Available at www.ElsevierMathematics.com
POWERED BY SCIENCE DIRECT®

ELSEVIER

Journal of Computational and Applied Mathematics 162 (2004) 263–272

JOURNAL OF COMPUTATIONAL AND APPLIED MATHEMATICS

www.elsevier.com/locate/cam

Generalized path-finding algorithms on semirings and the fuzzy shortest path problem

A. Boulmakoul

LIST Laboratory, Computer Sciences Department, Faculty of Sciences and Technology of Mohammedia, B.P. 146, Mohammedia, Morocco

Received 30 November 2001; received in revised form 10 November 2002

Abstract

A new dioïd structure $\langle \Omega_k, \oplus, \otimes, \varepsilon, e \rangle$ is proposed to solve a path-finding problem in a fuzzy graph. This algebraic structure is adapted precisely to solve the problem of the K-best fuzzy shortest paths. We demonstrate that the generalized Gauss–Seidel's algorithm always converges for the solving of the K-best fuzzy shortest paths problem on a valued fuzzy graph without cycles of negative weight. This work starts a safe extension of the path algebra paradigm to valued fuzzy graphs for the shortest path-finding problem.

Keywords: Fuzzy graphs; Path algebra; Dioïds; K-Best fuzzy shortest paths; Generalized Gauss–Seidel's algorithm

1. Introduction

The fuzzy shortest path-finding problem from a specified source node to the other nodes appears in several applications. In transportation systems area, their corresponding networks use fuzzy informations on the arcs, assumed to represent transportation time or economic cost than traffic flow, etc. These informations are soft and would be well presented by fuzzy numbers or fuzzy set based on fuzzy set theory [23].

The first works developed to solve the fuzzy shortest path problem have been initiated for the first time in [5,6,21]. Nevertheless, if the research of the shortest path length in a fuzzy graph is feasible, generally this path does not correspond to a real path in the considered fuzzy graph. This

E-mail address: boul@uh2m.ac.ma (A. Boulmakoul).

0377-0427/$ - see front matter
doi:10.1016/j.cam.2003.08.027

exception is explained by the particular behavior of the generalized min and max operators for the fuzzy numbers.

Dubois and Prade [6] comment the solution of the classical fuzzy shortest path problem through the use of extended sum, $\oplus$, and extended min and max, $\widetilde{\min}$, $\widetilde{\max}$. To solve the problem Floyd's and Ford's algorithms are applied. Unfortunately, this approach, even though it can determine the length of a fuzzy shortest path, it cannot find a fuzzy path which corresponds to this length in the fuzzy graph. This failure is a consequence of the classical operators extension min and max, according to the extension principle. From that principle, the extended $\widetilde{\min}$ or $\widetilde{\max}$ of several fuzzy numbers may not be one of those numbers. Some approaches based on the concept of α-cut [3,4,16,18] and other models based on the parametric orders [7] or relation order [20], did permit to reuse the classic methods in various fuzzy graphs applications on operations research field. With regard to the based methods on parametric or relation orders, a work proposed in [7] uses a modified Dijkstra's algorithm for valued fuzzy graphs, where valuations of the arcs are L-fuzzy numbers. An other algorithm has been proposed in [20] for the valued fuzzy graphs with L–R fuzzy numbers. The proposed algorithm defines a relation order between L–R fuzzy numbers. It is based on the multiple labeling method to obtain all nondominated paths. The multiple labeling can be considered as a generalization of Dijkstra's algorithm [20].

A formulation of the fuzzy shortest path problem not doing reference to the concept of α-cut or parametric orders, has been proposed in [12]. Klein's algorithm [12] is based on multi-criteria dynamic programming, and can find a path or paths for a level of membership set by a decision maker. This algorithm, however, assumes that the valued fuzzy graphs are acyclic graphs. To apply Klein's algorithm for other graphs, Klein [12] proposed a transformation for these graphs according to the following remark owed to Lawler [17]: each graph that has no cycles of negative weight can easily be converted to a directed acyclic graph. Nevertheless, the transformation procedure is NP-Hard in the general case [11]. Hence for the computational aspects, the Klein's algorithm is restricted to acyclic graphs. This algorithm, however, assumes that the network is layered and the number of layers is fixed. Klein's algorithm assumes also, that each arc can take an integer value for length between 1 and a fixed integer depending of the number of layers.

