Numerical wave interaction with tetrapods breakwater

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ABSTRACT: The paper provides some results of a new procedure to analyze the hydrodynamic aspects of the interactions between maritime emerged breakwaters and waves by integrating CAD and CFD. The structure is modeled in the numerical domain by overlapping individual three-dimensional elements (Tetrapods), very much like the real world or physical laboratory testing. Flow of the fluid within the interstices among concrete blocks is evaluated by integrating the RANS equations. The aim is to investigate the reliability of this approach as a design tool. Therefore, for the results' validation, the numerical run-up and reflection effects on virtual breakwater were compared with some empirical formulae and some similar laboratory tests. Here are presented the results of a first simple validation procedure. The validation shows that, at present, this innovative approach can be used in the breakwater design phase for comparison between several design solutions with a significant minor cost.

KEY WORDS: Volume of Fluid (VOF); Wave; Run up; Reflection; Rubble mound; Numerical simulations; Tetrapod; Flow 3D®; RANS equations.

INTRODUCTION

Coastal structures, and specially so rock mound breakwaters, are normally designed by using well proven formulas and by laboratory scale tests.

Recently 2D and 3D numerical simulation of Navier Stokes equation has been developed to the point that it can now be used as an affordable design tool to substitute or supplement tank experiments. The references on this topic are far too many to be examined in detail here, however it may be useful to recall some interesting examples of how these issues have been addressed both physically and numerically (developed by Lin and Liu, 1998; Giarrusso et al., 2003; Losada et al., 2008).

Mounds of rock or concrete blocks however still seem to defy the computational possibilities, due to the geometrical and hydrodynamical complexity of the problem. Water flows through the paths of the interstices within the blocks featuring strongly non stationary flow, free boundaries, turbulence, interaction with solid transport and complex geometry (Koutandos et al., 2006a; 2006b).

The currently used approach assumes that within the rubble mound the flow can be treated by using a classical “porous
media” methodology, i.e. by using, within the rubble mound, the equations that treat the filtration motion (Darcy or Forchheimer, if the head loss is linear or quadratic respectively).

In practice, an additional term is added to the equations to reproduce the interactions between the fluid and the inner flow paths using homogeneous coefficients for the entire filtration domain.

Such an approach was reported in Hsu et al. (2002), later implemented in the COBRAS numerical code and finally perfected by Lara et al. (2006).

The results obtained through these types of modeling, while certainly more reliable compared to the waterproof block model, present a number of drawbacks. First of all, this approach overlooks the convective aspects of the flow and the structure of turbulence; it is heavily reliant on the numerical parameters of the filtration equations and therefore requires a careful empirical calibration.

Only recently serious attempts have been made to model the detailed hydrodynamics of block mound structures on the basis of their real geometry by using advanced digital techniques: by using a fine computational grid, an adequate number of computational nodes is located within the interstices so that a complete solution of the full hydrodynamic equations is carried out, thus including convective effects and possibly also resolving the turbulence structure. All these aspects cannot be taken into account with the classical porous media approach, inadequate in such kind of situations.

Pioneering work with full simulation of such Flow Within the Armour Units (FWAU) method was carried out by using RANS-VOF, (Cavallaro et al., 2012; Dentale et al., 2012; 2013; 2014); Smoothed Particle Hydrodynamics (SPH) was applied to this problem by Altomare et al. (2012), while a somewhat similar approach involving CFD techniques in the interstices and numerical solid mechanics in the block themselves, is being attempted by Xiang et al. (2012).

The aim of the present work is to introduce a validation of the procedure against classical empirical formulas and physical tests for a structure with rubble mound in Tetrapods and to show how it is already a useful design tool in some complex configurations.

PROCEDURE

The innovative approach, should in principle be three-dimensional since the geometrical structure of the interstices among the blocks has inherently a very complex spatial structure; some successful attempts have indeed been made by the Authors in (Dentale et al., 2009) to develop equivalent 2D schemes, but they have not been followed in the present work.

