [17] $\mathcal{D}_{(k}Y_{m)n}=0$ for $Y_{mn}=-Y_{nm}$.

As follows from (4.14), (4.15), (4.33) and (4.38), $Y_{\alpha\alpha}$ and $F_{\alpha\alpha}$ constitute the Papapetrou two-form

$$\kappa_{\alpha\alpha}=\mathcal{M}F_{\alpha\alpha}+i\lambda^2Y_{\alpha\alpha}=\left(\mathcal{M}-\frac{\lambda^2}{\mathcal{G}^3}\right)F_{\alpha\alpha}. \tag{4.40}$$

5. Kerr black hole unfolded system from *AdS*$_4$

As a preparation to the description of the Kerr–Schild Ansatz in BHUS, let us reformulate the AdS_4 geometry in spinor notation. Eqs. (1.4)–(1.6) are equivalent to the following special case of the BHUS (4.2)–(4.9) with $f=0$ that describes the vacuum solution with vanishing Weyl tensor, i.e., empty AdS_4 space

$$DV_{\alpha\dot\alpha}=\frac12 h^\gamma{}_{\dot\alpha}\kappa_{0\gamma\alpha}+\frac12 h_\alpha{}^{\dot\gamma}\bar\kappa_{0\dot\alpha\dot\gamma}, \tag{5.1}$$

$$D\kappa_{0\alpha\alpha}=\lambda^2h_\alpha{}^{\dot\gamma}V_{\alpha\dot\gamma}, \tag{5.2}$$

$$D\bar\kappa_{0\dot\alpha\dot\alpha}=\lambda^2h^\gamma{}_{\dot\alpha}V_{\gamma\dot\alpha}, \tag{5.3}$$

$$Dh_{\alpha\dot\alpha}=0, \tag{5.4}$$

$$R_{\alpha\alpha}=d\Omega_{\alpha\alpha}+\Omega_\alpha{}^\beta\wedge\Omega_{\beta\alpha}=\frac{\lambda^2}{2}h_{\alpha\dot\alpha}\wedge h_\alpha{}^{\dot\alpha}, \tag{5.5}$$

$$\bar R_{\dot\alpha\dot\alpha}=d\bar\Omega_{\dot\alpha\dot\alpha}+\bar\Omega_{\dot\alpha}{}^{\dot\beta}\wedge\bar\Omega_{\dot\beta\dot\alpha}=\frac{\lambda^2}{2}h_{\alpha\dot\alpha}\wedge h^\alpha{}_{\dot\alpha}, \tag{5.6}$$

where $h_{\alpha\dot\alpha}$ is the AdS_4 vierbein, $\Omega_{\alpha\alpha}$ and $\bar\Omega_{\dot\alpha\dot\alpha}$ are components of Lorentz connection, D is the background Lorentz differential, $V_{\alpha\dot\alpha}$ is an AdS_4 Killing vector and $R_{\alpha\alpha}$, $\bar R_{\dot\alpha\dot\alpha}$ are the components of AdS_4 curvature two-form: $D^2\xi_{\alpha\dot\alpha}=\frac12 R_\alpha{}^\beta\xi_{\beta\dot\alpha}+\frac12\bar R_{\dot\alpha}{}^{\dot\beta}\xi_{\alpha\dot\beta}$.

The AdS_4 Killing projectors $\Pi^\pm_{0\alpha\beta}$ single out the null vector $k_{0\alpha\dot\alpha}$ that defines AdS_4 null geodesic congruence

$$k_0^{\alpha\dot\alpha}D_{\alpha\dot\alpha}k_{0\beta\dot\beta}=0, \tag{5.7}$$

where

$$k_{0\alpha\dot\alpha}=\frac{2}{(V^-V^+)}V^-_{\alpha\dot\alpha}. \tag{5.8}$$

Analogously, the Maxwell field generated by the null vector $k_{0\alpha\dot\alpha}$ is defined by

$$F_{0\alpha\alpha}=-\lambda^{-2}\mathcal{G}_0^3\kappa_{0\alpha\alpha}, \tag{5.9}$$

with

$$\mathcal{G}_0=-\frac{\lambda^2}{\sqrt{-\kappa_0^2}}. \tag{5.10}$$

Now we are in a position to show that the AdS_4 Kerr BH is a solution of BHUS (4.2)–(4.9) resulting at real $\mathcal{M}$ from some algebraic field redefinition

$$(\Omega_{\alpha\alpha},\bar\Omega_{\dot\alpha\dot\alpha},h_{\alpha\dot\alpha},V_{\alpha\dot\alpha},\kappa_{0\alpha\alpha})\to(\boldsymbol{\Omega}_{\alpha\alpha},\bar{\boldsymbol{\Omega}}_{\dot\alpha\dot\alpha},\mathbf{h}_{\alpha\dot\alpha},\mathcal{V}_{\alpha\dot\alpha},\kappa_{\alpha\alpha}), \tag{5.11}$$

that, in fact, extends the Kerr–Schild Ansatz to the full BHUS, expressing the AdS_4 Kerr BH entirely in terms of the AdS_4 background unfolded system.

