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An analytical method for finding exact solutions of modified Korteweg–de Vries equation

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A B S T R A C T

In this present work, we have studied new extension of the (G'/G)-expansion method for finding the solitary wave solutions of the modified Korteweg–de Vries (mKdV) equation. It has been shown that the proposed method is effective and can be used for many other nonlinear evolution equations (NLEEs) in mathematical physics. The obtained results show that the method is very powerful and convenient mathematical tool for nonlinear evolution equations in science and engineering.

1. Introduction

In the nonlinear sciences, it is well known that many NLEEs are widely used to describe the complex phenomena such as fluid mechanics, meteorology, plasma physics, optical fibers, biology, solid state physics, chemical kinematics, chemical physics, etc. The powerful and efficient methods to find analytic solutions and numerical solutions of nonlinear equations have drawn a lot of interest by diverse group of researchers. Many efficient analytic and numerical methods have been presented so far. During the research, searching for traveling wave solutions of NLEEs has been the main goal of many researchers, and many powerful methods for constructing exact solutions of nonlinear evolution equations have been established and developed. In order to better understand the nonlinear phenomena as well as further practical applications, it is important to seek their more exact traveling wave solutions. Even those special exact solutions that do not have a clear physical meaning can be used as test problems to verify the consistency and estimate errors of various numerical, asymptotic, and approximate analytical methods. Exact solutions can serve as a basis for perfecting and testing computer algebra software packages for solving NLEEs. It is significant that many equations of physics, chemistry, and biology contain empirical parameters or empirical functions. Exact solutions allow researchers to design and run experiments, by creating appropriate natural conditions, to determine these parameters or functions. Therefore, investigating exact traveling wave solutions is becoming successively attractive in nonlinear sciences day by day. However, not all equations posed of these models are solvable. As a result, many new techniques have been successfully developed by diverse groups of mathematicians and physicists, such as, the homotopy perturbation method [1–7], the (G'/G) -expansion method [8–12], the Kudryashov Method [13], the Exp-function method [14–16], the modified simple equation method [17–20], Hirota's bilinear transformation method [21,22], the exp $(-\Phi(\xi))$-expansion method [23], the Enhanced (G′/G)-expansion Method [24], Improved *F*-expansion method [25], the tanh-function method [26,27], etc.

Various ansatze have been proposed for seeking traveling wave solutions of nonlinear differential equations. The choice of an appropriate ansatz has a great significance in the direct methods. In 2008 Wang et al. [8] have established an ansatz method, named the (G'/G) -expansion method for seeking exact solutions of NLEEs, where $u(\xi) = \alpha_m \left(\frac{G'}{G}\right)^m + \cdots$ be the solution, $G = G(\xi)$ satisfies the ordinary differential equation $G''(\xi) + \lambda G'(\xi) + \mu G(\xi) = 0$, and λ and μ are arbitrary constants. In this paper, based on a new general ansatz, we introduce the extension of (G'/G) -expansion with new algebra expansion, which can be used to obtain explicit solutions of NLEEs. Here we assume the solution of NLEEs is of the form $u(\xi) = \sum_{i=0}^{n}\alpha_i(m + F(\xi))^i + \sum_{i=1}^{n}\beta_i(m + F(\xi))^{-i}$, where $F(\xi) = G'/G$, and $G = G(\xi)$ satisfies the ordinary differential equation $G''(\xi) + \lambda G'(\xi) + \mu G(\xi) = 0$, where λ and μ are arbitrary constants. From our observation we found that if we set $m = 0$ and leave

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out the portion $\sum_{i=1}^{n}\beta_i(m+F(\xi))^{-i}$ in our solution, then our solution coincides with the solution introduced by Wang et al. [8]. Hence we conclude that the basic (G'/G) -expansion method established by Wang et al. [8] is the particular case of our new extension of the (G'/G) -expansion method. Moreover, in this letter we have solved mKdV equation and found forty solutions, but by means of the basic (G'/G) -expansion method Wang et al. [8] found only three solutions.

The objective of this article is to present new extension of the (G'/G) -expansion method to construct the exact traveling wave solutions for NLEEs in mathematical physics via the mKdV equation.

