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Single transverse-spin asymmetry in semi-inclusive deep inelastic scattering

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Abstract

We study the single transverse-spin asymmetry in semi-inclusive hadron production in deep inelastic scattering. We derive the leading contribution to the asymmetry at moderate transverse momentum $P_{h\perp}$ of the produced hadron in terms of twist-three quark–gluon correlation functions, and compare with the result obtained from the approach based on factorization at fixed transverse momentum which involves asymmetric transverse-momentum and spin-dependent quark distributions. We verify that the two approaches yield identical results in this regime. Comparing with our earlier calculations for the single-spin asymmetry in the Drell–Yan process, we confirm the sign difference between the time-reversal-odd transverse-momentum-dependent quark distributions in the two processes.

1. The study of single-transverse spin asymmetries (SSAs) has been at the forefront of experimental and theoretical research in strong interaction physics ever since the first observation of strikingly large asymmetries in hadronic scattering in the 1970s [1]. The size of the asymmetries posed a significant challenge for QCD. With the advent of new experimental information from lepton scattering [2] and from RHIC [3], and with major recent theory advances, we are now beginning to obtain a much clearer picture of the possible origins of SSAs in QCD [4].

In particular, two types of mechanisms for generating SSAs in QCD had been identified in the literature: asymmetric transverse-momentum-dependent (TMD) parton distributions (the so-called Sivers functions [5]) or fragmentation functions (Collins functions [6]), and twist-three quark–gluon correlation functions (the so-called Efremov–Teryaev–Qiu–Sterman (ETQS) mechanism [7,8]), again either in the nucleon or in fragmentation [9]. These mechanisms have been applied to the SSAs for various processes in hadronic collisions and lepton–hadron scattering [5,6,8,9].

For a long time, despite a wide-spread belief that the two types of mechanisms were not completely unrelated, the precise connection between them remained obscure. Early efforts to link the two were made in [10–12]. In two recent publications [13], we have demonstrated that even though the two mechanisms each have their own domain of validity, they consistently describe the same physics in the kinematic regime where they both apply. We have shown this in [13] for the case of the SSA for Drell–Yan production of dilepton pairs with invariant mass Q and transverse momentum $q_\perp$. One reason for selecting the Drell–Yan process was that fragmentation (Collins) effects do not contribute to its SSA, and that it is therefore ideally suited for the study of "Sivers-type" effects. At large $q_\perp \sim Q$ where the ETQS mechanism applies, the resulting SSA is of twist-three nature. At small $q_\perp \ll Q$, the factorization in terms of transverse-momentum-dependent (TMD) distributions applies [14–17], among them the Sivers functions. If $q_\perp$ is much larger than Λ_{QCD}, the dependence of these functions on the transverse momentum can be computed

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using QCD perturbation theory. At the same time, the result obtained in the ETQS formalism may also be extrapolated into the regime $\Lambda_{\mathrm{QCD}} \ll q_\perp \ll Q$. We demonstrated in Ref. [13] that the result of this extrapolation is identical to that obtained using the TMD approach. In this sense, we have unified the two mechanisms widely held responsible for the observed SSAs.

In the present Letter, we extend our previous calculations to semi-inclusive hadron production in lepton–hadron deep inelastic scattering (SIDIS) [18], $e(\ell) + p(P) \to e(\ell') + h(P_h) + X$, which proceeds through exchange of a virtual photon with momentum $q_\mu = \ell_\mu - \ell'_\mu$ and invariant mass $Q^2 = -q^2$. A calculation for this process has also been reported in [19] where the twist-three effects in the parton distribution [8] as well as in the fragmentation function [9] were considered. In this Letter, we focus entirely on the single-spin asymmetries coming from the quark–gluon correlation function and/or the Sivers functions in the polarized proton (referred to as the "Sivers-type" SSA in the following) and do not discuss, for example, the contributions associated with the quark–gluon correlation in the fragmentation functions and the Collins effect [6,19]. Nonetheless, our calculation extends the work of [19]. The latter study focuses on the SSAs at large transverse momenta, $P_{h\perp} \sim Q$, with $P_{h\perp}$ the transverse momentum of the final-state hadron in the "hadron frame" defined below. Our primary interest in the present Letter, however, is in hadron production at intermediate transverse momenta, $\Lambda_{\mathrm{QCD}} \ll P_{h\perp} \ll Q$, where we will compare the results calculated from the two mechanisms. In [19], only the so-called "derivative" contributions were taken into account. These may or may not dominate the spin-dependent cross section at large $P_{h\perp} \sim Q$; however, when $P_{h\perp} \ll Q$, there are definitely other equally important contributions, which we will consider.

At first sight, an additional verification of the consistency of the two mechanisms in another physical process might appear to be of rather limited interest. However, there are several reasons why we believe that this is a valuable addition. Foremost, the SIDIS process is of greater current interest experimentally than Drell–Yan, with several experiments producing data for SSAs in lepton scattering [2]. We stress that, apart from clarifying the theoretical description of SSAs, our work also provides a detailed scheme for the practical analysis of single-spin asymmetries, since it addresses the asymmetries over the whole kinematic regime of transverse momentum. Secondly, as is well known by now, the peculiar gauge-link-dependence properties of the Sivers functions [20–22] predict a sign change of the functions when going from the Drell–Yan process to SIDIS. It is important to verify this sign change in an explicit calculation of a physical process, and our way of doing this is to confront our earlier Drell–Yan calculation with that for SIDIS. This provides a test of the QCD factorization and of the (non-)universality of spin-dependent TMD parton distributions.

The presentation of this Letter will very closely follow our previous work. We will start by calculating the SSA for SIDIS at large transverse momentum of the produced hadron, $P_{h\perp} \sim Q$. We will then expand the obtained result for $P_{h\perp} \ll Q$, in order to make contact with the expression provided by TMD factorization [16], and we will verify that also for SIDIS both approaches contain the same physics in the region $\Lambda_{\mathrm{QCD}} \ll P_{h\perp} \ll Q$. For this to hold true, the sign change mentioned above is vital.

