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Numerical solution of instability phenomenon arising in double phase flow through inclined homogeneous porous media☆

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Summary In the petroleum reservoir at an early stage the oil is recovered due to existing natural pressure and such type of oil recovery is referred as primary oil recovery. It ends when pressure equilibrium occurs and still large amount of oil remains in the reservoir. Consequently, secondary oil recovery process is employed by injection water into some injection wells to push oil towards the production well. The instability phenomenon arises during secondary oil recovery process. When water is injected into the oil filled region, due to the force of injecting water and difference in viscosities of water and native oil, protuberances occur at the common interface. It gives rise to the shape of fingers (protuberances) at common interface. The injected water shoots through inter connected capillaries at very high speed. It appears in the form of irregular trembling fingers, filled with injected water in the native oil field; this is due to the immiscibility of water and oil. The homogeneous porous medium is considered with a small inclination with the horizontal, the basic parameters porosity and permeability remain uniform throughout the porous medium. Based on the mass conservation principle and important Darcy's law under the specific standard relationships and basic assumptions considered, the governing equation yields a non-linear partial differential equation. The Crank—Nicolson finite difference scheme is developed and on implementing the boundary conditions the resulting finite difference scheme is implemented to obtain the numerical results. The numerical results are obtained by generating a MATLAB code for the saturation of water which decreases with the space variable and increases with time. The obtained numerical solution is efficient, accurate, and reliable, matches well with the physical phenomenon.

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Introduction

In primary oil recovery only 12—16% of oil is recovered due to the natural pressure without any pumping efforts at the wells. The part of the remaining oil is recovered by secondary oil recovery process, in which usually water is injected into oil formatted homogeneous porous medium to drive oil and consequently, oil is produced through production wells. The moment water is injected into the oil filled region protuberances occur at the common interface due to the force of injecting water and difference in viscosities of water and native oil. It gives rise to the shape of fingers (protuberances) at common interface. It appears in the irregular shape of trembling fingers in the oil field, due to the immiscibility of water and oil, therefore it is called fingering or instability phenomenon. In the statistical treatment of fingers, only the average behaviour of the two fluids involved is taken into consideration (Scheidegger and Johnson, 1961). The present study numerically describes the instability (fingering) phenomenon in double phase flow of two immiscible fluids (water and oil) through inclined homogeneous porous medium. An extensive study on this phenomenon with different view point is available in the standard literature (Babchin et al., 2008; Brailovsky et al., 2006; Chapwanya and Stockie, 2010; Scheidegger and Johnson, 1961).

Mathematical formulation

The seepage velocities of injected water V_w and native oil V_o are expressed by Darcy's law as

$$V_w = -\frac{k_w}{\delta_w} K \left(\frac{\partial P_w}{\partial x} + \rho_w g \sin\theta \right),$$

$$V_o = -\frac{k_o}{\delta_o} K \left(\frac{\partial P_o}{\partial x} + \rho_o g \sin\theta \right) \tag{1}$$

In Eq. (1) the relative permeabilities k_w and k_o are the functions of water saturation. The permeability K is assumed to be constant, the porous medium being homogenous. P_w, δ_w, ρ_w are the pressure, kinematic viscosity and density of water respectively whereas P_o, δ_o, ρ_o are the pressure, kinematic viscosity and density of oil respectively. θ is the angle of inclination of the considered homogeneous porous medium. The injected water and displaced oil also satisfy the equation of continuity for their constant phase densities as

$$\phi \frac{\partial S_w}{\partial t} + \frac{\partial V_w}{\partial x} = 0, \quad \phi \frac{\partial S_o}{\partial t} + \frac{\partial V_0}{\partial x} = 0 \tag{2}$$

where, ϕ is the porosity of the homogeneous porous medium.