The present work proposes a structure of dioïd (path algebra) to solve the fuzzy shortest path problem in a fuzzy graph. This structure agrees to solve the problem of the K-best fuzzy shortest paths. This first result generalizes Klein's work. This paper starts the extension of Gondran and Minoux path algebra given for the crisp case [10,11,19] to valued fuzzy graphs.

This paper is organized as follows: Foundations of path algebra are given succinctly in Section 2. In Section 3 we recall the basic concepts of fuzzy sets and fuzzy graphs, we give a summary description of the extension principle for the classic operators in the fuzzy context and we introduce a valued fuzzy graph example. These brief recalls are important for the formulation of the new structure of dioïd. Section 4 recalls the general Gauss–Seidel's algorithm for the resolution of the shortest path problem in crisp case. This algorithm is applied on the path algebra structure that we develop in the case of valued fuzzy graphs.

In the same section, we develop a dioïd structure to model the problem of the K-best fuzzy shortest paths, as well as the survey of convergence and complexity of the general algorithm using the proposed dioïd. A small illustration example is presented. In Section 5, a fuzzy graph conceptual modelling based on object-oriented paradigm is described. We also consider in the same section, the software development suited to the analysis of fuzzy graphs. This paper is concluded in Section 6.

2. Dioïds and the shortest path problem solving

The concept of dioïd has been proposed to designate an algebraic structure constituted by a set denoted by Ω which is provided with two internal operations $\oplus$ and $\otimes$ [15]. The dioïd structure has been transported from the matrix algebra to generalize known results in this algebra to graph theory. The definition of this concept is given here after.

Definition 1 (Semiring, Kuich and Salomaa [14], Yager [22]). By semiring $\langle \Omega, \oplus, \otimes, \varepsilon, e \rangle$ we mean a set Ω together with two binary operations $\oplus$ and $\otimes$ and two constant elements ε and e (ε and e are the neutral elements for $\oplus$ and $\otimes$, respectively), such that:

(i) $\langle \Omega, \oplus, \varepsilon \rangle$ is a commutative monoïd,
(ii) $\langle \Omega, \otimes, e \rangle$ is a monoïd,
(iii) the distribution laws $x \otimes (y \oplus z) = (x \otimes y) \oplus (x \otimes z)$ and $(x \oplus y) \otimes z = (x \otimes z) \oplus (y \otimes z)$,
(iv) $\varepsilon \otimes x = x \otimes \varepsilon = \varepsilon$ for every x.

It is known that each of the following is a semiring:
$\langle \mathfrak{R} \cup \{\infty\}, \min, +, \infty, 0 \rangle$, $\langle \mathfrak{R} \cup \{-\infty\}, \max, +, -\infty, 0 \rangle$,
$\langle \mathfrak{R}^+ \cup \{\infty\}, \max, \min, 0, \infty \rangle$, $\langle [0,1], \max, \cdot, \infty, 0, 1 \rangle$ where min and max are defined in the obvious fashion.

Definition 2 (Dioïd, path algebra, Gondran [9], Yager [22]). A dioïd or path algebra $\langle \Omega, \oplus, \otimes, \varepsilon, e \rangle$ is a semiring in which:

The canonical pre-order relation relatively to $\oplus$ (defined by $x \preccurlyeq y \Leftrightarrow \exists z : y = x \oplus z$) is an order relation, so it verifies $((x \preccurlyeq y) \wedge (y \preccurlyeq x) \Rightarrow x = y)$.

A dioïd is said commutative if the law $\otimes$ is commutative.

Definition 3 (Ring, Kuich and Salomaa [14]). We call a ring a semiring in which the basis set Ω has a structure of commutative group for the operation $\oplus$.

