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Tunnelling effect of charged and magnetized particles from the Kerr–Newman–Kasuya black hole

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Abstract

In this Letter, we extend the Parikh–Wilczek tunnelling framework to calculate the emission rate of a particle with electric and magnetic charges. We first reconstruct the electromagnetic field tensor and the Lagrangian of the field corresponding to the source with electric and magnetic charges. Then, in the background of Kerr–Newman–Kasuya black hole spacetime, we calculate the emission spectrum of the outgoing particles with electric and magnetic charges. For the sake of simplicity, we only consider the case that the rate of electric and magnetic charge of the emission particle is constant and equals that of the black hole. In this case, the emission spectrum deviates from the pure thermal spectrum, but it is consistent with an underlying unitary theory and takes the same functional form as that of uncharged massless particles. Finally, discussions about the result are presented.

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Keywords: Black hole; Hawking radiation; Quantum theory; Magnetic charge

1. Introduction

Parikh–Wilczek's tunnelling framework is a semiclassical method of calculating Hawking radiation, which was developed by Kraus and Wilczek and elaborated upon by Parikh and Wilczek [1–4]. The most difference from that of Hawking's original one is that the energy conservation was enforced in their computation. Thus, a corrected spectrum, which deviates from the exact thermal spectrum but consists with an underlying unitary theory, was obtained. Following this method, a number of static or stationary rotating black holes were studied [5–19]. The same result, that is, Hawking radiation is no longer pure thermal, unitary theory is satisfied and information is conserved, was obtained. In 2005 and 2006, Refs. [20–25] extended the method, and the emission rates of massive particles and charged particles were calculated, respectively. In this Letter, we attempt to extend this method to calculate the emission spectrum of charged and magnetized particles from the Kerr–Newman–Kasuya black hole. The major problem is how to treat the electromagnetic field created by the electric and magnetic charges. For the spacetime of Kerr–Newman–Kasuya black hole, electric and magnetic charges concentrate on the black hole, the outside of the hole is an electromagnetic vacuum. We can take the hole as a conducting sphere [26], and without loss of generality, we think that the rate of the densities of the electric and magnetic charges is constant. For the sake of simplicity, here we only consider the case that the rate of electric and magnetic charge of the emission particle is constant and equals that of the black hole. In this case, we can introduce an equivalent electromagnetic tensor, and rewrite the Lagrangian of the field. We find that the Lagrangian of the electromagnetic field can be

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expressed by a set of generalized coordinates $\widetilde{A}_\mu = (\widetilde{A}_t, \widetilde{A}_1, \widetilde{A}_2, \widetilde{A}_3)$. But these coordinates are cyclic coordinates. To eliminate the freedoms corresponding to these coordinates, we modify the Lagrangian function and use Parikh–Wilczek's method to get the corrected emission spectrum. Throughout the Letter, the geometrized units ($G \equiv c \equiv \hbar \equiv 1$) are used.

2. Maxwell equation and Lagrangian function corresponding to the source with electric and magnetic charges

In the following discussion, we first investigate the Maxwell equation corresponding to the source with electric and magnetic charges, and then write out the Lagrangian of the field.

For a source with electric and magnetic charge, the electromagnetic tensor is defined as

$$F_{\mu\nu} = \nabla_\nu A_\mu - \nabla_\mu A_\nu + G^+_{\mu\nu}, \tag{1}$$

where $G^+_{\mu\nu}$ is the Dirac string term [27]. Maxwell equation can be rewriten as

$$\nabla_\nu F^{\mu\nu} = 4\pi\rho_e u^\mu, \tag{2}$$

$$\nabla_\nu F^{+\mu\nu} = 4\pi\rho_g u^\mu, \tag{3}$$

where $F^{+\mu\nu}$ is the dual tensor of $F^{\mu\nu}$, ρ_e and ρ_g are the densities of electric and magnetic charges, respectively, while u^μ is the 4-velocity. In order to rewrite the Maxwell equations (2) and (3) to a simple form, we define a new real antisymmetric tensor

$$\widetilde{F}^{\mu\nu} = F^{\mu\nu}\cos\alpha + F^{+\mu\nu}\sin\alpha, \tag{4}$$

where α denotes a real constant angle. From (2) and (3) we get

$$\nabla_\nu \widetilde{F}^{\mu\nu} = 4\pi(\rho_e\cos\alpha + \rho_g\sin\alpha)u^\mu, \tag{5}$$

$$\nabla_\nu \widetilde{F}^{+\mu\nu} = 4\pi(-\rho_e\sin\alpha + \rho_g\cos\alpha)u^\mu. \tag{6}$$