The article is arranged as follows: In Section 2, the methodology is discussed. In Section 3, we apply this method to the nonlinear evolution equation pointed out above. In Section 4, results and **discussion** and in Section 5, conclusions are given.

2. Methodology

Suppose the general nonlinear partial differential equation,

$$P(u, u_t, u_x, u_{tt}, u_{tx}, u_{xx}, \ldots) = 0 \tag{1}$$

where $u = u(x,t)$ is an unknown function, P is a polynomial in $u(x,t)$ and its partial derivatives in which the highest order partial derivatives and the nonlinear terms are involved. The main steps of new extension of the (G'/G) -expansion method combined with the algebra expansion are as follows:

Step 1: The traveling wave variable ansatz.

$$\xi = x \pm \omega t, \qquad u(x,t) = u(\xi), \tag{2}$$

where $\omega \in \Re - \{0\}$ is the speed of the traveling wave, permits us to transform the Eq. (1) into the following ODE:

$$Q(u, u', u'', \ldots) = 0, \tag{3}$$

where the superscripts stand for the ordinary derivatives with respect to ξ.

Step 2: Suppose the traveling wave solution of Eq. (3) can be expressed by a polynomial in $F(\xi)$ as follows:

$$u(\xi) = \sum_{i=0}^{n}\alpha_i(m+F(\xi))^i + \sum_{i=1}^{n}\beta_i(m+F(\xi))^{-i}, \tag{4}$$

where $F(\xi) = G'/G$, α_n and β_n are not zero simultaneously. Also $G = G(\xi)$ satisfies the ordinary differential equation,

$$G''(\xi) + \lambda G'(\xi) + \mu G(\xi) = 0, \tag{5}$$

where λ and μ are arbitrary constants to be determined later.

The solutions of Eq. (5) can be written as follows:When $\Omega = \lambda^2 - 4\mu > 0$

$$F_1 = \frac{\sqrt{\Omega}}{2}\coth\left(A + \frac{\sqrt{\Omega}}{2}\xi\right) - \frac{\lambda}{2},$$

$$F_2 = \frac{\sqrt{\Omega}}{2}\tanh\left(A + \frac{\sqrt{\Omega}}{2}\xi\right) - \frac{\lambda}{2}.$$

When $\Omega = \lambda^2 - 4\mu < 0$

$$F_3 = \frac{\sqrt{\Omega}}{2}\cot\left(A + \frac{\sqrt{\Omega}}{2}\xi\right) - \frac{\lambda}{2},$$

$$F_4 = \frac{\sqrt{\Omega}}{2}\tan\left(A - \frac{\sqrt{\Omega}}{2}\xi\right) - \frac{\lambda}{2}.$$

When $\Omega = \lambda^2 - 4\mu = 0$

$$F_5 = \frac{B}{A + B\xi} - \frac{\lambda}{2}$$

Step 3: The positive integer n can be determined by considering the homogeneous balance between the highest order derivatives and the nonlinear terms appearing in Eq. (1) or Eq. (3). Moreover precisely, we define the degree of $u(\xi)$ as $D(u(\xi)) = n$ which gives rise to the degree of other expression as follows:

$$D\left(\frac{d^q u}{d\xi^q}\right) = n + q, \qquad D\left(u^p\left(\frac{d^q u}{d\xi^q}\right)^s\right) = np + s(n+q). \tag{6}$$

Therefore we can find the value of n in Eq. (4), using Eq. (6).

Step 4: Substituting Eq. (4) along with Eq. (5) into Eq. (3) together with the value of n obtained in step 3, we obtain polynomials in F^i and F^{-i} $(i = 0, 1, 2, 3, \cdots)$, then setting each coefficient of the resulted polynomial to zero, yields a system of algebraic equations for α_n, β_n and ω.

Step 5: Suppose the values of the constants α_n, β_n and ω can be determined by solving the system of algebraic equations obtained in step 4. Since the general solutions of Eq. (5) are known, substituting α_n, β_n and ω into Eq. (4), we obtain some exact traveling wave solutions of the nonlinear evolution Eq. (1).