Numerical reconstructions of the breakwater are thus produced by using a CAD software system for modeling 3D geometries; a data base of artificial blocks such as the cube, the modified cube, the Tetrapod, the Seabees (Brown and Dentale, 2013), the AccropodeTM and the Xblocons, has preliminarily been produced, while also natural rocks can be reproduced either by using spheres of various diameters or by randomly shaped blocks.

Breakwaters, both submerged and emerged, are numerically reconstructed by overlapping individual blocks under the conditions of gravity, collision and friction, according to the real geometry, very much like in the case of real constructions or laboratory test model.

In this case a classical breakwater is reconstructed by a CAD software with the following scheme:

- A waterproof core;
- A filter layer in natural stones;
- A toe protection in natural stones;
- An armour layer in Tetrapod reconstructed in respect of collision, gravity and interlocking forces.

Once the breakwaters geometry is defined (Fig. 1), its geometric configuration is imported into the CFD system. FLOW-3D® (Flow Science Inc.) was used for all calculations, like many other CFD systems employed for similar tasks, FLOW-3D® is based on the Reynolds Averaged Navier-Stokes (RANS) equations combined with the Volume of Fluid (VOF) method to apply the proper dynamic boundary conditions and to track the location of the fluid surfaces (Hirt and Nichols, 1981).
The flow is described by the general Navier-Stokes equations:

$$\frac{\partial u_i}{\partial x_i} = 0$$  \hspace{1cm} (1)

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho} \frac{\partial p}{\partial x_i} + \nu \frac{\partial^2 u_i}{\partial x_i \partial x_j} + g_i$$  \hspace{1cm} (2)

where $\nu$ is the molecular viscosity, $u_i$ is the $i$ th component of the instantaneous velocity in the pores, $p$ the instantaneous effective pressure and $g_i$ the $i$ th component of the gravitational force. Various turbulence models are available.

It has been thoroughly tested for coastal hydrodynamics problems, as shown in Chopakatla et al. (2008), Dentale et al. (2012, 2013, 2014), FLOW-3D®, as well as other RANS/VOF software systems, also incorporates a numerical procedure to define general geometric regions within rectangular grids, as it is essential for the construction of the breakwater block geometry. The turbulence model associated to the RANS equations is RNG for all simulations presented in this study.

A numerical wave flume was set up in order to carry out the numerical experiments described in the following; its cross section - as shown in Fig. 2 - is rather conventional, based as it is on typical experimental arrangements; its length is 170 m in x direction, 4.5 m in y direction and 18m in z direction. The water depth (d) in quiet conditions is 6 m.

The computational domain is divided into two sub-domains (Fig. 3): in a typical test case, after appropriate convergence tests, the mesh 1 (general mesh) for all the computations was chosen to be made up of 243.000 cells, $0.50 \times 0.50 \times 0.20$ m, while the local one (mesh 2) was 3.240.000 cells, $0.10 \times 0.10 \times 0.10$ m.

The computational burden is naturally very heavy: the computational time required for a simulation of 300 seconds in real time is approximately 12 hours with a machine type Processor Intel (R) Core (TM) i7 CPU, 2.67 GHz.

Since the more complex hydrodynamic interactions within the breakwater (mesh 2) obviously require a higher number of computational nodes; also, in order to fully accommodate the 3D block mound model, the virtual geometrical set up is wider than the actual computational domain.
Once the geometry of the structure, imported into the CFD, has been rebuilt and the size and the scope of the computing grids have been set, attacks wave were chosen.

![General and local mesh (mesh 1 and mesh 2).](image)

**Tests and Validation**

The simulated wave's attacks are of random type, the virtual wave generator generates wave's attacks according to JONSWAP spectrum requires two input parameters: wind speed and fetch.

It is important to consider, as already said above, that in numerical simulations - very much like in laboratory tests – a great deal of care should be take in order to correctly evaluate the incident wave height (in the following: \( H_i \)) by separating it from the reflected wave (in the following: \( H_r \)); in order to do so the water height time series were analyzed by using the two probes method as proposed by Goda and Suzuki (1976).

Table 1 shows the values of \( H_i \) (Incident wave) and \( T \) (wave period) (JONSWAP spectrum) that were used for the tests. These values were obtained using the method of Goda and Suzuki (1976) applied to all simulations.