2. The differential single-transverse-spin-dependent SIDIS cross section may be calculated from the formula

$$\frac{d\sigma(S_\perp)}{dx_B\,dy\,dz_h\,d^2\vec{P}_{h\perp}} = \frac{2\pi\alpha_{\mathrm{em}}^2}{Q^4} y L_{\mu\nu}(\ell, q) W^{\mu\nu}(P, S_\perp, q, P_h), \tag{1}$$

where α_{em} is the electromagnetic coupling and $x_B \equiv Q^2/2P\cdot q$, $z_h \equiv P\cdot P_h/P\cdot q$, $y \equiv P\cdot q/P\cdot\ell$. We also introduce $S_{ep} = (P+\ell)^2$, the electron–proton center-of-mass energy squared. $L_{\mu\nu}$ and $W^{\mu\nu}$ are the leptonic and hadronic tensors, respectively. The latter depends on the transverse proton spin vector, $S_\perp$. We consider scattering of unpolarized leptons by virtual-photon exchange, in which case the leptonic tensor is given by

$$L^{\mu\nu}(\ell, q) = 2\big(\ell^\mu \ell'^\nu + \ell^\mu \ell'^\nu - g^{\mu\nu} Q^2/2\big). \tag{2}$$

The hadronic tensor has the following expression in QCD:

$$W^{\mu\nu}(P, S_\perp, q, P_h) = \frac{1}{4z_h} \sum_X \int \frac{d^4\xi}{(2\pi)^4} e^{iq\cdot\xi} \langle PS|J_\mu(\xi)|XP_h\rangle\langle XP_h|J_\nu(0)|PS\rangle, \tag{3}$$

where J^μ is the quark electromagnetic current and X represents all other final-state hadrons other than the observed particle h.

It is convenient to write the momentum of the virtual photon in terms of the incoming and outgoing hadron momenta in SIDIS,

$$q^\mu = q_t^\mu + \frac{q\cdot P_h}{P\cdot P_h} P^\mu + \frac{q\cdot P}{P\cdot P_h} P_h^\mu, \tag{4}$$

where q_t^μ is transverse to the momenta of the initial and final hadrons, $q_t^\mu P_\mu = q_t^\mu P_{h\mu} = 0$. q_t is a space-like vector; we define

$$\vec{q}_\perp^{\,2} \equiv -q_t^2 = Q^2\left[1 + \frac{1}{x_B}\frac{q\cdot P_h}{P\cdot P_h}\right]. \tag{5}$$

The hadronic tensor $W^{\mu\nu}$ in Eq. (3) can be decomposed in terms of five parity and current conserving tensors $\mathcal{V}_i^{\mu\nu}$ [18]:

$$W^{\mu\nu} = \sum_{i=1}^{5} \mathcal{V}_i^{\mu\nu} W_i, \tag{6}$$

where the W_i are structure functions which may be projected out from $W^{\mu\nu}$ by $W_i = W_{\alpha\beta}\tilde{\mathcal{V}}_i^{\alpha\beta}$, with the corresponding inverse tensors $\tilde{\mathcal{V}}_i$ [18]. Both $\mathcal{V}_i$ and $\tilde{\mathcal{V}}_i$ can be constructed from four orthonormal basis vectors [18]:

$$T^\mu = \frac{1}{Q}\left(q^\mu + 2x_B P^\mu\right), \qquad X^\mu = \frac{1}{q_\perp}\left[\frac{P_h^\mu}{z_h} - q^\mu - \left(1 + \frac{q_\perp^2}{Q^2}\right)x_B P^\mu\right],$$
$$Y^\mu = \epsilon^{\mu\nu\rho\sigma} Z_\nu X_\rho T_\sigma, \qquad Z^\mu = -\frac{q^\mu}{Q}, \tag{7}$$

with $q_\perp \equiv \sqrt{\vec{q}_\perp^{\,2}}$ and normalizations $T^\mu T_\mu = 1$, $X^\mu X_\mu = Y^\mu Y_\mu = Z^\mu Z_\mu = -1$. In the following, we will only consider the contributions associated with the tensor $\mathcal{V}_1$. The tensor $\mathcal{V}_5$ does not contribute when contracted with a symmetric $L_{\mu\nu}$. The other three may contribute at large transverse momentum $P_{h\perp} \sim Q$ and should be included in phenomenological analyses [19]. However, as we discussed in the Introduction, we are primarily interested in hadron production in an intermediate transverse momentum region, $\Lambda_{\rm QCD} \ll P_{h\perp} \ll Q$. Here, $\mathcal{V}_1$ alone provides the leading contribution. This is known from the literature [18] for the unpolarized cross section, and we have verified it by explicit calculation for the (Sivers-type) single-transverse-spin dependent polarized cross section. The tensors $\mathcal{V}_1$ and $\tilde{\mathcal{V}}_1$ are given by [18]

$$\mathcal{V}_1^{\mu\nu} = X^\mu X^\nu + Y^\mu Y^\nu, \qquad \tilde{\mathcal{V}}_1^{\mu\nu} = \frac{1}{2}\left(2T^\mu T^\nu + X^\mu X^\nu + Y^\mu Y^\nu\right). \tag{8}$$