3. Application

In the present work, we consider the following mKdV equation [25] with parameters of the form,

$$u_t - u^2 u_x + \delta u_{xxx} = 0, \tag{7}$$

where δ is a nonzero constant.

The mKdV equation is identical to the KdV equation in that both are completely integrable and each has infinitely many conserved quantities. The mKdV equation gives algebraic solitons solutions in the form of a rational function. The modified KdV equation describes nonlinear wave propagation in systems with polarity symmetry. The mKdV equation appears in applications such as electric circuits and multi-component plasmas, electrodynamics, electro-magnetic waves in size-quantized films, traffic flow and elastic media [28].

The traveling wave transformation equation $u(\xi) = u(x,t)$, $\xi = x - \omega t$ transforms Eq. (7) to the following ordinary differential equation:

$$-\omega u' - u^2 u' + \delta u''' = 0. \tag{8}$$

Now integrating Eq. (8) with respect to ξ once, we have

$$\delta u'' - \omega u - \frac{u^3}{3} + C = 0 \tag{9}$$

where C is a constant of integration. Balancing the highest-order derivative term u'' and the nonlinear term u^3 from Eq. (9), yields $3n = n + 2$, which gives $n = 1$.

Hence for $n = 1$ Eq. (4) reduces to

$$u(\xi) = \alpha_0 + \alpha_1(m + F) + \beta_1(m + F)^{-1}. \tag{10}$$

Now substituting Eq. (10) along with Eq. (5) into Eq. (9), we get a polynomial in $F(\xi)$.. Equating the coefficient of same power of $F(\xi)$, we attain the following system of algebraic equations:

$$6\delta\alpha_1 - \alpha_1^3 = 0$$

$$18\delta\alpha_1 m - 6\alpha_1^3 m + 9\delta\alpha_1\lambda - 3\alpha_0\alpha_1^2 = 0$$

$$6\delta\alpha_1\mu - 15\alpha_0\alpha_1^2 m - 3\alpha_1^2\beta - 3\omega\alpha_1 + 18\delta\alpha_1 m^2 - 15\alpha_1^3 m^2 + 27\delta\alpha_1\lambda m + 3\delta\alpha_1\lambda^2 - 3\alpha_0^2\alpha_1 = 0$$

$$-12\alpha_0^2\alpha_1 m + 9\delta\alpha_1\lambda^2 m + 3\delta\alpha_1\lambda\mu - \alpha_0^3 - 3\omega\alpha_0 + 18\delta\alpha_1\mu m - 12\omega\alpha_1 m - 20\alpha_1^3 m^3 + 27\delta\alpha_1\lambda m^2 - 6\delta\beta_1 m + 3c - 30\alpha_0\alpha_1^2 m^2 + 3\delta\beta_1\lambda - 12\alpha_1^2 m\beta_1 - 6\alpha_0\alpha_1\beta + 6\delta\alpha_1 m^3 = 0$$

$$-18\alpha_1^2 m^2\beta_1 + 9\delta\alpha_1\lambda m^3 - 18\alpha_0^2\alpha_1 m^2 - 15\alpha_1^3 m^4 - 3\alpha_0^3 m + 18\delta\alpha_1\mu m^2 + 9\delta\alpha_1\lambda^2 m^2 - 3\alpha_0^2\beta_1 + 3\delta\beta_1\lambda^2 + 9\delta\alpha_1\lambda\mu m - 3\omega\beta_1 + 6\delta\beta_1\mu - 30\alpha_0\alpha_1^2 m^3 - 9\delta\beta_1\lambda m - 9\omega\alpha_0 m - 18\alpha_0\alpha_1 m\beta_1 - 18\omega\alpha_1 m^2 - 3\alpha_1\beta_1^2 + 9cm = 0$$