Remark 4 (Minoux [19]). $\langle \mathbb{Z}, +, \times, 0, 1 \rangle$ is a ring but is not a dioïd, $\langle \mathbb{N}, +, \times, 0, 1 \rangle$ is a dioïd. It is therefore the presence of a relation order intrinsically bound to the operation $\oplus$ that constitutes the main distinction between rings and dioïds.

Dioïds have been used for the formulation of graph path-finding problems. The solving of an operations research problem (classic problems) consists to determining an algebraic structure based on dioïds and applying some general algorithms. Let us recall some classic dioïds examples conceived to solve path-finding problems: The structure $\langle \mathfrak{R} \cup \{\infty\}, \min, +, \infty, 0 \rangle$ correspond to the classical shortest path problem, $\langle \mathfrak{R} \cup \{-\infty\}, \max, +, -\infty, 0 \rangle$ correspond to the classical longest path problem [11,19].

3. Concepts of fuzzy sets and fuzzy graphs

This section introduces the basic concepts of fuzzy sets used through out this paper.

3.1. Concepts of fuzzy sets

Let Ω denote a universal set, then the characteristic function of a crisp set Ω assigns a value of either 1 or 0 to each individual in the universal set. This function can be generalized such that the values assigned to the elements of Ω fall within a specified range.

Such a function is called a membership function and the set defined by it is a fuzzy set. The membership function $\mu_{\tilde{A}}$ by which a fuzzy set $\tilde{A}$ is usually defined as the form $\mu_{\tilde{A}}: \Omega \to [0,1]$ [13].

Given a crisp universal set Ω, let $P(\Omega) = [0,1]^{\Omega}$ denote the fuzzy sets of Ω. The support of a fuzzy set $\tilde{A}$ in the universal set Ω is obtained by the function $\Theta(\tilde{A}) = \{x \in \Omega | \mu_{\tilde{A}}(x) \succ 0\}$. Let us introduce a special notation that is often used in the literature for defining fuzzy sets with a finite support. Assume that $x_i \in \Theta(\tilde{A})$ and that μ_i is its grade of membership in $\tilde{A}$, then $\tilde{A}$ is written as $\tilde{A} = \sum_{i=1}^{n} \mu_i' x_i$.

Similarly, when Ω is an interval of real numbers, a fuzzy set $\tilde{A}$ is often written as $\tilde{A} = \int_{\Omega} \mu_{\tilde{A}}(x)' x$

3.2. The extension principle

One of the most basic concepts of fuzzy set theory which can be used to generalize crisp mathematical concepts to fuzzy sets is the extension principle [6]. Let Ω be a cartesian product of n universes $\Omega = \Omega_1 \times \cdots \times \Omega_n$, and $\tilde{A}_1, \ldots, \tilde{A}_n$ be n fuzzy sets in $\Omega_1, \ldots, \Omega_n$, respectively.

ϖ is a mapping from Ω to a universe Λ, $y = \varpi(x_1, \ldots, x_n)$. Then the extension principle allows us to define a fuzzy set $\tilde{B}$ in Λ by

$\tilde{B} = \{(y, \mu_{\tilde{B}}(y)) | y = \varpi(x_1, \ldots, x_n), (x_1, \ldots, x_n) \in \Omega\}$,

where $\mu_{\tilde{B}}(y) = \mathrm{Sup}_{(x_1,\ldots,x_n) \in \varpi^{-1}(y)}(\mathrm{Min}\{\mu_{\tilde{A}_1}(x_1), \ldots, \mu_{\tilde{A}_n}(x_n)\})$

if $\varpi^{-1}(y) \neq \varnothing$ else $\mu_{\tilde{B}}(y) = 0$,

where ϖ^{-1} is the inverse of ϖ.

The extension principle will be used in Section 4 to define the dioïd structure based on two internal operations $\oplus$ and $\otimes$.