The definitions (7) for the coordinate vectors still leave freedom to associate the axes with specific momentum directions. In the following, we will perform our calculations in the so-called *hadron frame*, where the virtual photon and target proton are taken to have a spatial component only in the z-direction [18]:

$$P^\mu = P^+ p^\mu, \qquad q^\mu = -x_B P^+ p^\mu + \frac{Q^2}{2x_B P^+} n^\mu, \tag{9}$$

where the light-cone momenta are defined as $P^\pm = (P^0 \pm P^3)/\sqrt{2}$, and $p^\mu = (1^+, 0^-, 0_\perp)$, $n^\mu = (0^+, 1^-, 0_\perp)$ are two light-like vectors with $p \cdot n = 1$. Usually one chooses the photon to have a vanishing energy component, corresponding to $P^+ = Q/\sqrt{2}x_B$. In the hadron frame, the final state hadron will have the momentum

$$P_h^\mu = \frac{x_B \vec{P}_{h\perp}^2}{z_h Q^2} P^+ p^\mu + z_h \frac{Q^2}{2x_B P^+} n^\mu + P_{h\perp}^\mu, \tag{10}$$

where z_h has been defined above. Using the expression for q^μ in (4), one can show that in this frame $q_\perp = P_{h\perp}/z_h$ with $P_{h\perp} = \sqrt{\vec{P}_{h\perp}^2}$. The differential unpolarized and single-transverse-spin-dependent cross sections will be calculated in terms of $q_\perp$, and their dependence on $P_{h\perp}$ follows immediately. In the following, we will use both $q_\perp$ and $P_{h\perp}$ when discussing the transverse momentum in SIDIS, keeping in mind that they are essentially the same in the hadron frame.

Substituting the tensors in Eq. (8) into (6) and into the formula (1) for the differential cross section, we obtain

$$\frac{d\sigma(S_\perp)}{dx_B\,dy\,dz_h\,d^2\vec{P}_{h\perp}} = \frac{4\pi\alpha_{\rm em}^2 S_{ep}}{Q^4}\left\{2x_B\left(1 - y + y^2/2\right)W_1\right\}. \tag{11}$$

At large $P_{h\perp}$, we may use collinear factorization which expresses W_1 in terms of a convolution of parton distribution functions, fragmentation functions for the produced hadron, and hard partonic cross sections. The lowest-order (LO) contributions to the latter arise from the processes $\gamma^* q \to qg$ (or $\gamma^*\bar{q} \to \bar{q}g$) and $\gamma^* g \to q\bar{q}$. In the present study, we are mainly interested in hadron production in the forward direction of the polarized beam, where initial valence quarks dominate the SSA, and where final-state quark fragmentation dominates over gluon fragmentation. Therefore we will consider only the $\gamma^* q \to qg$ channel, with the quark fragmenting into the observed hadron and the gluon "not observed". Also, we will eventually be interested in the extrapolation of our results to $P_{h\perp} \ll Q$. Here, the process $\gamma^* q \to qg$ further dominates over $\gamma^* q \to gq$ (with the quark "not observed") and $\gamma^* g \to q\bar{q}$, by a logarithm $\ln(Q^2/\vec{P}_{h\perp}^2)$. We note, however, that in general at large $\vec{P}_{h\perp}$, and/or when smaller x are relevant, the process $\gamma^* g \to q\bar{q}$, initial anti-quarks, and gluon fragmentation may all make non-negligible contributions. The result for the unpolarized SIDIS cross section through $\gamma^* q \to qg$ scattering reads:

$$\left.\frac{d\sigma}{dx_B\,dy\,dz_h\,d^2\vec{P}_{h\perp}}\right|_{\mathcal{V}_1} = \frac{4\pi\alpha_{\rm em}^2 S_{ep}}{z_h^2 Q^4}\frac{\alpha_s}{2\pi^2}C_F\int\frac{dx\,dz}{xz}q(x)\hat{q}(z)x_B\left(1 - y + \frac{y^2}{2}\right)\hat{\sigma}_{\rm unp}\delta\left(\vec{q}_\perp^{\,2} - \frac{Q^2(1-\xi)(1-\hat{\xi})}{\xi\hat{\xi}}\right), \tag{12}$$

where the contribution $\hat{\sigma}_{\rm unp}$ associated with the tensor structure $\mathcal{V}_1$ has been given in the literature [18]:

$$\hat{\sigma}_{\rm unp} = \xi\hat{\xi}\left[\frac{1}{Q^2\vec{q}_\perp^{\,2}}\left(\frac{Q^4}{\xi^2\hat{\xi}^2} + \left(Q^2 - \vec{q}_\perp^{\,2}\right)^2\right) + 6\right]. \tag{13}$$

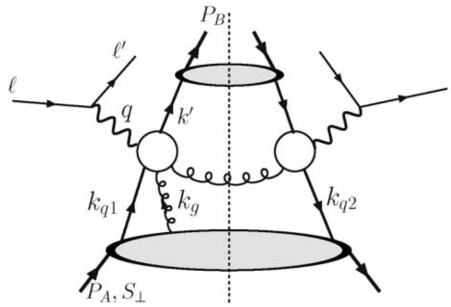

Fig. 1. A generic Feynman diagram contributing to the Sivers-type single-transverse-spin asymmetry for inclusive hadron production in semi-inclusive deep inelastic scattering.

Here $q(x)$ denotes the quark parton distribution function with x the initial-state hadron's momentum fraction carried by the quark, and $\hat{q}(z)$ the fragmentation function for a quark going into the observed hadron with z times the quark's momentum. The variables ξ and $\hat{\xi}$ in Eq. (12) are defined as $\xi = x_B/x$ and $\hat{\xi} = z_h/z$, respectively. We have for simplicity suppressed their dependence on a factorization scale, and also a sum over all quark and anti-quark flavors in Eq. (12). Finally, α_s denotes the strong coupling constant, and $C_F = 4/3$.