$$9\delta\alpha_1\lambda\mu m^2 - 18\alpha_0\alpha_1\beta_1 m^2 - 12\omega\alpha_1 m^3 - 6\alpha_1 m\beta_1^2 - 9\omega\alpha_0 m^2 + 3\delta\alpha_1\lambda^2 m^3 - 6\delta\beta_1\mu m + 6\delta\alpha_1\mu m^3 - 6\alpha_0^2\beta_1 m - 12\alpha_0^2\alpha_1 m^3 + 9\delta\beta_1\lambda\mu + 9cm^2 - 6\alpha_1^3 m^5 - 3\delta\beta_1\lambda^2 m - 12\alpha_1^2 m^3\beta_1 - 3\alpha_0\beta_1^2 - 3\alpha_0^3 m^2 - 6\omega\beta_1 m - 15\alpha_0\alpha_1^2 m^4 = 0$$

$$-3\alpha_0\alpha_1^2 m^5 - 3\alpha_1 m^2\beta_1^2 + 6\delta\beta_1\mu^2 - 6\alpha_0\alpha_1 m^3\beta_1 - 3\delta\beta_1\lambda\mu m - 3\omega\beta_1 m^2 - \beta_1^3 - 3\alpha_0^2\beta_1 m^2 + 3\delta\alpha_1\lambda\mu m^3 - 3\alpha_0\beta_1^2 m - 3\omega\alpha_0 m^3 + 3cm^3 - 3\alpha_1^2 m^4\beta_1 - \alpha_0^3 m^3 - 3\alpha_0^2\alpha_1 m^4 - \alpha_1^3 m^6 - 3\omega\alpha_1 m^4 = 0$$

Solving the above system of equations for α_0, α_1, β_1, ω, m and c, we get the following values:

Set-01: $c=0, \omega=-\frac{1}{2}(\lambda^2-4\mu)\cdot\delta, m=m, \alpha_0=\mp\frac{1}{2}\sqrt{6\delta}(2m-\lambda)$, $\alpha_1=0, \beta_1=\pm\sqrt{6\delta}(m^2-m\lambda+\mu)$.

Set-02: $c=0, \omega=-\frac{1}{2}(\lambda^2-4\mu)\cdot\delta, m=m, \alpha_0=\mp\frac{1}{2}\sqrt{6\delta}(2m-\lambda)$, $\alpha_1=\mp\sqrt{6\delta}, \beta_1=0$.

Set-03: $c=0, \omega=(\lambda^2-4\mu)\cdot\delta, m=\frac{1}{2}\lambda, \alpha_0=0, \alpha_1=\pm\sqrt{6\delta}$, $\beta_1=\mp\frac{1}{4}\sqrt{6\delta}(\lambda^2-4\mu)$.

Set-04: $c=-2\sqrt{6}\delta^{\frac{3}{2}}(\lambda^2 m-3m^2\lambda+2\mu m+2m^3-\lambda\mu)$, $\omega=\left(6\lambda m-4\mu-6m^2-\frac{1}{2}\lambda^2\right)\delta$, $m=m, \alpha_0=\mp\frac{1}{2}\sqrt{6\delta}(2m-\lambda), \alpha_1=\pm\sqrt{6\delta}$, $\beta_1=\pm\sqrt{6\delta}(m^2-m\lambda+\mu)$.

3.1. Hyperbolic function solutions

When $\Omega=\lambda^2-4\mu>0$, we get the following solutions.

Family 1 :

$$u_{1,2}(\xi)=\mp\left(\frac{1}{2}\sqrt{6\delta}(2m-\lambda)-\frac{\sqrt{6\delta}\left(m^2-m\lambda+\mu\right)}{m+\frac{1}{2}\sqrt{\Omega}\tanh\left(A+\frac{1}{2}\sqrt{\Omega}\xi\right)-\frac{1}{2}\lambda}\right),$$

$$u_{3,4}(\xi)=\mp\left(\frac{1}{2}\sqrt{6\delta}(2m-\lambda)-\frac{\sqrt{6\delta}\left(m^2-m\lambda+\mu\right)}{m+\frac{1}{2}\sqrt{\Omega}\coth\left(A+\frac{1}{2}\sqrt{\Omega}\xi\right)-\frac{1}{2}\lambda}\right),$$

where $\xi=x+\frac{1}{2}(\lambda^2-4\mu)\delta t$.