3.3. Basic concepts of fuzzy graphs

Definition 5 (Fuzzy graph). Let Ω be a finite set which is assumed equal to $\{1, \ldots, n\}$. The triplet $G(\Omega, \sigma, \mu)$ will be called a fuzzy graph on Ω where:

$\sigma: \Omega \to [0,1]$ and stands for the membership level of each node;

and $\mu: \Omega \times \Omega \to [0,1]$ and stands for the membership level of each arc [4].

Definition 6 (Valuation on fuzzy graphs). Let $G(\Omega, \sigma, \mu)$ be a fuzzy graph and assume that nodes and arcs are crisp, for every $(i,j) \in \Omega^2$ we define the valuation associated with (i,j), such as a function ϕ, such that [4]: $\phi: \Omega^2 \to \aleph$, where $\aleph$ is an ordered set. The structure $G(\Omega, \sigma, \mu, \phi)$ will be called valued fuzzy graph.

Example 7. Fig. 1 gives an example of valued fuzzy graph, where:

$\Omega = \{1,2,3,4,5,6\}$, $\phi(1,2) = \{.1'1, .5'2, .8'3\}$,

$\phi(1,3) = \{.2'1, .5'2, .6'3\}$, $\phi(1,4) = \{.3'1, .1'2, .6'3\}$,

$\phi(5,2) = \{.1'1, .3'2, .4'3\}$, $\phi(2,4) = \{.2'1, .6'2, .9'3\}$,

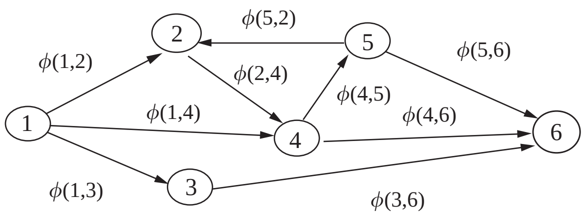

Fig. 1. Fuzzy graph.

$\phi(4,5) = \{.3'1, .3'2, .2'3\}, \quad \phi(3,6) = \{.5'1, .6'2, .6'3\},$
$\phi(4,6) = \{.3'1, .6'2, .1'3\}, \quad \phi(5,6) = \{.1'1, .3'2, .4'3\}.$

4. Path algebra and the K-best fuzzy shortest paths problem

In this section we introduce a new dioïd structure (path algebra) to solve the K-best fuzzy shortest path-finding problem. All technical constructions are given in the following subsection.

4.1. Dioïd of the K-best fuzzy shortest paths problem

Let Ω_k be the set of all fuzzy sets of $\mathbb{N}$ the set of natural numbers, with its support's cardinality less or equal than k. The couple $(\mathbb{N} \cup \{+\infty\}, \leqslant)$ represent an ordered structure, where the relation $\leqslant$ defines the natural order on the set $\mathbb{N}$.

We have $\Omega_k = \{\tilde{A} \in [0,1]^{\mathbb{N}} \mid \|\Theta(\tilde{A})\| \leqslant k\}$, $\|\Theta(\tilde{A})\|$ denotes the cardinal of the set $\Theta(\tilde{A})$. The symbols $\tilde{\cup}$ and $\tilde{\cap}$ denote, respectively, the union and intersection operators defined on fuzzy sets [6,13]. The symbol $\mp$ denotes the addition operator defined on fuzzy sets.

Let the operator $\Pi_k(\,)$ defined on the crisp subsets of $\mathbb{N}$ the set of natural numbers, denotes the select or sorting operator that returns only the k first-ordered element of the considered set. The selection operator $\Pi_k(\,)$ uses the order relation that is defined on natural numbers. We define the operator $[\,]_k$ on subsets of Ω_k by

$$[\tilde{A}]_k = \Pi_k(\Theta(\tilde{A})) \tilde{\cap} \tilde{A},$$

where $\forall \omega \in \Omega \to \mu_{[\tilde{A}]_k}(\omega) = 1_{\Pi_k(\Theta(\tilde{A}))}(\omega) \times \mu_{\tilde{A}}(\omega)$,

where $1_A(\,)$ denotes the classical indicator function of the set A. ($\forall \omega \in \Omega$, if $\omega \in A$ then $1_A(\omega) = 1$ else $1_A(\omega) = 0$).