In the Fig. 4 the results of turbulent energy are shown, in particular is shown the calculation grid in mesh 2 for the innovative approach.

A consistent turbulent kinetic energy develops among the flow paths inside the blocks, mostly due to the strong velocity gradients. This influences the wave profile evolution at the breakwater, giving a different shape from the one obtained with the “porous media” model (Dentale et al., 2014), which obviously not only cannot reconstruct the dynamic effects inside the permeable layer, but also produces an entirely different turbulence structure outside it.

The final aim of the new computational procedure is to provide a design tool, and therefore a proper calibration should in principle involve a comparison between real and simulated fluid forces acting on the blocks within the mound. However, while the latter can be certainly computed with the methods we just described, there is no way to differentiate between the fluid actions and the forces exerted on a single block by the neighboring ones, unless the structural dynamic problem of the block is tackled with simultaneously. This approach, now being pursued by Latham et al. (2008) and Xiang et al. (2012) for very simple groups of block, is not yet mature to provide a proper assessment of the method. At present, the only way very to verify the new procedure is to make use of global parameters such as the reflection coefficients and the run-up heights, on which plenty of experimental data is available in the technical literature.
Table 1 Wave characteristics at toe breakwater.

<table>
<thead>
<tr>
<th>ID</th>
<th>Simulation</th>
<th>$Fe$ (km)</th>
<th>$U$ (m/s)</th>
<th>$T$ (s)</th>
<th>$H_s$ (m)</th>
<th>$H$ (m)</th>
<th>$L$ (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>NS1</td>
<td>5</td>
<td>30</td>
<td>3.19</td>
<td>0.47</td>
<td>0.16</td>
<td></td>
<td>15.64</td>
</tr>
<tr>
<td>NS2</td>
<td>5</td>
<td>40</td>
<td>3.58</td>
<td>0.78</td>
<td>0.22</td>
<td></td>
<td>19.24</td>
</tr>
<tr>
<td>NS3</td>
<td>5</td>
<td>50</td>
<td>3.97</td>
<td>0.90</td>
<td>0.30</td>
<td></td>
<td>22.86</td>
</tr>
<tr>
<td>NS4</td>
<td>20</td>
<td>15</td>
<td>4.2</td>
<td>0.76</td>
<td>0.26</td>
<td></td>
<td>24.98</td>
</tr>
<tr>
<td>NS5</td>
<td>20</td>
<td>20</td>
<td>4.62</td>
<td>1.04</td>
<td>0.37</td>
<td></td>
<td>28.80</td>
</tr>
<tr>
<td>NS6</td>
<td>20</td>
<td>25</td>
<td>4.95</td>
<td>1.21</td>
<td>0.44</td>
<td></td>
<td>31.75</td>
</tr>
<tr>
<td>NS7</td>
<td>20</td>
<td>30</td>
<td>5.27</td>
<td>1.20</td>
<td>0.42</td>
<td></td>
<td>34.57</td>
</tr>
<tr>
<td>NS8</td>
<td>20</td>
<td>40</td>
<td>5.56</td>
<td>1.29</td>
<td>0.49</td>
<td></td>
<td>37.09</td>
</tr>
<tr>
<td>NS9</td>
<td>100</td>
<td>6</td>
<td>5.18</td>
<td>1.00</td>
<td>0.37</td>
<td></td>
<td>33.78</td>
</tr>
<tr>
<td>NS10</td>
<td>100</td>
<td>9</td>
<td>5.77</td>
<td>1.24</td>
<td>0.47</td>
<td></td>
<td>38.90</td>
</tr>
<tr>
<td>NS11</td>
<td>100</td>
<td>12.5</td>
<td>6.64</td>
<td>1.39</td>
<td>0.55</td>
<td></td>
<td>46.28</td>
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<td>NS12</td>
<td>100</td>
<td>16</td>
<td>7.26</td>
<td>1.55</td>
<td>0.66</td>
<td></td>
<td>51.43</td>
</tr>
<tr>
<td>NS13</td>
<td>100</td>
<td>20</td>
<td>7.44</td>
<td>2.01</td>
<td>1.14</td>
<td></td>
<td>52.92</td>
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<tr>
<td>NS14</td>
<td>250</td>
<td>5</td>
<td>6.5</td>
<td>1.24</td>
<td>0.48</td>
<td></td>
<td>45.10</td>
</tr>
<tr>
<td>NS15</td>
<td>250</td>
<td>8</td>
<td>7.71</td>
<td>1.72</td>
<td>0.75</td>
<td></td>
<td>55.13</td>
</tr>
<tr>
<td>NS16</td>
<td>250</td>
<td>12</td>
<td>8.72</td>
<td>1.83</td>
<td>1.11</td>
<td></td>
<td>63.35</td>
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<tr>
<td>NS17</td>
<td>500</td>
<td>3</td>
<td>6.98</td>
<td>1.24</td>
<td>0.50</td>
<td></td>
<td>49.11</td>
</tr>
<tr>
<td>NS18</td>
<td>500</td>
<td>5</td>
<td>8.63</td>
<td>1.93</td>
<td>0.85</td>
<td></td>
<td>62.62</td>
</tr>
<tr>
<td>NS19</td>
<td>500</td>
<td>7</td>
<td>9.19</td>
<td>1.96</td>
<td>0.95</td>
<td></td>
<td>67.14</td>
</tr>
</tbody>
</table>