The main objective of this Letter is to calculate the single-transverse-spin dependent cross section $\Delta\sigma(S_\perp) = [\sigma(S_\perp) - \sigma(-S_\perp)]/2$. At large transverse momentum $P_{h\perp} \gg \Lambda_{\rm QCD}$, the corresponding SSA is generated by the ETQS mechanism in terms of twist-three transverse-spin dependent quark–gluon correlation functions [8]. The difference between the physics of the unpolarized cross section and the transverse-spin dependent one is that the latter involves an additional polarized gluon from the polarized proton which interacts with partons in the hard part, and hence is a twist-three observable. In Fig. 1, we show a generic Feynman diagram for such a contribution. The lower shaded oval of the diagram represents the transverse-spin-dependent quark–gluon correlation function for the polarized-proton target [8]:

$$T_F(x_1, x_2) \equiv \int \frac{d\zeta^- d\eta^-}{4\pi} e^{i(k_{q1}^+ \eta^- + k_g^+ \zeta^-)} \epsilon_\perp^{\beta\alpha} S_{\perp\beta} \langle PS|\bar{\psi}(0)\mathcal{L}(0,\zeta^-)\gamma^+ g F^+{}_\alpha(\zeta^-)\mathcal{L}(\zeta^-,\eta^-)\psi(\eta^-)|PS\rangle, \tag{14}$$

where the sums over color and spin indices are implicit, $|PS\rangle$ denotes the proton state, ψ the quark field, and $F^+{}_\alpha$ the gluon field tensor. In order to form a gauge-invariant expression for the non-perturbative structure represented by the lower shaded oval in Fig. 1, the gluon field connecting the upper and lower parts has to be converted to either a covariant derivative or to $F^{+\alpha}$ in the leading-order perturbative expansion [8]. A set of quark–gluon correlation functions can be constructed in this way, which may contribute to the SSAs in hard-scattering processes [8,19]. However, by explicit calculations, we found that the "Sivers-type" SSAs generated by terms other than the one in Eq. (14) either vanish or are suppressed by powers of $q_\perp/Q$ in the low-transverse-momentum limit. Therefore, in the following calculations, we will focus on the contributions from T_F in Eq. (14). In the above definition, $x_1 = k_{q1}^+/P^+$ and $x_2 = k_{q2}^+/P^+$ are the fractions of the polarized proton's light-cone momentum carried by the initial quark lines in Fig. 1, while $x_g = k_g^+/P^+ = x_2 - x_1$ is the fractional momentum carried by the gluon; $\mathcal{L}$ is the light-cone gauge link, $\mathcal{L}(\zeta_2, \zeta_1) = \exp(-ig\int_{\zeta_2}^{\zeta_1} d\xi^- A^+(\xi^-))$, that makes the correlation operator gauge-invariant, and $\epsilon_\perp^{\alpha\beta}$ is the 2-dimensional Levi-Civita tensor with $\epsilon_\perp^{12} = 1$.

The strong interaction phase necessary for a non-vanishing SSA arises from the interference between the imaginary part of the partonic scattering amplitude with the extra gluon and the real scattering amplitude without a gluon in Fig. 1. The imaginary part is due to the pole of the parton propagator associated with the integration over the gluon momentum fraction x_g. Depending on which propagator's pole contributes, $\Delta\sigma(S_\perp)$ may get contributions from $x_g = 0$ ("soft-pole") [8] and $x_g \neq 0$ ("hard-pole") [13, 23,24]. When calculating the partonic scattering amplitudes, we have to attach the polarized gluon to any propagator of the hard part contained in the light circles in the diagram of Fig. 1. If the polarized gluon attaches to the outgoing quark in the final state, the on-shell propagation of the quark line will generate a soft gluonic pole. A hard pole arises when internal quark propagators go on-shell with non-zero x_g. In Figs. 2 and 3 we show the relevant soft- and hard-pole partonic diagrams, respectively. There are a total of eight diagrams contributing to the soft-pole part, four of which we show in Fig. 2. The remaining four diagrams can be obtained by attaching the gluon on the right side of the cut. There are twelve diagrams for the hard-pole contributions, and again only half of them are shown in Fig. 3. We note that only diagrams with an s-channel quark propagator can have a hard pole. All diagrams in Figs. 2 and 3 are crossed versions of the ones needed for the SSA in the Drell–Yan process considered in [13].

The calculations of the soft-pole and hard-pole contributions follow the same procedure as we used for the Drell–Yan process [13]. We only give a brief outline here and refer the reader to this reference for details. We perform our calculations in a covariant gauge. The collinear expansion is the central step in obtaining the final results. For example, in the diagrams of Figs. 2 and 3, the dominant component of the momentum of the polarized gluon is $x_g P + k_{g\perp}$. The contribution to the single-transverse-spin asymmetry arises from terms linear in $k_{g\perp}$ in the expansion of the partonic scattering amplitudes. One important contribution of the

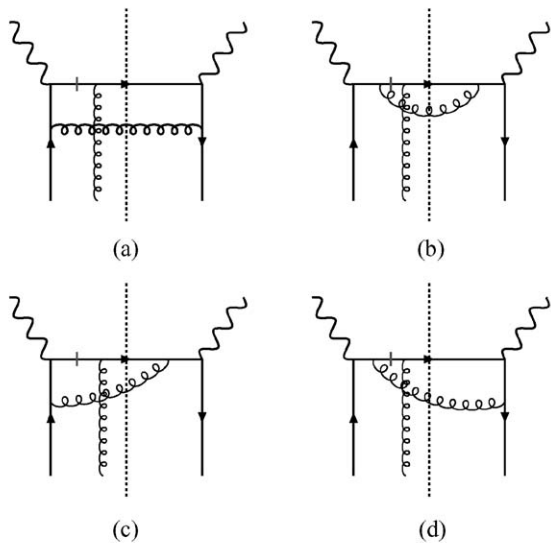

Fig. 2. Feynman diagrams making soft-pole contributions to the single-transverse-spin-dependent cross section. The bars indicate the propagators where a soft pole arises. The "mirror" diagrams for which the additional initial gluon attaches on the right of the cut are not shown, but are included in the calculations.