Stages of construction of the structure $\langle \Omega_k, \oplus, \otimes, \varepsilon, e \rangle$ are developed below.

The operation $\oplus$: Let $\tilde{A} \in \Omega_k$ and $\tilde{B} \in \Omega_k$ then we define the fuzzy set $\tilde{A} \oplus \tilde{B}$ by $\tilde{A} \oplus \tilde{B} = [\tilde{A} \tilde{\cup} \tilde{B}]_k$. The grade of membership of the elements of $\tilde{A} \oplus \tilde{B}$ are given by

$$\begin{aligned} \forall \omega \in \Omega_k \to \mu_{\tilde{A} \oplus \tilde{B}}(\omega) &= \mu_{[\tilde{A} \tilde{\cup} \tilde{B}]_k}(\omega) = 1_{\Pi_k(\Theta(\tilde{A} \tilde{\cup} \tilde{B}))}(\omega) \times \mu_{\tilde{A} \tilde{\cup} \tilde{B}}(\omega) \\ &= 1_{\Pi_k(\Theta(\tilde{A} \tilde{\cup} \tilde{B}))}(\omega) \times \max(\mu_{\tilde{A}}(\omega), \mu_{\tilde{B}}(\omega)). \end{aligned}$$

The operation $\otimes$: Let $\tilde{A} \in \Omega_k$ and $\tilde{B} \in \Omega_k$ then we define the fuzzy set $\tilde{A} \otimes \tilde{B}$ by $\tilde{A} \otimes \tilde{B} = [\tilde{A} \mp \tilde{B}]_k$. According to the extension principle, the grade of membership of the elements of $\tilde{A} \otimes \tilde{B}$ are given by

$$\forall \omega \in \Omega_k \to \mu_{\tilde{A}\otimes\tilde{B}}(\omega) = \mu_{[\tilde{A}\mp\tilde{B}]_k}(\omega) = 1_{\Pi_k(\Theta(\tilde{A}\mp\tilde{B}))}(\omega) \times \mu_{\tilde{A}\mp\tilde{B}}(\omega)$$
$$= 1_{\Pi_k(\Theta(\tilde{A}\mp\tilde{B}))}(\omega) \times \max_{\omega=x+y}(\min(\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(y))).$$

Example 8. Let $\tilde{A} = \{.1'1, .5'2, .8'3\}$, $\tilde{B} = \{.2'1, .5'2, .6'3\}$ and $k = 3$. Then while using definitions above, we obtain

$$\tilde{A} \oplus \tilde{B} = [\tilde{A}\tilde{\cup}\tilde{B}]_3 = [\{.2'1, .5'2, .8'3\}]_3 = \{.2'1, .5'2, .8'3\},$$

$$\tilde{A} \otimes \tilde{B} = [\tilde{A} \mp \tilde{B}]_3 = [\{.1'2, .2'3, .5'4, .5'5, .5'6\}]_3 = \{.1'2, .2'3, .5'4\}.$$

Proposition 9 (The K-best fuzzy shortest paths dioïd). *The structure* $\langle \Omega_k, \oplus, \otimes, \varepsilon, e\rangle$ *developed above and which is proposed for the K-best fuzzy shortest paths problem, corresponds to a dioïd structure, where* $\varepsilon = \emptyset$ *and* $e = \{1/0\}$.

Proof. The structure $\langle \Omega_k, \oplus, \otimes, \varepsilon, e\rangle$ verifies the path algebra (or dioïd) properties. The triplet $([0,1]^{\Omega}, \mp, \tilde{\cup})$ is a semiring, therefore the distributivity of the operation $\otimes$ relative to the operation $\oplus$ can be deduced. $\square$

4.2. Generalized Gauss–Seidel's path-finding algorithm

Let $G(\Omega, \sigma, \mu, \phi)$ be a valued fuzzy graph. Assume that the structure $(\Omega, \oplus, \otimes)$ is a path algebra. In the case of a graph without p-absorbing cycles the general algorithm is given below (see algorithm 1) [10,11,19]. Here, $\pi(i)$ is the length of the shortest path or paths from vertex 1 to vertex i. Through the general algorithm, vertex i is labeled with $\pi(i)$, and the labels allows the determination of the path.