Family 2 :

$$u_{5,6}(\xi)=\mp\left(\frac{1}{2}\sqrt{6\delta}(2m-\lambda)-\sqrt{6\lambda}\left(m+\frac{1}{2}\sqrt{\Omega}\tanh\left(A+\frac{1}{2}\sqrt{\Omega}\xi\right)\right)-\frac{1}{2}\lambda\right),$$

$$u_{7,8}(\xi)=\mp\left(\frac{1}{2}\sqrt{6\delta}(2m-\lambda)-\sqrt{6\lambda}\left(m+\frac{1}{2}\sqrt{\Omega}\coth\left(A+\frac{1}{2}\sqrt{\Omega}\xi\right)\right)-\frac{1}{2}\lambda\right),$$

where $\xi=x+\frac{1}{2}(\lambda^2-4\mu)\delta t$.

Family 3 :

$$u_{9,10}(\xi)=\pm\left(\frac{1}{2}\sqrt{6\delta}\sqrt{\Omega}\tanh\left(A+\frac{1}{2}\sqrt{\Omega}\xi\right)-\frac{1}{2}\frac{\sqrt{6\delta}\left(\lambda^2-4\mu\right)}{\sqrt{\Omega}\tanh\left(A+\frac{1}{2}\sqrt{\Omega}\xi\right)}\right),$$

$$u_{11,12}(\xi)=\pm\left(\frac{1}{2}\sqrt{6\delta}\sqrt{\Omega}\coth\left(A+\frac{1}{2}\sqrt{\Omega}\xi\right)-\frac{1}{2}\frac{\sqrt{6\delta}\left(\lambda^2-4\mu\right)}{\sqrt{\Omega}\coth\left(A+\frac{1}{2}\sqrt{\Omega}\xi\right)}\right),$$

where $\xi=x-(\lambda^2-4\mu)\cdot\delta t$.

Family 4 :

$$u_{13,14}(\xi)=\mp\left(\frac{1}{2}\sqrt{6\delta}(2m-\lambda)-\sqrt{6\delta}\left(m+\frac{1}{2}\sqrt{\Omega}\tanh\left(A+\frac{1}{2}\sqrt{\Omega}\xi\right)-\frac{1}{2}\lambda\right)-\frac{\sqrt{6\delta}\left(m^2-m\lambda+\mu\right)}{m+\frac{1}{2}\sqrt{\Omega}\tanh\left(A+\frac{1}{2}\sqrt{\Omega}\xi\right)-\frac{1}{2}\lambda}\right),$$

$$u_{15,16}(\xi)=\mp\left(\frac{1}{2}\sqrt{6\delta}(2m-\lambda)-\sqrt{6\delta}\left(m+\frac{1}{2}\sqrt{\Omega}\coth\left(A+\frac{1}{2}\sqrt{\Omega}\xi\right)-\frac{1}{2}\lambda\right)-\frac{\sqrt{6\delta}\left(m^2-m\lambda+\mu\right)}{m+\frac{1}{2}\sqrt{\Omega}\coth\left(A+\frac{1}{2}\sqrt{\Omega}\xi\right)-\frac{1}{2}\lambda}\right),$$

where $\xi=x-\left(6\lambda m-4\mu-6m^2-\frac{1}{2}\lambda^2\right)\delta t$.

3.2. Trigonometric function solutions

When $\Omega=\lambda^2-4\mu<0$, we get the following solutions.

Family 5 :

$$u_{17,18}(\xi)=\mp\left(\frac{1}{2}\sqrt{6\delta}(2m-\lambda)-\frac{\sqrt{6\delta}\left(m^2-m\lambda+\mu\right)}{m+\frac{1}{2}\sqrt{\Omega}\tan\left(A+\frac{1}{2}\sqrt{\Omega}\xi\right)-\frac{1}{2}\lambda}\right),$$

$$u_{19,20}(\xi)=\mp\left(\frac{1}{2}\sqrt{6\delta}(2m-\lambda)-\frac{\sqrt{6\delta}\left(m^2-m\lambda+\mu\right)}{m+\frac{1}{2}\sqrt{\Omega}\cot\left(A+\frac{1}{2}\sqrt{\Omega}\xi\right)-\frac{1}{2}\lambda}\right),$$

where $\xi=x+\frac{1}{2}(\lambda^2-4\mu)\delta t$.