Fig. 4 Snapshot of turbulent energy (joules/kg) in local mesh.
Run up validation

The wave run up level is one of the most important factors affecting the design of coastal structures because it determines the design crest level of the structure in cases where no overtopping is accepted (Wang et al., 2011), (Kobayashi et al., 2012), and is also a good calibration parameter, since a great deal of experimental evidence is available in the form of comprehensive formulas.

The values of run up were measured according to the scheme shown in Fig. 5, through the snapshot of the central section of breakwater, with a frequency of 0.5 seconds, and the value of the corresponding run up was measured. Particularly the run up measured is the distance between SWL and the highest point of contact with the breakwater.

For each simulation, then, approximately 601 run up values have been measured. From the latter have been extracted the so called run-up statistics:

- Run up 2%: Average of the highest 2% of the numerical measured Run up values;
- Run up 10%: Average of the highest 10% of the numerical measured Run up values;
- Run up 1/3: Average of the highest third of the numerical measured Run up values;
- Run up medium: Average of all numerical measured Run up values;

![Fig. 5 Definition of measured run up (left) and example of the graph relative to the values of the run-up for the simulation (right).](image)

In order to quantify the distortion, the mean error and the regression coefficient were calculated 2%, 10%, medium and significant Run up, and compared with the results by Van der Meer and Stam (1992) and Burcharth (1998). The run up determined by Van der Meer & Stam formula are the significant run up (\( R_{u\,1/3} \)), \( R_{u\,1/10} \), \( R_{u\,2\%} \) and \( R_{u\,medium} \), while the run up obtained by Burcharth formula is \( R_{u\,2\%} \).

The regression coefficient \( R^2 \) is the usual “goodness of fit parameter”; “distortion” is defined as \( |1 - \alpha|\% \), “\( \alpha \)” being the slope of the straight line, so \( \alpha = 1 \) and distortion = 0 stand for perfect agreement. Also is defined a mean error as a ratio of the predictions by the empirical formulas and the results of numerical simulations in order to obtain a rough estimate of the differences. Obviously, the more this ratio is close to 1, the more the numerical model approximates the empirical formula.

\[
\text{Mean error} = \frac{1}{n} \sum_{k=1}^{n} \frac{X_{fk}}{Y_{nk}}
\]

where
- \( X_{fk} = k \)-th run up as calculated by the literature formula;
- \( Y_{nk} = k \)-th run up calculated by the numerical simulation;

Fig. 6 shows the regression between numerical results and formulae results.
Fig. 6 Examples of correlations between equation and new numerical approach for run up - burcharth and van der meer formula (top left), van der meer and stam formula for Run up 1/10 (top right), van der meer and stam formula for run up 1/3 (lower left), van der meer and stam formula for run up medium (lower right).