If we let

$$\rho_e\cos\alpha + \rho_g\sin\alpha = \rho_h, \tag{7}$$

$$-\rho_e\sin\alpha + \rho_g\cos\alpha = 0, \tag{8}$$

that is, $\rho_e/\rho_g = \cot\alpha$, then, the Maxwell equation can be written as

$$\nabla_\nu \widetilde{F}^{\mu\nu} = 4\pi\rho_h u^\mu, \tag{9}$$

$$\nabla_\nu \widetilde{F}^{+\mu\nu} = 0, \tag{10}$$

that is

$$\frac{\partial}{\partial x^\nu}\left(\sqrt{-g}\,\widetilde{F}^{\mu\nu}\right) = 4\pi\sqrt{-g}\,J^\mu, \tag{11}$$

where $J^\mu = \rho_h u^\mu$. Obviously, Eqs. (9)–(11) are similar to the Maxwell equation corresponding to the source with only electric charges. As mentioned above, for the Kerr–Newman–Kasyer black hole spacetime, electric and magnetic charges concentrate on the black hole. According to the No-hair Theorem, most of the information about the hole was lost. Without loss of generality, we can take the hole as a conducting sphere and suppose that the densities of electric and magnetic charges satisfy Eqs. (7) and (8), that is $\rho_e/\rho_g = \cot\alpha$, then we have

$$Q_h^2 = Q_e^2 + Q_g^2, \tag{12}$$

where Q_e and Q_g are the electric and magnetic charges of the hole, respectively, while Q_h is the equivalent charge corresponding to the density ρ_h. Similarly, we can construct the Lagrangian of the electromagnetic field as follows

$$L_h = -\frac{1}{4}\widetilde{F}_{\mu\nu}\widetilde{F}^{\mu\nu}. \tag{13}$$

In Eq. (13) the corresponding generalized coordinates are

$$\widetilde{A}_\mu = \left(\widetilde{A}_t, \widetilde{A}_1, \widetilde{A}_2, \widetilde{A}_3\right), \tag{14}$$

which satisfy

$$\widetilde{F}_{\mu\nu} = \nabla_\nu\widetilde{A}_\mu - \nabla_\mu\widetilde{A}_\nu. \tag{15}$$

3. Phase velocity and electromagnetic potential

The spacetime of the Kerr–Newman–Kasyer black hole can be written as the following [29,30]

$$ds^2 = -\left(1 - \frac{2Mr - (Q_e^2 + Q_g^2)}{\rho^2}\right) dt_k^2 + \frac{\rho^2}{\Delta} dr^2 + \rho^2 d\theta^2 + \left[\left(r^2 + a^2\right)\sin^2\theta + \frac{(2Mr - (Q_e^2 + Q_g^2))a^2 \sin^4\theta}{\rho^2}\right] d\varphi^2 - \frac{2(2Mr - (Q_e^2 + Q_g^2))a \sin^2\theta}{\rho^2} dt_k\, d\varphi, \tag{16}$$

where

$$\rho^2 \equiv r^2 + a^2 \cos^2\theta, \tag{17}$$

$$\Delta \equiv r^2 + a^2 + \left(Q_e^2 + Q_g^2\right) - 2Mr. \tag{18}$$

Considering Eq. (12) the line element can be rewritten as

$$ds^2 = -\left(1 - \frac{2Mr - Q_h^2}{\rho^2}\right) dt_k^2 + \frac{\rho^2}{\Delta} dr^2 + \rho^2 d\theta^2 + \left[\left(r^2 + a^2\right)\sin^2\theta + \frac{(2Mr - Q_h^2)a^2 \sin^4\theta}{\rho^2}\right] d\varphi^2 - \frac{2(2Mr - Q_h^2)a \sin^2\theta}{\rho^2} dt_k\, d\varphi. \tag{19}$$

The event horizon $r = r_H$ is given by

$$r_H = M + \sqrt{M^2 - a^2 - Q_h^2}, \tag{20}$$

and the 4-dimensional electromagnetic potential is

$$\widetilde{A}_a = -\rho^{-2} Q_h r\left[(dt)_a - a \sin^2\theta (d\varphi)_a\right]. \tag{21}$$

To calculate the emission rate, we should adopt a good behaved coordinate system at the event horizon.