Family 6 :

$$u_{21,22}(\xi)=\mp\left(\frac{1}{2}\sqrt{6\delta}(2m-\lambda)-\sqrt{6\delta}\left(m+\frac{1}{2}\sqrt{\Omega}\tan\left(A+\frac{1}{2}\sqrt{\Omega}\xi\right)\right)-\frac{1}{2}\lambda\right),$$

$$u_{23,24}(\xi)=\mp\left(\frac{1}{2}\sqrt{6\delta}(2m-\lambda)-\sqrt{6\delta}\left(m+\frac{1}{2}\sqrt{\Omega}\cot\left(A+\frac{1}{2}\sqrt{\Omega}\xi\right)\right)-\frac{1}{2}\lambda\right),$$

where $\xi=x+\frac{1}{2}(\lambda^2-4\mu)\delta t$.

Family 7 :

$$u_{25,26}(\xi)=\pm\left(\frac{1}{2}\sqrt{6\delta}\sqrt{\Omega}\tan\left(A+\frac{1}{2}\sqrt{\Omega}\xi\right)-\frac{1}{2}\frac{\sqrt{6\delta}\left(\lambda^2-4\mu\right)}{\sqrt{\Omega}\tan\left(A+\frac{1}{2}\sqrt{\Omega}\xi\right)-\frac{1}{2}\lambda}\right),$$

$$u_{27,28}(\xi)=\pm\left(\frac{1}{2}\sqrt{6\delta}\sqrt{\Omega}\cot\left(A+\frac{1}{2}\sqrt{\Omega}\xi\right)-\frac{1}{2}\frac{\sqrt{6\delta}\left(\lambda^2-4\mu\right)}{\sqrt{\Omega}\cot\left(A+\frac{1}{2}\sqrt{\Omega}\xi\right)-\frac{1}{2}\lambda}\right),$$

where $\xi=x-(\lambda^2-4\mu)\delta t$.

Family 8 :

$$u_{29,30}(\xi)=\mp\left(\frac{1}{2}\sqrt{6\delta}(2m-\lambda)-\sqrt{6\delta}\left(m+\frac{1}{2}\sqrt{\Omega}\tan\left(A+\frac{1}{2}\sqrt{\Omega}\xi\right)-\frac{1}{2}\lambda\right)-\frac{\sqrt{6\delta}\left(m^2-m\lambda+\mu\right)}{m+\frac{1}{2}\sqrt{\Omega}\tan\left(A+\frac{1}{2}\sqrt{\Omega}\xi\right)-\frac{1}{2}\lambda}\right),$$

$$u_{31,32}(\xi)=\mp\left(\frac{1}{2}\sqrt{6\delta}(2m-\lambda)-\sqrt{6\delta}\left(m+\frac{1}{2}\sqrt{\Omega}\cot\left(A+\frac{1}{2}\sqrt{\Omega}\xi\right)-\frac{1}{2}\lambda\right)-\frac{\sqrt{6\delta}\left(m^2-m\lambda+\mu\right)}{m+\frac{1}{2}\sqrt{\Omega}\cot\left(A+\frac{1}{2}\sqrt{\Omega}\xi\right)-\frac{1}{2}\lambda}\right),$$

where $\xi = x - (6\lambda m - 4\mu - 6m^2 - \frac{1}{2}\lambda^2)\delta t$.

3.3. Rational function solutions

When $\Omega = \lambda^2 - 4\mu = 0$, we get the following solutions.

Family 9 : $u_{33,34}(\xi) = \mp\left(\frac{1}{2}\sqrt{6\delta}(2m-\lambda) - \frac{\sqrt{6\delta}(m^2 - m\lambda + \mu)}{m + \frac{B}{A+B\xi} - \frac{1}{2}\lambda}\right)$,

where $\xi = x + \frac{1}{2}(\lambda^2 - 4\mu)\delta t$.

Family 10 : $u_{35,36}(\xi) = \mp\left(\frac{1}{2}\sqrt{6\delta}(2m-\lambda) - \sqrt{6\lambda}\left(m + \frac{B}{A+B\xi} - \frac{1}{2}\lambda\right)\right)$,

where, $\xi = x + \frac{1}{2}(\lambda^2 - 4\mu)\delta t$.