Algorithm 1 (Gauss–Seidel's algorithm, Gondran and Minoux [10, 11]). *Let* $G(\Omega, \sigma, \mu, \phi)$ *be a valued fuzzy graph, where* $\phi(i,i) = \varepsilon$.

The function Γ *gives the set of successors of each node* $i \in \Omega$.

(α): $\pi(1) = e$, $\pi(i) = \phi(1,i)$ *for* $i \geqslant 2$,

(β): *at step* k,

for $i = 1 \ldots n$ **do**

$$\pi(1) = \sum_{j\in\Gamma^{-1}(1)}^{\oplus} [\pi(j) \otimes \phi(j,1) \oplus e],$$

$$\pi(i) = \sum_{j\in\Gamma^{-1}(i)}^{\oplus} [\pi(j) \otimes \phi(j,i)] \ (i \geqslant 2).$$

(χ): **repeat** (β) **until** *stabilization of* $\pi(i)$.

4.2.1. Algorithm complexity analysis

Let n and m denote, respectively, the number of nodes and arcs of a valued fuzzy graph without p-absorbing cycles. The first loop (α) uses O(n) operations, there are $(n-1)$ iterations in the loop (β) with m $(\oplus, \otimes)$ operations. Hence we have O($n \times m$) operations at this step. The operation $\oplus$ needs k comparisons and the operation $\otimes$ uses $k \times \log(k)$ operations (for a sorting algorithm). Hence in the algorithm the complexity of the operation $\otimes$ is $\Theta(k + k^2 + k \times \log(k))$.

Proposition 10 (Convergence of the algorithm). *The generalized Gauss–Seidel's algorithm always converges in $\langle \Omega_k, \oplus, \otimes, \varepsilon, e \rangle$ path algebra, and solves the K-best fuzzy shortest paths problem on a valued fuzzy graph without p-absorbing cycles.*

Proof. It is known that if the graph doesn't contain cycles of negative length, then the weight of every cycle is $(K-1)$-*regular* [10] in the dioïd $\langle \Omega_k, \oplus, \otimes, \varepsilon, e \rangle$ associated to the K-best fuzzy shortest paths problem. Once the operation $\otimes$ is commutative, then we can deduce from the Gondran's theorem [11,8] the convergence of the algorithm. □

All labels $\pi(i)$ are fuzzy sets of Ω_k. These labels give the fuzzy lengths of paths going from the departure node 1 to any node i of the fuzzy graph.

Example 11. We describe in this example the main stages of the algorithm. First note that in this illustration, node 1 correspond to the source node, and $k = 3$. By applying the algorithm using the K-best fuzzy shortest paths dioïd (see algorithm 1) on the valued fuzzy graph given in Fig. 1.

With these considerations we obtain the following results:

Initialization step: $\pi(1) = e = \{1'0\}, \pi(2) = \{.1'1, .5'2, .8'3\}$,

$\pi(3) = \{.2'1, .5'2, .6'3\}$, $\pi(4) = \{.3'1, .1'2, .6'3\}$,

$\pi(5) = \pi(6) = \varepsilon = \emptyset$.

After convergence the labels are given by:

$\pi(1) = \{1'0, 1'0, 1'0\}, \pi(2) = \{.1'1, .5'2, .8'3\}, \pi(3) = \{.2'1, .5'2, .6'3\}$,

$\pi(4) = \{.3'1, .1'2, .6'3\}, \pi(5) = \{.3'2, .3'3, .3'3\}, \pi(6) = \{.3'2, .5'2, .5'3\}$. The K-best fuzzy shortest path length is then given by: $\pi(6) = \{.3'2, .5'3, .5'3\}$.