Substituting the value of the seepage velocities V_w and V_o from Eq. (1) into Eq. (2), applying standard relation $P_c(S_w) = P_o - P_w$ and then eliminating $\frac{\partial P_w}{\partial x}$, from (2) we obtain,

$$\phi \frac{\partial S_w}{\partial t} = \frac{\partial}{\partial x} \left(K \frac{k_w}{\delta_w} \left(\frac{\partial P_o}{\partial x} - \frac{\partial P_c}{\partial x} \right) \right) + \rho_w g \sin\theta \frac{\partial}{\partial x} \left(\frac{k_w}{\delta_w} K \right) \tag{3}$$

Using the relation $S_w + S_o = 1$, we have simplified form as

$$\frac{\partial}{\partial x} \left(K \left(\frac{k_w}{\delta_w} + \frac{k_o}{\delta_o} \right) \frac{\partial P_o}{\partial x} - K \frac{k_w}{\delta_w} \left(\frac{\partial P_c}{\partial x} \right) \right) + \rho_w g \sin\theta \frac{\partial}{\partial x} \left(\frac{k_w}{\delta_w} K \right) + \rho_o g \sin\theta \frac{\partial}{\partial x} \left(\frac{k_o}{\delta_o} K \right) = 0 \tag{4}$$

Integrating Eq. (4) with respect to x and simplifying we have,

$$\phi \frac{\partial S_w}{\partial t} + \frac{1}{2} \frac{\partial}{\partial x} \left(K \frac{k_w}{\delta_w} \frac{\partial P_c}{\partial S_w} \frac{\partial S_w}{\partial x} \right) = \rho_w g \sin\theta \frac{\partial}{\partial x} \left(K \cdot \frac{k_w}{\delta_w} \right) \tag{5}$$

following Scheidegger and Johnson approximation (Scheidegger and Johnson, 1961).

Using the standard relationships for relative permeability and capillary pressure (Brailovsky et al., 2006; Chapwanya and Stockie, 2010; Scheidegger and Johnson, 1961), the resulting governing equation for the instability phenomenon in homogeneous porous medium inclined at a small angle is given by,

$$\frac{\partial S_w}{\partial T} = \frac{\partial}{\partial X} \left(S_w \frac{\partial S_w}{\partial X} \right) + C \frac{\partial S_w}{\partial X} \tag{6}$$

where, $X = x/L$, $T = K\beta t/(2\delta_w L^2 \phi)$ are the dimensionless variables and $C = 2L\rho_w g \sin\theta/\beta$.

The following initial and boundary conditions are considered:

$$S_w(X,0) = 0, \quad 0 \le X \le 1$$
$$S_w(0,T) = 1, \quad T > 0 \tag{7}$$
$$S_w(1,T) = 0, \quad T > 0$$

Numerical solution

The resulting finite difference scheme obtained for (6) is given by (8) and (8.1), for $2 \le i \le (R-1)$, where, $r = \Delta T/(\Delta X)^2$. For $i = 1$ and $i = R$ the resulting schemes can be obtained after implementing the boundary conditions from (7) (Von Rosenberg, 1969).

$$\left[S_{w_{i,n+1/2}} - \frac{1}{4}\left(S_{w_{i+1,n+1/2}} - S_{w_{i-1,n+1/2}} \right) - \frac{C}{4\Delta X} \right] S_{w_{i-1,n+1}} + \left[-2S_{w_{i,n+1/2}} - \frac{2}{r} \right] S_{w_{i,n+1}} + \left[S_{w_{i,n+1/2}} + \frac{1}{4}\left(S_{w_{i+1,n+1/2}} - S_{w_{i-1,n+1/2}} \right) + \frac{C}{4\Delta X} \right] S_{w_{i+1,n+1}}$$
$$= -\left[S_{w_{i,n+1/2}} - \frac{1}{4}\left(S_{w_{i+1,n+1/2}} - S_{w_{i-1,n+1/2}} \right) - \frac{C}{4\Delta X} \right] S_{w_{i-1,n}} + \left[2S_{w_{i,n+1/2}} - \frac{2}{r} \right] S_{w_{i,n}} - \left[S_{w_{i,n+1/2}} + \frac{1}{4}\left(S_{w_{i+1,n+1/2}} - S_{w_{i-1,n+1/2}} \right) + \frac{C}{4\Delta X} \right] S_{w_{i+1,n}} \tag{8}$$

with

$$S_{w_{i,n+1/2}} = S_{w_{i,n}} + \frac{r}{2} \left[S_{w_{i,n}} \left(S_{w_{i+1,n}} - 2S_{w_{i,n}} + S_{w_{i-1,n}} \right) + \frac{1}{4} \left(S_{w_{i+1,n}} - S_{w_{i-1,n}} \right)^2 + \frac{C\Delta X}{2} \left(S_{w_{i+1,n}} - S_{w_{i-1,n}} \right) \right] \tag{8.1}$$

Table 1 Saturation of water for instability phenomenon in inclined homogeneous porous medium for $C = 0.0001$.