Here, we take the Painlevé–Kerr–Newman–Kasuya coordinate system, and the line element in this coordinate system can be obtained from Ref. [17]. Namely,

$$ds^2 = \hat{g}_{00}\, dt^2 + 2\sqrt{\hat{g}_{00}(1 - g_{11})}\, dt\, dr + dr^2 + \left[\hat{g}_{00} G(r,\theta)^2 + g_{22}\right] d\theta^2 + 2\hat{g}_{00} G(r,\theta)\, dt\, d\theta + 2\sqrt{\hat{g}_{00}(1 - g_{11})} G(r,\theta)\, dr\, d\theta, \tag{22}$$

where

$$\hat{g}_{00} = -\frac{\rho^2 \Delta}{(r^2 + a^2)^2 - \Delta a^2 \sin^2\theta}, \tag{23}$$

$$g_{11} = \frac{\rho^2}{\Delta}, \tag{24}$$

$$g_{22} = \rho^2. \tag{25}$$

The components of the electromagnetic potential in the Painlevé–Kerr–Newman–Kasuya coordinate system is

$$\widetilde{A}_0 = -\rho^{-2} Q_h r\left[1 - a\Omega \sin^2\theta\right], \quad \widetilde{A}_1 = \widetilde{A}_2 = 0. \tag{26}$$

From (26) we obtain the electromagnetic potential on the event horizon

$$\widetilde{A}_0|_{r_H} = -\widetilde{V}_0 = -\frac{Q_h r_H}{r_H^2 + a^2}, \quad \widetilde{A}_1|_{r_H} = \widetilde{A}_2|_{r_H} = 0. \tag{27}$$

4. Emission rate

In our discussion, the outgoing particle is charged and magnetized. But, as mentioned above, we can treated it as an equivalent electric charge q_h. Similar to Ref. [21], we treat the equivalent charged particle as a de Broglie wave and we can easily obtain the expression of $\dot{r}$. Namely,

$$\dot{r} = v_p = -\frac{1}{2}\frac{\hat{g}_{00}}{\hat{g}_{01}} = \frac{\Delta}{2}\sqrt{\frac{\rho^2}{(\rho^2 - \Delta)[(r^2 + a^2)^2 - \Delta a^2 \sin^2\theta]}}. \tag{28}$$

As described in Ref. [21], we should take into account the self-gravitation of the tunnelling particle with energy ω and equivalent electric charge q_h. That is, we should replace M and Q_h with $M-\omega$ and Q_h-q_h in (22) and (28), respectively.

Considering the matter-gravity system which consists of the black hole and the electromagnetic field outside the hole, the Lagrangian function of the matter-gravity system should be written as

$$L = L_m + L_h, \tag{29}$$

where $L_h = -\frac{1}{4}\widetilde{F}_{\mu\nu}\widetilde{F}^{\mu\nu}$ is the Lagrangian function of the electromagnetic field corresponding to the generalized coordinates $\widetilde{A}_\mu = (\widetilde{A}_t, 0, 0)$ in the Painlevé–Kerr–Newman–Kasuya coordinate system [28]. When a equivalent charged particle tunnels out, the system transit from one state to another. But from the expression of L_h we find that $\widetilde{A}_\mu = (\widetilde{A}_t, 0, 0)$ is a cyclic coordinate. Moreover, in the Painlevé–Kerr–Newman–Kasuya coordinate system, φ does not appear in the line element expressions (22). That is to say, φ is also a cyclic coordinate to the Lagrangian function L. To eliminate these two freedoms completely, the action for the classically forbidden trajectory should be written as