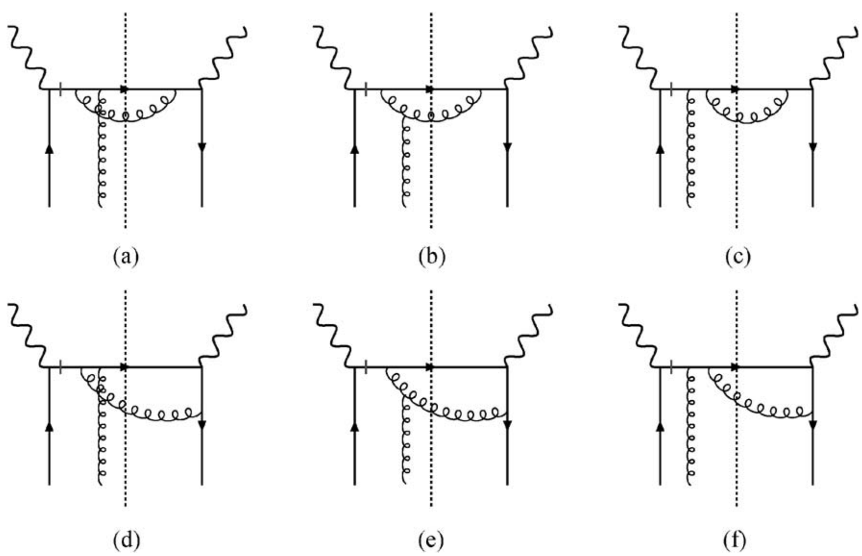

Fig. 3. Same as Fig. 2, but for the hard-pole contributions.

$k_{g\perp}$ expansion comes from the on-shell condition for the outgoing "unobserved" gluon, whose momentum depends on $k_{g\perp}$. This leads to a term involving the *derivative* of the correlation function T_F. In addition, the soft and hard poles in the diagrams may also arise as double poles [8], which will lead to a derivative contribution as well. The hard-pole contributions by the individual diagrams in Fig. 3 also give derivative terms. However, the derivative contributions cancel out in their sum, similar to what we found for the Drell–Yan case in [13]. For example, the derivative contribution from Fig. 3(a) is canceled out by part of 3(b), 3(c) by another part of 3(b). The remaining contributions contain only non-derivative terms. We note that in order to obtain the correct result for the hard-pole contributions, it is crucial to sum only over physical polarization states of the "unobserved" gluon in the Feynman diagrams.

Combining the contributions by all the diagrams, we find for the single-transverse-spin-dependent cross section:

$$\frac{d\Delta\sigma(S_\perp)}{dx_B\,dy\,dz_h\,d^2\vec{P}_{h\perp}}\bigg|_{\mathcal{V}_1} = -\frac{4\pi\alpha_{\rm em}^2 S_{ep}}{z_h^3 Q^4}\epsilon^{\alpha\beta}S_\perp^\alpha P_{h\perp}^\beta \frac{\alpha_s}{2\pi^2}\int\frac{dx\,dz}{xz}\hat{q}(z)\delta\left(\vec{q}_\perp^{\,2}-\frac{Q^2(1-\xi)(1-\hat{\xi})}{\xi\hat{\xi}}\right)x_B\left(1-y+\frac{y^2}{2}\right)$$
$$\times\left\{\left(x\frac{\partial}{\partial x}T_F(x,x)\right)\left(\frac{1}{2N_c}\right)\frac{1-\xi}{\hat{\xi}\vec{q}_\perp^{\,2}}\hat{\sigma}_{\rm unp}+\left(-\frac{1}{2N_c}\right)T_F(x,x)\frac{\xi}{Q^2}\left[\frac{1+\hat{\xi}^2}{(1-\xi)^2(1-\hat{\xi})^2}\right.\right.$$
$$\left.\left.+\frac{2\hat{\xi}(2-3\hat{\xi})+(1-2\xi)(1+6\hat{\xi}^2-6\hat{\xi})}{(1-\hat{\xi})^2}\right]+T_F(x,x_B)\left(\frac{1}{2N_c}+C_F\hat{\xi}\right)\frac{\xi}{Q^2}\frac{1+\hat{\xi}^2\xi}{(1-\xi)^2(1-\hat{\xi})^2}\right\}, \quad (15)$$

where $\hat{\sigma}_{\text{unp}}$ has been defined in Eq. (13). Here we have kept only the contribution associated with the tensor structure $\mathcal{V}_1$. All other terms have been neglected, because they are suppressed by powers of $q_\perp/Q$ in the limit of $q_\perp \ll Q$. Similar to the Drell–Yan process, the hard part for the derivative term is proportional to the unpolarized cross section. The last term in the above equation comes from the hard-pole contributions, which are characterized by the dependence on the full quark–gluon correlation function $T_F(x, x_B)$. We have also performed all calculations in a frame where the initial proton and the produced hadron are collinear and move in the z-direction and found identical results for both the soft-pole and the hard-pole contributions.

The derivative contribution in Eq. (15) agrees with that derived in [19]. Our non-derivative terms for the soft-pole and the hard-pole contributions are new, however. Even though the derivative contribution is expected to dominate in some kinematic situations [8], the non-derivative parts become of equal importance for $q_\perp \ll Q$, as we shall see shortly. Since it is our goal in this Letter to match the result obtained within the ETQS formalism to the one based on TMD factorization, it is crucial that we keep the non-derivative parts. Note that the bulk of the SIDIS event rate in experiments is generally located at relatively modest $q_\perp$.

The angular correlation between the observed hadron's transverse momentum $P_{h\perp}$ and the target proton's polarization vector $S_\perp$ as shown in Eq. (15) is characteristic of the contribution from the quark–gluon correlation in the proton. Other contributions, like the twist-three quark–gluon correlation in the fragmentation function, will lead to a different angular correlation between these two [19]. Because of their different angular dependence, these contributions can be easily disentangled experimentally (see, e.g., [2]).