Let the sets of fields $(\boldsymbol{\Omega}_{\alpha\alpha},\bar{\boldsymbol{\Omega}}_{\dot\alpha\dot\alpha},\mathbf{h}_{\alpha\dot\alpha},\mathcal{V}_{\alpha\dot\alpha},\kappa_{\alpha\alpha})$ and $(\Omega_{\alpha\alpha},\bar\Omega_{\dot\alpha\dot\alpha},h_{\alpha\dot\alpha},V_{\alpha\dot\alpha},\kappa_{0\alpha\alpha})$ be the Lorentz connections, vierbeins, Killing vectors and Papapetrou fields of the BHUS (4.2)–(4.9) and of the vacuum equations (5.1)–(5.6), respectively. The explicit map between these two unfolded systems is

$$\boldsymbol{\Omega}_{\alpha\dot\alpha|\gamma\gamma}=\Omega_{\alpha\dot\alpha|\gamma\gamma}-\frac{\mathcal{M}\mathcal{G}_0^2}{\sqrt{-\kappa_0^2}}\kappa_{0\gamma\gamma}k_{0\alpha\dot\alpha}-\frac{\mathcal{M}\mathcal{G}_0}{2}(\mathcal{G}_0+\bar{\mathcal{G}}_0)k_{0\gamma}{}^{\dot\gamma}V_{\alpha\dot\gamma}k_{0\gamma\dot\alpha}, \tag{5.12}$$

$$\mathbf{h}_{\alpha\dot\alpha}=h_{\alpha\dot\alpha}+\frac{\mathcal{M}}{4}(\mathcal{G}_0+\bar{\mathcal{G}}_0)h^{\beta\dot\beta}k_{0\beta\dot\beta}k_{0\alpha\dot\alpha}, \tag{5.13}$$

$$\mathcal{V}_{\alpha\dot\alpha} = V_{\alpha\dot\alpha} + \frac{\mathcal{M}}{2}(\mathcal{G}_0 + \bar{\mathcal{G}}_0)k_{0\alpha\dot\alpha}, \tag{5.14}$$

$$\kappa_{\alpha\alpha} = \left(1 - \mathcal{M}\lambda^{-2}\mathcal{G}_0^3\right)\kappa_{0\alpha\alpha}. \tag{5.15}$$

As a consequence of (5.13), the metric has the Kerr–Schild form

$$g_{mn} = \eta_{mn} + \mathcal{M}(\mathcal{G}_0 + \bar{\mathcal{G}}_0)k_{0m}k_{0n}. \tag{5.16}$$

Another consequence is the invariance of the null congruence (5.8) and function $\mathcal{G}_0$ under the deformation

$$\mathcal{G}_0 = \mathcal{G}, \qquad k_{0\alpha\dot\alpha} = k_{\alpha\dot\alpha}. \tag{5.17}$$

Note that (5.17) along with (4.34) entail the equality $F_{0\alpha\alpha} = F_{\alpha\alpha}$ and thus determine the Weyl tensor (4.35) in terms of AdS_4 geometry and some its Killing vector.

Let us sketch the main steps of the derivation of the relations (5.12)–(5.15) in some more detail. To see that (5.12)–(5.15) indeed relate two unfolded systems and to simplify the subsequent analysis, we start with the vacuum system (5.1)–(5.6) and suppose that the deformation is geodesic, i.e., (5.11) leaves the Kerr–Schild vector and the function $\mathcal{G}_0$ invariant (5.17), checking this afterwards.