$$S = \int_{t_i}^{t_f} \big(L - P_{\widetilde{A}_t}\dot{\widetilde{A}}_t - P_\varphi\dot{\varphi}\big)\,dt, \tag{30}$$

which is related to the emission rate of the tunnelling particle by

$$\Gamma \sim e^{-2\,\mathrm{Im}\,S}. \tag{31}$$

Therefore, the imaginary part of the action is

$$\mathrm{Im}\,S = \mathrm{Im}\left\{\int_{r_i}^{r_f}\left[P_r - \frac{P_{\widetilde{A}_t}\dot{\widetilde{A}}_t}{\dot{r}} - \frac{P_\varphi\dot{\varphi}}{\dot{r}}\right]dr\right\} = \mathrm{Im}\left\{\int_{r_i}^{r_f}\left[\int_{(0,0,0)}^{(P_r,P_{\widetilde{A}_t},P_\varphi)} dP'_r - \frac{\dot{\widetilde{A}}_t}{\dot{r}}\,dP'_{\widetilde{A}_t} - \frac{\dot{\varphi}}{\dot{r}}\,dP'_\varphi\right]dr\right\}, \tag{32}$$

where $P_{\widetilde{A}_t}$ and P_φ are the canonical momentums conjugate to $\widetilde{A}_t$ and φ, respectively. Similar to Ref. [21], we treat the black hole as a rotating sphere and consider the particle self-gravitation. Therefore,

$$\dot{\varphi} = \Omega'_H, \tag{33}$$

and

$$J' = \big(M - \omega'\big)a = P'_\varphi, \tag{34}$$

where Ω'_H is the dragged angular velocity of the event horizon. The imaginary part of the action can be rewritten as

$$\mathrm{Im}\,S = \mathrm{Im}\left\{\int_{r_i}^{r_f}\left[\int_{(0,0,J)}^{(P_r,P_{\widetilde{A}_t},J-\omega a)} dP'_r - \frac{\dot{\widetilde{A}}_t}{\dot{r}}\,dP'_{\widetilde{A}_t} - \frac{\Omega'_H}{\dot{r}}\,dJ'\right]dr\right\}. \tag{35}$$

We now eliminate the momentum in favor of energy by using Hamilton's equations

$$\dot{r} = \frac{dH}{dP_r}\bigg|_{(r;\widetilde{A}_t,P_{\widetilde{A}_t};\varphi,P_\varphi)} = \frac{d(M-\omega')}{dP_r} = \frac{dM'}{dP_r}, \tag{36}$$

$$\dot{\widetilde{A}}_t = \frac{dH}{dP_{\widetilde{A}_t}}\bigg|_{(\widetilde{A}_t;r,P_r;\varphi,P_\varphi)} = \frac{V'_0\,dQ'_h}{dP_{\widetilde{A}_t}} = \frac{(Q_h-q'_h)r_H}{r_H^2+a^2}\cdot\frac{d(Q_h-q'_h)}{dP_{\widetilde{A}_t}}. \tag{37}$$

Note that to derive (37) we have treated the black hole as a charged conducting sphere [26].

Substituting (36) and (37) into (35) and considering the self-gravitation of the outgoing particle, we have

$$\mathrm{Im}\,S = \mathrm{Im}\int_{r_i}^{r_f}\left[\int \frac{2\sqrt{(\rho^2-\Delta')[(r^2+a^2)^2-\Delta' a^2\sin^2\theta]}}{\Delta'\sqrt{\rho^2}}\left(dM' - \frac{Q'_h r'_H}{r'^2_H+a^2}\,dQ'_h - \Omega'_H\,dJ'\right)\right]dr, \tag{38}$$

where

$$\Delta' = r^2 + a^2 + Q'^2_h - 2M'r = \big(r - r'_+\big)\big(r - r'_-\big), \tag{39}$$

$$r'_\pm = \big(M-\omega'\big) \pm \sqrt{\big(M-\omega'\big)^2 - a^2 - \big(Q_h - q'_h\big)^2}, \tag{40}$$