Family 11 : $u_{37,38}(\xi) = \pm\left(\frac{\sqrt{6\delta}B}{A+B\xi} - \frac{1}{4}\frac{\sqrt{6\delta}(\lambda^2 - 4\mu)\cdot(A+B\xi)}{B}\right)$,

where $\xi = x + (\lambda^2 - 4\mu)\delta t$.

Family 12 :

$$u_{39,40}(\xi) = \mp\left(\frac{1}{2}\sqrt{6\delta}(2m-\lambda) - \sqrt{6\delta}\left(m + \frac{B}{A+B\xi} - \frac{1}{2}\lambda\right) - \frac{\sqrt{6\delta}(m^2 - m\lambda + \mu)}{m + \frac{B}{A+B\xi} - \frac{1}{2}\lambda}\right),$$

where $\xi = x - (6\lambda m - 4\mu - 6m^2 - \frac{1}{2}\lambda^2)\delta t$.

Remark: All these obtained solutions have been verified with Maple by substituting them back into the original equations and found correct.

4. Results and discussions

In this section we will discuss about the nature of some obtained solutions of Eq. (7) by selecting particular values of the parameters existing in the exact solutions using the mathematical software Maple 13, which are represented in Figs. 1–4. From our obtained solutions we observe that solutions from Family 1 to Family 4 are hyperbolic function solutions for $\lambda^2 - 4\mu > 0$, from Family 5 to Family 8 are trigonometric function solutions for $\lambda^2 - 4\mu < 0$ and from Family 9 to Family 12 are rational function solutions for $\lambda^2 - 4\mu = 0$.

Solution $u_1(\xi)$ shows Kink shaped soliton profile for $\lambda = 1, \mu = -1, A = 0, \delta = 1, m = 3$ within the interval $-3 \leqslant x, t \leqslant 3$, which is represented in Fig. 1.

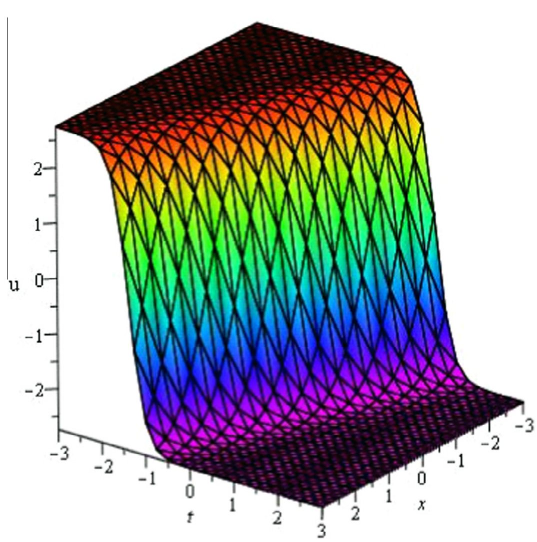

Fig. 1. Kink profile of solutions $u_1(\xi)$ of mKDV equation for $\lambda = 1, \mu = -1, A = 0, \delta = 1, m = 3$ within the interval $-3 \leqslant x, t \leqslant 3$.

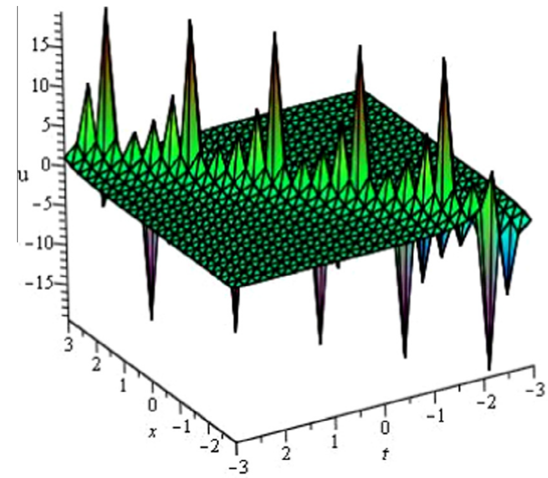

Fig. 2. Soliton profile of solutions $u_9(\xi)$ of mKDV equation for $\lambda = 3, \mu = 1, A = 0, \delta = 0.25$ within the interval $-3 \leqslant x, t \leqslant 3$.