In general, the trend is satisfactory, and also, at present, the model intends to provide a tool to support the physical modeling in the preliminary design phase, without replacing the latter and, therefore, the results shown in Fig. 7 are considered acceptable. Regarding the results not well correlated presented in Fig. 7 in the upper left, one must take into account that the data taken into consideration are relative to 2% highest sample, then the average is referred at about 5 values for each simulation, giving rise to a dispersion consequently greater.

Table 2 Summarizes the results of the comparisons developed.

<table>
<thead>
<tr>
<th>Author, Year</th>
<th>Formula</th>
<th>Distortion (%)</th>
<th>$R^2$</th>
<th>Mean error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Van der Meer &amp; Stam, 1992</td>
<td>$R_{e,7} \leq d$</td>
<td>12.2</td>
<td>0.65</td>
<td>1.14</td>
</tr>
<tr>
<td></td>
<td>$R_{e,10} \leq 1.45$</td>
<td>11.1</td>
<td>0.87</td>
<td>0.91</td>
</tr>
<tr>
<td></td>
<td>$R_{e,\text{max}} \leq 0.82$</td>
<td>5.2</td>
<td>0.92</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>$R_{e} \leq 1.35$</td>
<td>14.9</td>
<td>0.90</td>
<td>0.95</td>
</tr>
<tr>
<td>Burcharth &amp; Van der Meer, 1998</td>
<td>$R_{e,2} = (A\xi + C)\gamma_{\alpha}\gamma_{\beta}$</td>
<td>16.1</td>
<td>0.66</td>
<td>1.20</td>
</tr>
</tbody>
</table>
Furthermore, the purpose of the presented validation, is not to obtain identical parameters in values, but similar trends, such as to say that the numerical model can be used to support the physical modeling, as a useful tool in preliminary design phase to allow a selection of design alternatives.

Consistency between numerical and experimental evidence is fully satisfactory, especially considering that no ad hoc calibration parameter was used for the flow in the rock mound.

**Reflection validation**

Wave reflection coefficient, i.e. the ratio between the reflected and incident wave $K_r = H_r / H_i$, is also an useful validation parameter, as well as having some practical design application.

Computed data for $H_i$ and $H_r$, derived through by the same Goda and Suzuki’s procedure discussed above, were therefore used to compare $K_r$ against experimental tests.

Fig. 7 provides two examples of correlation based on Hughes and Fowler Formula (1995) (left) and Zanuttigh and Van der Meer Formula (2006) (right).

![Graph 1](image1)

**Fig. 7 Examples of correlations between Literature's Formula and new numerical approach for reflection coefficient ($K_r$) - Hughes and Fowler Formula (1995) (left), Zanuttigh and Van der Meer Formula (2006) (Right).**

In Figs. 8 and 9 the numerical results for $K_r$ are shown over the graphs proposed by Zanuttigh and Van der Meer (2006), which reports a substantial number of experimental tests carried out in scale models or prototypes.

In the following graph, on the x axis is represented the Irribarren parameter, obtained by the equation:

$$\xi = \frac{tg \beta}{\sqrt{\frac{H}{L}}}$$

where:

$tg \beta = \frac{2}{3}$ ;

$H_i$ and $L$ are obtained as above described.
The dispersion of the numerical and experimental results is contained, and it's a function of the large number of parameters involved and the inherent randomness of the phenomena, all the results can be found in the same range of parameters. In order to provide a more precise validation, with the same procedure shown above for the run up, comparisons between numerical
$K_r$ and literature formulas (Seelig and Ahrens, 1981; Burger, 1988; Postma, 1989; Hughes and Fowler, 1995; Van der Meer, 1992; Zanuttigh and Van der Meer, 2006) were carried out and are summarized in Table 3:

Table 3 Kr validation.