A decision maker can then find that the first best shortest possible path has length 2 with a grade membership of 0.3. This path corresponds to the path: $[.3'2]$ $(1 \to 4 \to 6)$ in the original valued fuzzy graph (Fig. 1). If the decision maker takes a threshold of membership of 0.5, then he could choose the path of length 3 which correspond to the path $[.5'3]$ $(1 \to 3 \to 6)$. Our approach in this paper is to first present the model based on the path algebra.

As proposed in [12], this approach finds a fuzzy number representing the fuzzy shortest path length but it also associates an effective path to that number. The membership grades are associated to the arcs and “nondominated” paths are found.

5. Fuzzy graphs and object oriented modelling

Our main practical objective is to develop a *Fuzzy Graph Object Component*. The software component could be integrated in many applications. To this end, we have used mainly the

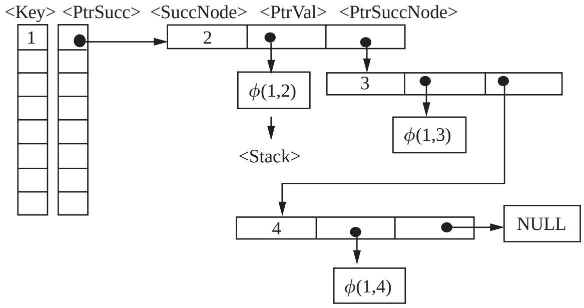

Fig. 2. Class diagram of the fuzzy graphs software component (UML notation).

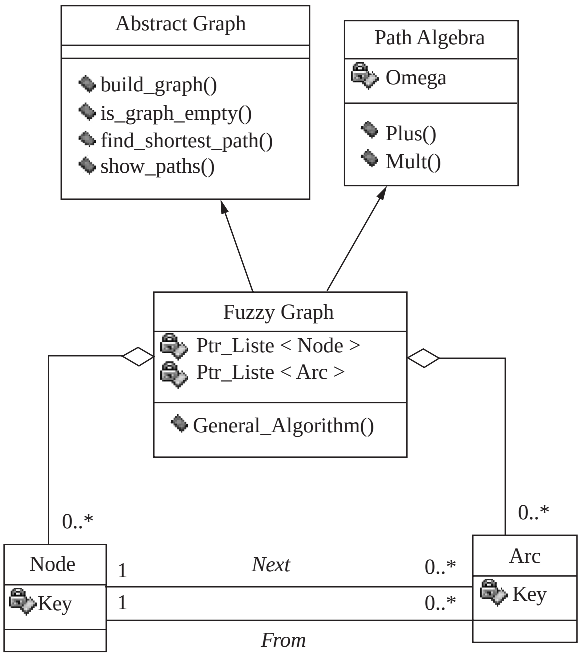

Fig. 3. A directed valued fuzzy graph and its linked implementation.

object-oriented-programming methodologies. We use the C++ programming language for the implementation of concepts.

Frequently basic Abstract Data Type are implemented in the Standard Template Library (STL). STL is assumed to become a part of the C++ standard library and therefore it is an ideal basis when writing portable programs.

Unfortunately, STL has no support for graphs Abstract Data Type. We decided to implement a fuzzy graph library based on STL. The built library contains the classes needed to work with fuzzy graphs (see Figs. 2 and 3), nodes and edges and some basic fuzzy path-finding algorithms (Klein's algorithm, Dijkstra's algorithm for parametric and relation orders, generalized Gauss–Seidel's algorithm with the corresponding path algebra for the K-best shortest paths problem).

6. Conclusion

In this paper we provide a new path algebra structure for the K-best fuzzy shortest paths problem on the valued fuzzy graphs. The main result introduced in this paper concerns the extension of the path-finding algorithms to valued fuzzy graphs. More precisely, we have used the generalized Gauss–Seidel's algorithm with the proposed path algebra to solve the K-best fuzzy shortest paths problem. Future investigations would involve the analysis of the obtained paths and the development of procedures to help decision maker for choosing paths. Further orientations would be considered to improve the computing time aspect. The transposition of results given in this paper to solve some operational problems is foreseeable (scheduling problem with duration expressed with fuzzy variables, fuzzy routing problem in the area of telecommunication and transportation integrating the fuzzy modelling [1,2]).