Let us look for the Kerr–Schild deformation of the background vierbein field

$$\mathbf{h}_{\alpha\dot\alpha} = h_{\alpha\dot\alpha} + \frac{\mathcal{M}}{4}(\mathcal{G} + \bar{\mathcal{G}})h^{\beta\dot\beta}k_{\beta\dot\beta}k_{\alpha\dot\alpha}. \tag{5.18}$$

Imposing the zero-torsion condition $\mathcal{D}\mathbf{h}_{\alpha\dot\alpha} = 0$ on the deformed vierbein (5.18), one obtains the deformed Lorentz connection $\boldsymbol{\Omega}_{\alpha\alpha} = \Omega_{\alpha\alpha} + \omega_{\alpha\alpha}$ with

$$\omega_{\alpha\dot\alpha|\gamma\gamma} = -\mathbf{h}^i{}_\gamma{}^{\dot\gamma} D_{\gamma\dot\gamma}\mathbf{h}_{i\alpha\dot\alpha}. \tag{5.19}$$

It follows then that

$$\omega_{\alpha\dot\alpha|\gamma\gamma} = -\frac{\mathcal{M}}{2}D_{\gamma\dot\gamma}\left((\mathcal{G} + \bar{\mathcal{G}})k_\gamma{}^{\dot\gamma}k_{\alpha\dot\alpha}\right). \tag{5.20}$$

Using (4.27) and (4.28) one observes that the background Lorentz derivative D in (5.20) can be replaced by the deformed one $\mathcal{D}$. Using (4.2), (4.3) and (4.16) we obtain (5.12).

Now it is straightforward to check that the Weyl tensor admits the following representation

$$C_{\alpha\alpha\alpha\alpha} = \mathcal{D}_{\alpha\dot\alpha}\omega_\alpha{}^{\dot\alpha}{}_{|\alpha\alpha}. \tag{5.21}$$

Note that here $\mathcal{D}$ can again be replaced by D, i.e., the terms quadratic in ω cancel.

The substitution of $h_{\alpha\dot\alpha}$ (5.18) and $\Omega_{\alpha\alpha} = \boldsymbol{\Omega}_{\alpha\alpha} - \omega_{\alpha\alpha}$ into the AdS_4 unfolded system (5.1)–(5.6) yields BHUS (4.2)–(4.9) with the Killing vector and the Killing two-form transformed according to (5.14) and (5.15). Finally, it is not hard to verify that (5.17) is consistent with (5.14) and (5.15).

Thus, the BHUS that describes the AdS_4 BH geometry with non-trivial Weyl tensor results from the algebraic field redefinition of the AdS_4 vacuum vierbein and connection, providing Kerr–Schild type vacuum solution of Einstein equations.

The identification with the standard BH description of Section 2 requires

$$\mathcal{G} = \frac{1}{Q}, \qquad \mathcal{M} = M, \tag{5.22}$$

with Q (2.8) and the Killing vector $\mathcal{V}^i$ (2.13). Then the Kerr–Schild vector k^i determined from (5.8) coincides with (2.5), whereas n^i, which arises from (5.1)–(5.6) with the aid of the discrete symmetry $\tau_{-1}\circ\rho$, coincides with the one from Section 2. Moreover, the metric (5.16) is identical to standard Kerr–Schild representation (2.1).

The Schwarzschild case with $a = 0$ is singled out by the condition

$$\mathcal{V}^{+-}_{\alpha\dot\alpha} = \mathcal{V}^{-+}_{\alpha\dot\alpha} = 0. \tag{5.23}$$

6. Black hole massless fields

Let us show how the BHUS reproduces solutions to free massless field equations in AdS_4 for all integer spins via a Kerr–Schild type algebraic field redefinition. Here we assume $\mathcal{M}$ to be real.

Consider the traceless symmetric tensor

$$\varphi_{mm} = \frac{1}{2}(\mathcal{G} + \bar{\mathcal{G}})k_m k_m. \tag{6.1}$$

Taking into account (4.2), (4.3) and (4.5), the straightforward calculation yields

$$\mathcal{D}^n\mathcal{D}_n\varphi_{mm} - 2\mathcal{D}^n\mathcal{D}_m\varphi_{mn} = -6\lambda^2\varphi_{mm}. \tag{6.2}$$

In this way we obtain Einstein equations for the Kerr–Schild decomposition (5.16), or spin-2 free field equations. Note that one can use D instead of $\mathcal{D}$ in (6.2).

The Maxwell case is analogous. It is a simple consequence of (4.34) that the vector field $\varphi_m = \frac{1}{2}(\mathcal{G} + \bar{\mathcal{G}})k_m$ satisfies (4.37).