<table>
<thead>
<tr>
<th>Author</th>
<th>Year</th>
<th>Formula</th>
<th>Mean error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ahrens &amp; Seelig</td>
<td>1981</td>
<td>$K_r = \frac{0.6\xi^2}{6.6 + \xi^2}$</td>
<td>1.12</td>
</tr>
<tr>
<td>Burger</td>
<td>1988</td>
<td>$K_r = \frac{0.6\xi^2}{12 + \xi^2}$</td>
<td>0.89</td>
</tr>
<tr>
<td>Postma</td>
<td>1989</td>
<td>$K_r = 0.125\xi^2$</td>
<td>0.90</td>
</tr>
<tr>
<td>Van der Meer</td>
<td>1992</td>
<td>$K_r = 0.07\left(P^{0.08} + \xi\right)$</td>
<td>0.94</td>
</tr>
<tr>
<td>Hughes &amp; Fowler</td>
<td>1995</td>
<td>$K_r = \frac{1}{1 + 7.1\xi^{0.8}}$</td>
<td>1.04</td>
</tr>
<tr>
<td>Zanuttigh &amp; Van der Meer</td>
<td>2006</td>
<td>$K_r = \tgh\left(0.12\xi^{0.87}\right)$</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Another interesting comparison can be made by using the relative water depth $k_0d$ as an independent parameter (here $k_0 = 2\pi / L_0$ and is the depth): Fig. 10 shows the results of $K_r$ vs. $k_0d$ used for constructing a new formula, based on empirical tests with regular and random waves, of Muttray et al. (2006) for a given breakwater type of construction (Xblok® single layer). On this graph are reported the numerical results of the innovative approach for a further comparison. The results appear qualitatively positive, but need further laboratory testing, as already explained above; in particular, for a relative depth greater than 0.8, the results obtained from Muttray are only 3, while the results of new numerical approach, also for other types of block are about 200 and show a trend of higher reflection coefficient.

Fig. 10 Numerical $K_r$ vs. $k_0d$ - Numerical and physical data (Muttray et al., 2006) for breakwater scheme.
FORCES ON BLOCKS

One of the most important perspectives of the new numerical approach is certainly the computation of hydrodynamical loads on single blocks in order to improve the safety and the cost effectiveness of coastal structure.

It is possible to evaluate, through the CFD software, the temporal evolution of the total hydrodynamic forces (pressure and shear) on a single block (Fig. 13), these results do not completely solve the problem of evaluating the stability of an armor block (Burcharh and Thompson, 1983; Nielsen and Burcharh, 1983), which also depends on the structural connection between the blocks (pull out) (Muttray et al., 2005), but they do provide some important pointers. Accordingly, it is intended to identify some “pilot” blocks in the armour layer (Fig. 11) on which to perform the calculation of the hydrodynamic forces acting (Fig. 12).

In the first place the stability of the single element can be defined by comparing the force with the rock weight; if such force exceeds the block’s weight, the element is potentially at risk, and as its balance within the breakwater is only guaranteed by the interlocking forces. This makes it possible to calculate a minimum block size, and also identify which of the elements would be most subjected to damage caused by extreme hydrodynamic action.

Another important result is that the highest forces are experienced by blocks nearer to the average waterline: an aspect which was already known by the construction practice but had never been quantified before and which might lead do some design improvement.
CONCLUSIONS AND PERSPECTIVES

A new approach has been set up and tested to evaluate wave actions on rock mound breakwaters in Tetrapods within 3D-RANS-VOF hydrodynamical simulation.

Unlike the conventional procedure, whereby the flow within the rock mound is treated as a simple seepage flow, the water movement between the blocks is dealt with the full Navier Stokes equations.

A virtual structure is modeled, as it happens in real construction practice, by overlapping individual 3D elements, and a sufficiently thin numerical grid is fitted to evaluate the flow in the passages between the blocks.

An assessment of the procedure, carried out against well proven experimental result on wave reflection and run-up, has shown that the methodology described here can be successfully used without any need to calibrate physical parameters.

Tests have also been performed to evaluate the time-varying hydrodynamic forces on single blocks; while a direct experimental check of these latter result is still impossible, physically sound conclusions and design hints have been already been obtained.

By appropriately combining and tuning modern CAD and CFD techniques a relatively easy - if computationally expensive - tool has been created to investigate the interaction between a rubble mound and the wave motion thus filling as much as possible the gap between empirical formulae and physical laboratory.

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