The results we have shown above in Eqs. (12) and (15) are valid when both $P_{h\perp}, Q \gg \Lambda_{\text{QCD}}$. In order to make contact with the TMD factorization formalism, we shall now extrapolate them into the region of $\Lambda_{\text{QCD}} \ll P_{h\perp} \ll Q$. This is also the region exclusively dominated by the contributions associated with the tensor $\mathcal{V}_1$ that we have considered. In doing the expansion, we only keep the terms leading in $P_{h\perp}/Q$, and neglect all higher powers. For small $P_{h\perp}/Q$, the delta function in Eqs. (12) and (15) can be expanded as [25]

$$\delta\left(\vec{q}_\perp^{\,2} - \frac{Q^2(1-\xi)(1-\hat{\xi})}{\xi\hat{\xi}}\right) = \frac{\xi\hat{\xi}}{Q^2}\left\{\frac{\delta(\xi-1)}{(1-\hat{\xi})_+} + \frac{\delta(\hat{\xi}-1)}{(1-\xi)_+} + \delta(\xi-1)\delta(\hat{\xi}-1)\ln\frac{Q^2}{\vec{q}_\perp^{\,2}}\right\}. \tag{16}$$

Inserting this expression into Eq. (12), we find for the small-$P_{h\perp}$ behavior of the unpolarized differential cross section [25]:

$$\begin{aligned}\frac{d\sigma}{dx_B\,dy\,dz_h\,d^2\vec{P}_{h\perp}} = {} & \frac{4\pi\alpha_{\text{em}}^2 S_{ep}}{Q^4}\frac{\alpha_s}{2\pi^2}\frac{1}{\vec{P}_{h\perp}^2}C_F\int\frac{dx\,dz}{xz}q(x)\hat{q}(z)\left\{\frac{1+\xi^2}{(1-\xi)_+}\delta(\hat{\xi}-1)\right.\\ & \left. + \frac{1+\hat{\xi}^2}{(1-\hat{\xi})_+}\delta(\xi-1) + \delta(\xi-1)\delta(\hat{\xi}-1)\ln\frac{z_h^2Q^2}{\vec{P}_{h\perp}^2}\right\}.\end{aligned} \tag{17}$$

Similarly, for the single-transverse-spin-dependent cross section, we have

$$\frac{d\Delta\sigma(S_\perp)}{dx_B\,dy\,dz_h\,d^2\vec{P}_{h\perp}} = -\frac{4\pi\alpha_{\text{em}}^2 S_{ep}}{Q^4}\epsilon^{\alpha\beta}S_\perp^\alpha\frac{z_hP_{h\perp}^\beta}{(\vec{P}_{h\perp}^2)^2}\frac{\alpha_s}{2\pi^2}\int\frac{dx\,dz}{xz}\hat{q}(z)\left\{\delta(\hat{\xi}-1)A + \delta(\xi-1)B\right\}, \tag{18}$$

where

$$\begin{aligned}A = {} & \frac{1}{2N_C}\left\{\left[x\frac{\partial}{\partial x}T_F(x,x)\right]\left(1+\xi^2\right) + T_F(x, x-\hat{x}_g)\frac{1+\xi}{(1-\xi)_+}\right.\\ & \left. + T_F(x,x)\frac{(1-\xi)^2(2\xi+1)-2}{(1-\xi)_+}\right\} + C_FT_F(x, x-\hat{x}_g)\frac{1+\xi}{(1-\xi)_+},\end{aligned} \tag{19}$$

$$B = C_FT_F(x,x)\left[\frac{1+\hat{\xi}^2}{(1-\hat{\xi})_+} + 2\delta(\hat{\xi}-1)\ln\frac{z_h^2Q^2}{\vec{P}_{h\perp}^2}\right], \tag{20}$$

with $\hat{x}_g \equiv (1-\xi)x = x - x_B$. We stress that both soft poles and hard poles contribute to this result. The T_F function for the hard-pole contribution reduces to $T_F(x,x)$ at $\xi = 1$ which is crucial for obtaining the correct structure of the small-$P_{h\perp}$ limit of the cross section. Because the contributions from all tensor structures other than $\mathcal{V}_1$ vanish in the limit of $P_{h\perp} \ll Q$, the above results are the final results for the unpolarized and (Sivers-type) single-transverse-spin-dependent cross sections in this kinematical regime.

Comparing the small-$P_{h\perp}$ behavior in Eqs. (18), (20) to the one we obtained for the Drell–Yan process at low pair transverse momentum $q_\perp \ll Q$ [13], we find that the hard partonic parts are the same, with however an opposite sign. This sign difference comes from the fact that in the Drell–Yan SSA the strong interaction phase arises from initial-state interactions, while in DIS it is due to final-state interactions. Of course, the real physical asymmetries will also depend on the size of the unpolarized quark distribution and fragmentation functions and not differ just by a sign. It is interesting to note that this universality (up to a sign) of the Drell–Yan and the SIDIS twist-three partonic cross sections only happens at low transverse momentum. At $q_\perp \sim Q$, there is no connection between the two processes at all. The universality of the partonic hard parts at low transverse momentum is actually a manifestation of the TMD factorization at $P_{h\perp} \ll Q$, and of the universality of the TMD quark distributions and fragmentation functions. We will discuss this further in the following section.