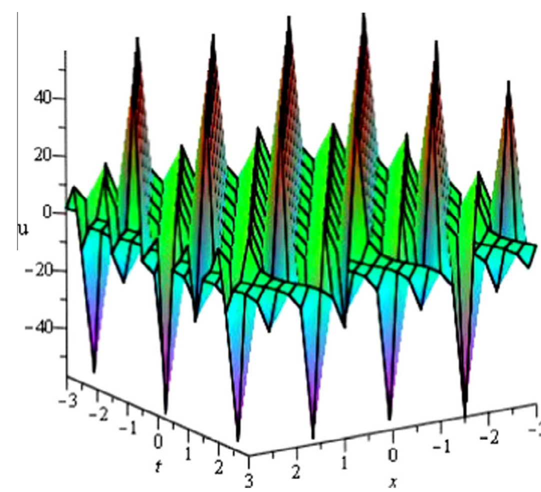

Fig. 3. Periodic profile of solutions $u_{21}(\xi)$ of mKDV equation for $\lambda = 1, \mu = 2, A = 0, \delta = 2, m = 1$ within the interval $-3 \leqslant x, t \leqslant 3$.

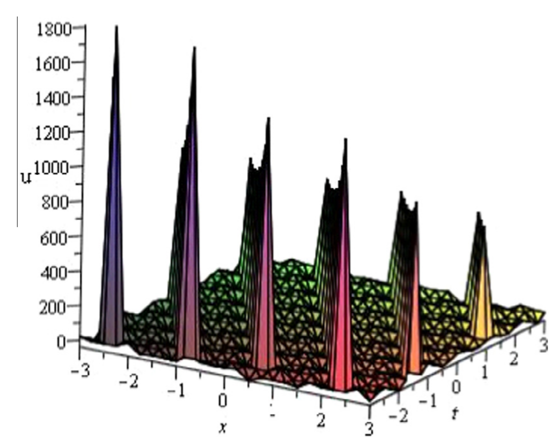

Fig. 4. Periodic profile of solutions $u_{35}(\xi)$ of mKDV equation for $\lambda = 1, \mu = 2, A = 0.50, \delta = 1, m = 0$ within the interval $-3 \leqslant x, t \leqslant 3$.

Solution $u_9(\xi)$ provides singular soliton profile for the particular values of $\lambda = 3, \mu = 1, A = 0, \delta = 0.25$ within the interval $-3 \leqslant x, t \leqslant 3$, which are represented in Fig. 2.

Fig. 3 represents periodic solution of mKdV equation for $\lambda = 1, \mu = 2, A = 0, \delta = 2, m = 1$ within the interval $-3 \leqslant x, t \leqslant 3$ (Only shows the shape of $u_{21}(\xi)$.

Fig. 4 also represents periodic solution of mKdV equation for $\lambda = 1, \mu = 2, A = 0.50, \delta = 1, m = 0$ within the interval $-3 \leqslant x, t \leqslant 3$ (Only shows the shape of $u_{35}(\xi)$).

Some of our obtained traveling wave solutions are represented in the following figures:

5. Conclusions

In summary, the new extension of the (G'/G) -expansion method has been applied to find out exact solutions of nonlinear equations with the aid of the computer software Maple. This method allows to carry out the solution process of nonlinear wave equations more thoroughly and conveniently by computer algebra systems such as the Maple and Mathematica. We have successfully obtained some exact traveling wave solutions of the mKdV equation with parameters. When the parameters are taken as special values, the solitary wave solutions and periodic wave solutions are originated from the exact solutions. We believe that the obtained solutions will serve as a very important milestone in the study of nonlinear phenomena arising in Mathematical Physics and Engineering fields. This work shows that the new extension of the (G'/G) -expansion method is sufficient, effective and suitable for solving other nonlinear evolution equations, it deserves further applying and studying as well. Further studies for finding exact solutions to NLEEs those arise in mathematical physics and engineering can be a possible future direction.

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