	$T = 0.001$	$T = 0.10$	$T = 0.20$	$T = 0.30$
$X = 0$	1	1	1	1
$X = 0.1$	0.0022	0.8751	0.9120	0.9280
$X = 0.2$	0.0000	0.7091	0.7977	0.8356
$X = 0.3$	0.0000	0.5274	0.6749	0.7374
$X = 0.4$	0.0000	0.3335	0.5439	0.6334
$X = 0.5$	0.0000	0.1385	0.4056	0.5240
$X = 0.6$	0.0000	0.0029	0.2623	0.4095
$X = 0.7$	0.0000	0.0000	0.1202	0.2911
$X = 0.8$	0.0000	0.0000	0.0087	0.1715
$X = 0.9$	0.0000	0.0000	0.0000	0.0585
$X = 1$	0	0	0	0

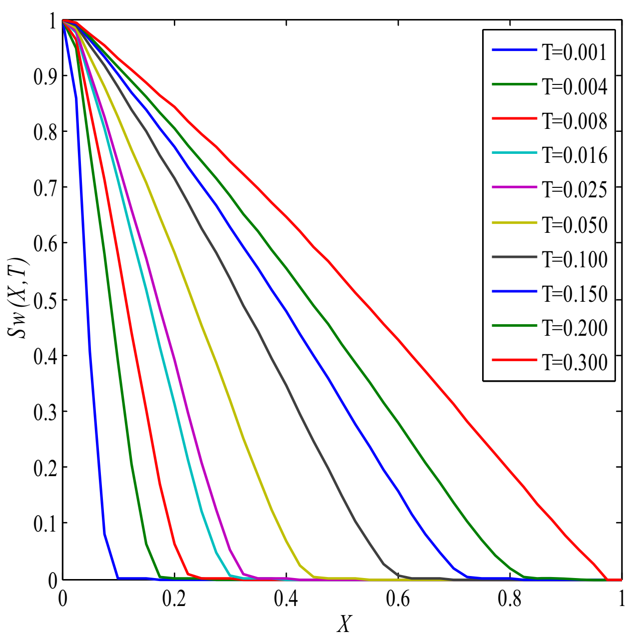

Figure 1 Water saturation profile for instability phenomenon in inclined homogeneous porous media for various values of time for $C = 0.0001$.

The numerical results for the above Crank—Nicolson finite difference scheme for the nonlinear Eq. (6) governing the instability phenomenon in homogeneous porous media with small inclination is obtained by generating a MATLAB code.

Results and discussion

The numerical results for the saturation of water are shown in Table 1 and its graphical representation is shown in Fig. 1. It can be seen that the saturation of water advances with time and decreases with the space variable. This shows that the physical fact of the problem is preserved. The numerical values are shown in Table 1 for which the numerical results behave well with problem. It was observed that for $T > 0.3125$, the numerical results, does not behave well. With the suitable choice of the parameters involved in the problem the corresponding value of time with respective dimensionless number T can be calculated and the sensitivity of parameters can be equally studied.

Conclusion

In the present paper the governing non-linear equation representing the instability phenomenon in homogeneous porous medium with a small inclination is solved by using Crank—Nicolson scheme. The precise set of initial and boundary conditions have been employed to derive numerical solution. The obtained results behave well with the physical phenomena of the problem. It was observed that the saturation of water increases with time and decreases with the space variable. This shows that water advances with the time and the oil recovery factor can be calculated with respect to obtained results. The sensitivity of parameters can be equally studied and the present method can be conveniently applied to other non-linear equations subject to specific initial and boundary conditions.

References

Babchin, A., Brailovsky, I., Gordon, P., Sivashinsky, G., 2008. On fingering instabilities in immiscible displacement. Phys. Rev. E. (03), 77.

Brailovsky, I., Babchin, A., Frankle, M., Sivashinsky, G., 2006. Instability in water-oil displacement. Transport Porous Media (63), 380—393.

Chapwanya, M., Stockie, J., 2010. Numerical simulations of gravity-driven fingering in unsaturated porous media using a non-equilibrium model. Water Resour. Res. (46), 1—15.

Scheidegger, A., Johnson, E., 1961. The statistical behaviour of instabilities in displacement process in porous media. Can. J. Phys. (39), 326—334.

Von Rosenberg, 1969. Methods for the Numerical Solution of Partial Differential Equations. American Elsevier Publishing Company Inc., New York, pp. 1—67.