3. When $P_{h\perp} \ll Q$, the transverse-momentum-dependent factorization formalism applies [16], according which the differential SIDIS cross section may be written as

$$\frac{d\sigma(S_\perp)}{dx_B\,dy\,dz_h\,d^2\vec{P}_{h\perp}} = \sigma_0 \times \left[F_{UU}^{(1)} + \sin(\phi_h - \phi_S)|S_\perp| F_{UT}^{(1)}\right], \tag{21}$$

where $\sigma_0 = 4\pi\alpha_{\rm em}^2 S_{ep}/Q^4 \times (1-y+y^2/2)x_B$, and where ϕ_S and ϕ_h are the azimuthal angles of the proton's transverse polarization vector and the transverse momentum vector of the final-state hadron, respectively. Again, we only keep the terms we are interested in: F_{UU} corresponds to the unpolarized cross section, and $F_{UT}^{(1)}$ to the Sivers function contribution to the single-transverse-spin asymmetry. Other contributions, for example those related to the Collins effect [6], may be incorporated similarly [16]. F_{UU} and $F_{UT}^{(1)}$ depend on the kinematical variables, x_B, z_h, Q^2, y, and $P_{h\perp}$. According to the TMD factorization formalism, these structure functions can be factorized into products of TMD parton distributions and fragmentation functions, and soft and hard parts. For example, $F_{UU}^{(1)}$ has the following factorized form [16]:

$$F_{UU}\big(x_B, z_h, Q^2, P_{h\perp}\big) = \sum_{q=u,d,s,\ldots} e_q^2 \int d^2\vec{k}_\perp\, d^2\vec{p}_\perp\, d^2\vec{\lambda}_\perp\, q(x_B, k_\perp)\hat{q}(z_h, p_\perp)\big(S(\vec{\lambda}_\perp)\big)^{-1} \\ \times H_{UU}\big(Q^2\big)\delta^{(2)}(z_h\vec{k}_\perp + \vec{p}_\perp + \vec{\lambda}_\perp - \vec{P}_{h\perp}), \tag{22}$$

where q and $\hat{q}$ denote the unpolarized TMD quark distributions and fragmentation functions, respectively. H is a hard factor and is entirely perturbative. It is a function of $Q \gg P_{h\perp}$ only. The soft-factor S is a vacuum matrix element of Wilson lines and captures the effects of soft gluon radiation. Since the soft-gluon contributions in the TMD distribution and fragmentation have not been subtracted, the soft factor enters with inverse power. We have not displayed the dependence of the TMD quark distribution (fragmentation) functions on the variable $\zeta^2 = (2v\cdot P)^2/v^2$ ($\hat{\zeta}^2 = (2\tilde{v}\cdot P_h)^2/\tilde{v}^2$), which serves to regulate their light-cone singularities. Here, v and $\tilde{v}$ are vectors off the light-cone. We finally introduce the soft-gluon rapidity cut-off $\rho = \sqrt{(2v\cdot\tilde{v})^2/v^2\tilde{v}^2}$, on which the soft factor depends. In a special coordinate frame, one may choose $x_B^2\zeta^2 = \hat{\zeta}^2/z_h^2 = \rho Q^2$ [16]. There is also explicit renormalization scale dependence of the various factors in the factorization formula which, too, has been omitted for simplicity.

Similarly to Eq. (22), the contribution to the Sivers single-transverse-spin asymmetry can be factorized as

$$F_{UT}^{(1)} = \sum_{q=u,d,s,\ldots} e_q^2 \int d^2\vec{k}_\perp\, d^2\vec{p}_\perp\, d^2\vec{\lambda}_\perp \frac{\vec{k}_\perp\cdot\hat{\vec{P}}_{h\perp}}{M_P} q_T(x_B, k_\perp)\hat{q}(z_h, p_\perp)\big(S(\vec{\lambda}_\perp)\big)^{-1} \\ \times H_{UT}^{(1)}\big(Q^2\big)\delta^{(2)}(z_h\vec{k}_\perp + \vec{p}_\perp + \vec{\lambda}_\perp - \vec{P}_{h\perp}), \tag{23}$$

where $\hat{\vec{P}}_{h\perp}$ is a unit vector in direction of $\vec{P}_{h\perp}$ and q_T is the Sivers TMD quark distribution. The proton mass M_P is used to normalize the Sivers function and the unpolarized TMD quark distribution to the same mass dimension. For the operator definition of the Sivers function, see for example [13].

In order to make contact with the result for the ETQS formalism of the previous section, we compute the various factors in the factorization formulas (22), (23) at large transverse momentum ($P_{h\perp} \gg \Lambda_{\rm QCD}$), where their dependence on $P_{h\perp}$ is perturbative. The unpolarized quark distribution and fragmentation functions at large $P_{h\perp}$ can be expressed in terms of their respective $k_\perp$-integrated distributions, multiplied by perturbatively calculable coefficients. Their expressions are well known (see, for example, Ref. [16]). For the quark distribution function, one has:

$$q(x_B, k_\perp) = \frac{\alpha_s}{2\pi^2}\frac{1}{\vec{k}_\perp^2} C_F \int \frac{dx}{x} q(x)\left[\frac{1+\xi^2}{(1-\xi)_+} + \delta(\xi-1)\left(\ln\frac{x_B^2\zeta^2}{\vec{k}_\perp^2} - 1\right)\right], \tag{24}$$

where $q(x)$ is the integrated quark distribution and $\xi = x_B/x$. Likewise, the TMD quark fragmentation function is given by

$$\hat{q}(z_h, p_\perp) = \frac{\alpha_s}{2\pi^2}\frac{1}{\vec{p}_\perp^2} C_F \int \frac{dz}{z}\hat{q}(z)\left[\frac{1+\hat{\xi}^2}{(1-\hat{\xi})_+} + \delta(\hat{\xi}-1)\left(\ln\frac{\hat{\zeta}^2}{\vec{p}_\perp^2} - 1\right)\right], \tag{25}$$

where $\hat{q}(z)$ is the integrated quark fragmentation function and $\hat{\xi} = z_h/z$. At large transverse momentum, the soft factor $S(\lambda_\perp)$ in the factorization formula can also be calculated perturbatively [16],

$$S(\lambda_\perp) = \frac{\alpha_s}{2\pi^2}\frac{1}{\vec{\lambda}_\perp^2} C_F\big(\ln\rho^2 - 2\big). \tag{26}$$