Things change for the scalar $\varphi = \frac{1}{2}(\mathcal{G} + \bar{\mathcal{G}})$. Using (4.2), (4.3), (4.16) and (4.37) one finds

$$\mathcal{D}^{\alpha\dot\alpha}\mathcal{D}_{\alpha\dot\alpha}\mathcal{G} = -4\mathcal{G}^2\sqrt{-\kappa^2}. \tag{6.3}$$

The substitution of (4.15) yields

$$\mathcal{D}^{\alpha\dot{\alpha}}\mathcal{D}_{\alpha\dot{\alpha}}\mathcal{G} = 4\lambda^2\mathcal{G} - 4\mathcal{M}\mathcal{G}^4. \tag{6.4}$$

Thus, it reduces to $D^{\alpha\dot{\alpha}}D_{\alpha\dot{\alpha}}\varphi = 4\lambda^2\varphi$ only in the AdS_4 limit with $\mathcal{M} = 0$, $\mathcal{D} = D$. This equation describes propagation of a massless scalar in AdS_4. Using differential properties of $k_{\alpha\dot{\alpha}}$, one can obtain

$$\mathcal{D}_m\big(k^m k^n \mathcal{D}_n(\mathcal{G} + \bar{\mathcal{G}})^2\big) = 4\big(\mathcal{G}^4 + \bar{\mathcal{G}}^4\big). \tag{6.5}$$

Hence, from (6.4) it follows

$$\mathcal{D}^m\mathcal{D}_m\varphi = 2\lambda^2\varphi - 2\mathcal{M}\mathcal{D}_m\big(\varphi^{mn}\mathcal{D}_n\varphi\big). \tag{6.6}$$

The second term on the r.h.s. represents the nonlinear correction to a massless scalar propagating in the Kerr–Schild background. Thus, the fields $\varphi, \varphi_m, \varphi_{mm}$ verify the free massless equations in AdS_4 space for spins 0,1 and 2, respectively, [27,30]. Using BHUS one can show that[4]

$$\varphi_{m(s)} = \frac{1}{2}(\mathcal{G} + \bar{\mathcal{G}})k_m \cdots k_m \tag{6.7}$$

gives a Kerr–Schild solution for a massless integer spin-s equation in AdS_4 background

$$D^n D_n \varphi_{m(s)} - s D_n D_m \varphi^n{}_{m(s-1)} = -2(s-1)(s+1)\lambda^2\varphi_{m(s)}. \tag{6.8}$$

In the Kerr–Schild background (6.8) changes to

$$\mathcal{D}^n\mathcal{D}_n\varphi_{m(s)} - s\mathcal{D}_n\mathcal{D}_m\varphi^n{}_{m(s-1)} = -2(s-1)(s+1)\lambda^2\varphi_{m(s)} - \mathcal{M}(s-1)(s-2)\mathcal{D}_n\big(\varphi^{nr}\mathcal{D}_r\varphi_{m(s)}\big). \tag{6.9}$$

Note that the interaction term in (6.9) vanishes only for $s = 1$ and $s = 2$.

This result suggests that the Kerr–Schild Ansatz should admit an extension to the nonlinear equations of 4*d* massless fields of all spins (see [5] and references therein).

7. Summary and discussion

We have shown that AdS_4 Kerr BH admits a simple description in terms of unfolded field equations generated by a Killing vector of the background AdS_4 space. We considered the case of four dimensions using the spinor formalism.

Our aim was to find the description of the AdS_4 Kerr BH that does not refer to a particular coordinate system. Such a description is given in terms of a coordinate-independent unfolded system of differential equations that encodes all properties of the AdS_4 Kerr BH.

The proposed approach allowed us to show how seemingly different structures resided in the 4*d* BH such as, e.g., the existence of Yano–Killing tensor, the decomposition of the curvature tensor into Maxwell tensor and the Papapetrou field, naturally arise from the AdS_4 geometry with a distinguished Killing vector. An interesting direction for the further study is to explore the BH unfolded system on its own right, including, in particular, the case of complex deformation parameter $\mathcal{M}$ which, as we expect, should correspond to the Taub-NUT case. More generally, such analysis can provide a powerful technique for identification of BH solutions via algebraic properties of the corresponding solutions.

We believe that the proposed construction allows an extension to the nonlinear HS field equations also formulated in the unfolded form (see [5,7,8] for reviews). It is worth to note that the proposed BH unfolded system allows us to build solutions not only to linearized Einstein equations which reduce in this case to Pauli–Fierz equations but also to Fronsdal equations of all massless integer spins propagating in AdS_4. This fact looks particularly encouraging from the HS gauge theory perspective. Indeed, this implies that AdS_4 Kerr BH naturally fits linearized HS gauge theory through an algebraic field redefinition of the vacuum solution. The analysis of the BH solution in the nonlinear HS gauge theory is, however, beyond the scope of this Letter and will be given elsewhere.

Acknowledgements

This research was supported in part by INTAS Grants Nos. 03-51-6346 and 05-7928, RFBR Grant No. 05-02-17654, LSS No. 4401.2006.2. M.V. acknowledges partial support from the Alexander von Humboldt Foundation Grant PHYS0167. A.M. acknowledges financial support from Landau Scholarship Foundation and from Dynasty Foundation.

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