References

[1] A. Boulmakoul, R. Laurini, M.A. Janati, First specifications of a telegeomonitoring system for the transportation of hazardous materials, Comput. Environ. Urban Systems 23 (1999) 259–270.
[2] A. Boulmakoul, Z. Zeitouni, R. Laurini, M.A. Aufaure, Spatial decision support system for hazardous materials transportation planning, IFAC Transportation Systems'97, Chania, Greece, June 16–18, 1997, pp. 611–616.
[3] S. Chanas, M. Delgado, J.L. Verdegay, M.A. Vila, Fuzzy optimal flow on imprecise structures, European J. Oper. Res. 83 (1995) 568–580.
[4] M. Delgado, J.L. Verdegay, M.A. Vila, On valuation and optimisation problems in fuzzy graphs: a general approach and some particular cases, ORSA J. Comput. 2 (1) (1990) 74–83.
[5] D. Dubois, H. Prade, Algorithmes de plus courts chemins pour traiter des donnees floues, RAIRO/Oper. Res. 2 (2) (1978) 213–227.
[6] D. Dubois, H. Prade, Fuzzy Sets and Systems, Academic Press, New York, 1980.
[7] N. Furukawa, A parametric total order on fuzzy numbers and a fuzzy shortest route problem, Optimization 30 (1994) 367–377.
[8] M. Gondran, Algebre lineaire et cheminement dans un graphe, RAIRO/Oper. Res. 1 (1975) 77–99.
[9] M. Gondran, Path algebra and algorithms, in: B. Roy (Ed.), Combinatorial Programming: Methods and Applications, Reidel, Dordrecht, 1975, pp. 137–148.
[10] M. Gondran, M. Minoux, Linear algebra in dioïds: a survey of recent results, Ann. Discrete Math. 19 (1984) 147–164.
[11] M. Gondran, M. Minoux, Graphes et algorithmes, 3eme Edition, Eyrolles, Paris, 1995.
[12] C.M. Klein, Fuzzy shortest paths, Fuzzy Sets and Systems 39 (1991) 27–41.
[13] G.J. Klir, T.A. Folger, Fuzzy Sets, Uncertainty and Information, Prentice-Hall, Englewood Cliffs, NJ, 1988.
[14] L.T. Koczy, Fuzzy graphs in the evaluation and optimisation of networks, Fuzzy Sets and Systems 46 (1992) 307–319.
[15] W. Kuich, A. Salomaa, Semirings, Automata, Languages, Vol. 5, Springer, Berlin, 1986.
[16] J. Kuntzmann, Theorie des reseaux, DUNOD, Paris, 1972.
[17] E. Lawler, Combinatorial optimization, Networks and Matroids, Holt, Reinehart & Winston, New York, 1976.
[18] K.C. Lin, M.S. Chern, The fuzzy shortest path problem and its most vital arcs, Fuzzy Sets and Systems 58 (1993) 343–353.

[19] M. Minoux, Structures algebriques generalisees des problemes de cheminements dans les graphes: theoremes, algorithmes et applications, RAIRO Rech. Oper. 10 (6) (1976) 33–62.
[20] S. Okada, T. Soper, A shortest path problem on a network with fuzzy arc lengths, Fuzzy Sets and Systems 109 (2000) 129–140.
[21] H. Prade, Using fuzzy set theory in a scheduling problem: a case study, Fuzzy Sets and Systems 2 (1979) 153–165.
[22] R.A. Yager, On general class of fuzzy connectivities, Fuzzy Sets and Systems 4 (1980) 235–242.
[23] L.A. Zadeh, Fuzzy sets, Inform. Control 8 (1965) 338–353.