Similarly, one may evaluate the Sivers function at large $k_\perp$. Because it is (naively) time-reversal-odd, the only contribution comes from the twist-three quark–gluon correlation function T_F in Eq. (14). The calculation follows the same procedure as for our calculation for the Drell–Yan process in [13]. The Feynman diagrams are the same, the only difference being that the gauge-link propagators each have an opposite sign for their imaginary part. Carrying out the calculations accordingly, we find

$$q_T(x_B, k_\perp) = -\frac{\alpha_s}{4\pi^2}\frac{2M_P}{(\vec{k}_\perp^2)^2}\int\frac{dx}{x}\left\{A + C_F T_F(x,x)\delta(\xi-1)\left(\ln\frac{x_B^2\zeta^2}{\vec{k}_\perp^2} - 1\right)\right\}, \tag{27}$$

where A has been defined in Eq. (19) and where $\xi = x_B/x$. Indeed, as expected [20–22], we find that the Sivers function in DIS is the same as that in the Drell–Yan process, but with an opposite sign. As is well known now [20–22], this sign difference comes from the different directions of the gauge links for the two processes: in DIS the gauge link arises from final-state interactions and runs to positive light-cone infinity, while in Drell–Yan it is due to initial-state interactions and goes to $-\infty$.

In order to calculate the explicit $P_{h\perp}$-dependence generated by the TMD factorization, we let one of the transverse momenta $\vec{k}_\perp$, $\vec{p}_\perp$, and $\vec{\lambda}_\perp$ be of the order of $\vec{P}_{h\perp}$ and the others much smaller. When $\vec{\lambda}_\perp$ is large, for example, we neglect $\vec{k}_\perp$ and $\vec{p}_\perp$ in the delta function, and the integrations over these momenta yield either the ordinary quark distribution, or a $k_\perp$ moment of the Sivers function. The latter is related to the twist-three correlation [10]:

$$\int d^2\vec{k}_\perp\, q(x,k_\perp) = q(x), \qquad \int d^2\vec{k}_\perp \frac{\vec{k}_\perp^2}{M_P} q_T(k_\perp, x) = -T_F(x,x), \tag{28}$$

where the minus sign on the right-hand-side of the second equation is again due to the direction of the DIS gauge link. In case $\vec{\lambda}_\perp$ is neglected in the delta function, one makes use of the relation [16] $\int d^2\vec{\lambda}_\perp\, S(\lambda_\perp) = 1$. From the factorization formulas in Eqs. (22), (23) we then obtain the following results for the unpolarized and single-transverse-spin-dependent cross sections:

$$\frac{d\sigma}{dx_B\,dy\,dz_h\,d^2\vec{P}_{h\perp}} = \sigma_0\frac{\alpha_s}{2\pi^2}C_F\frac{1}{\vec{P}_{h\perp}^2}\int\frac{dx\,dz}{xz}q(x)\hat{q}(z)\left\{\frac{1+\xi^2}{(1-\xi)_+}\delta(\hat{\xi}-1) + \frac{1+\hat{\xi}^2}{(1-\hat{\xi})_+}\delta(\xi-1) + \delta(\xi-1)\delta(\hat{\xi}-1)\ln\frac{Q^2 z_h^2}{\vec{P}_{h\perp}^2}\right\}, \tag{29}$$

$$\frac{d\Delta\sigma(S_\perp)}{dx_B\,dy\,dz_h\,d^2\vec{P}_{h\perp}} = \sigma_0\frac{\alpha_s}{2\pi^2}\epsilon_{\alpha\beta}S_\perp^\alpha\frac{-z_h P_{h\perp}^\beta}{(\vec{P}_{h\perp}^2)^2}\int\frac{dx\,dz}{xz}\hat{q}(z)\left\{\delta(\hat{\xi}-1)A + \delta(\xi-1)B\right\}, \tag{30}$$

where A and B are defined as in Eqs. (19), (20). In the above equations, the dependence on the regulators ζ, $\hat{\zeta}$ and ρ for the light-cone singularities in the various TMD functions in Eqs. (24)–(27) has canceled, since we have combined the functions into a physical quantity. To verify the cancellation, one needs the relation $\zeta^2\hat{\zeta}^2 x_B^2/z_h^2\rho^2 = Q^4$. The latter is frame-independent, and so is therefore the cancellation itself. It is evident that the above results reproduce the differential cross sections in Eqs. (17), (18).

The above results demonstrate, in an explicit form, the consistency between the TMD factorization formalism and the twist-three quark–gluon correlation approach at intermediate transverse momentum ($\Lambda_{\mathrm{QCD}} \ll P_{h\perp} \ll Q$) to the lowest non-trivial order in α_s. We expect this conclusion to hold also at higher orders in perturbation theory, where however a number of new issues may arise. For example, one will need to pay attention to the regularization of the integrands in Eq. (28) at low $k_\perp$ through virtual diagrams. Moreover a careful tracking of the renormalization and factorization scales will be important to obtain consistent results.

4. In conclusion, we have demonstrated in this Letter that the two mechanisms for the Sivers-type single-transverse-spin asymmetry in semi-inclusive deeply-inelastic scattering are the same at moderate transverse momentum, $\Lambda_{\mathrm{QCD}} \ll P_{h\perp} \ll Q$. This provides an additional test of the unification of the mechanisms discussed in [13]. It will be important to carry out a relevant experimental test of this unification. Furthermore, our calculation also explicitly exemplifies the process-dependence of the functions generating single-transverse-spin asymmetries. We finally note that another interesting SSA phenomenon in semi-inclusive DIS processes is associated with the so-called Collins effect [6]. A similar connection between the twist-three quark–gluon correlation mechanism in fragmentation [9] and the Collins function should exist. An extension to this case will be considered elsewhere.

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