$$r_i = M + \sqrt{M^2 - a^2 - Q_h^2}, \tag{41}$$

$$r_f = M - \omega + \sqrt{(M-\omega)^2 - a^2 - (Q_h - q_h)^2}. \tag{42}$$

We see that $r = r'_+ = (M-\omega') + \sqrt{(M-\omega')^2 - a^2 - (Q_h - q'_h)^2}$ is a pole. The integral can be evaluated by deforming the contour around the pole, so as to ensure that positive energy solution decay in time. Doing the r integral first we obtain

$$\operatorname{Im} S = -\frac{1}{2} \int_{(M,Q_h)}^{(M-\omega,Q_h-q_h)} \frac{4\pi(M'^2 + M'\sqrt{M'^2 - a^2 - Q_h'^2} - \frac{1}{2}Q_h'^2)}{\sqrt{M'^2 - a^2 - Q_h'^2}} \left(dM' - \frac{Q'_h r'_H}{r_H'^2 + a^2} dQ'_h - \Omega'_H \, dJ' \right) dr. \tag{43}$$

Finishing the integration we get

$$\begin{aligned} \operatorname{Im} S &= \pi \left[M^2 - (M-\omega)^2 + M\sqrt{M^2 - a^2 - Q_h^2} - (M-\omega)\sqrt{(M-\omega)^2 - a^2 - (Q_h - q_h)^2} - \frac{1}{2}\big(Q_h^2 - (Q_h - q_h)^2\big) \right] \\ &= -\frac{1}{2}\Delta S_{BH}. \end{aligned} \tag{44}$$

In fact, if we bear in mind that

$$T' = \frac{\sqrt{M'^2 - a^2 - Q_h'^2}}{4\pi(M'^2 + M'\sqrt{M'^2 - a^2 - Q_h'^2} - \frac{1}{2}Q_h'^2)}, \tag{45}$$

we easily get

$$\frac{1}{T'}\big(dM' - V'_0 \, dQ'_h - \Omega'_H \, dJ'\big) = dS'. \tag{46}$$

That is, (44) is a natural result of the first law of black hole thermodynamics.

The tunnelling rate is therefore

$$\Gamma \sim \exp[-2\operatorname{Im} S] = e^{\Delta S_{BH}}. \tag{47}$$

Obviously, the emission spectrum (47) deviates from the pure thermal spectrum but consists with an underlying unitary theory and takes the same functional form as that of uncharged massless particles.

5. Discussions

5.1. The first law of black hole thermodynamics

In previous literature, the first law of black hole thermodynamics only consider the change of the electric charges. In our discussion, the change of the magnetic charge is also taken into account, and the general form of the first law can be obtained easily.

In fact, from Eqs. (7) and (8) we have

$$\cos\alpha \, dQ_e + \sin\alpha \, dQ_g = dQ_h, \tag{48}$$

$$-\sin\alpha \, dQ_e + \cos\alpha \, dQ_g = 0. \tag{49}$$

Substituting (48) and (49) into (46) yields

$$dS'_{BH} = \frac{1}{T'} dM' - \frac{\Omega'}{T'} dJ' - \frac{V'_e}{T'} dQ'_e - \frac{V'_g}{T'} dQ'_g, \tag{50}$$

where

$$V'_e = \frac{Q_e r_H}{r_H^2 + a^2}, \tag{51}$$

$$V'_g = \frac{Q_g r_H}{r_H^2 + a^2}. \tag{52}$$

Eq. (50) is the general differential form of the first law of black hole thermodynamics.

5.2. Rediscussion on the information puzzle

Eq. (50) means that the emission process satisfies the first law of black hole thermodynamics. Similar discussion to Ref. [24] let us know that in Parikh–Wilczek tunnelling framework, the emission process is treated as a reversible process. That is to say, Parikh–Wilczek's result that Hawking radiation satisfy the unitary theory and information about the hole is conserved is only suitable for reversible precess. Considering that a black hole has a negative heat capacity, the black hole cannot get a thermal equilibrium with the outside. Therefore, the process of the emission of particles from a black hole will be irreversible. That is, Parikh–Wilczek's tunnelling framework did not prove the information conservation in black hole radiation.

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