Physics Letters B 664 (2008) 112–115

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

The ambiguity-free four-dimensional Lorentz-breaking Chern–Simons action

F.A. Brito [a], J.R. Nascimento [b], E. Passos [b], A.Yu. Petrov [b,*]

[a] Departamento de Física, Universidade Federal de Campina Grande, Caixa Postal 10071, 58109-970 Campina Grande, Paraíba, Brazil
[b] Departamento de Física, Universidade Federal da Paraíba, Caixa Postal 5008, 58051-970 João Pessoa, Paraíba, Brazil

ARTICLE INFO

Article history:
Received 4 April 2008
Received in revised form 29 April 2008
Accepted 5 May 2008
Available online 9 May 2008
Editor: M. Cvetič

ABSTRACT

The four-dimensional Lorentz-breaking finite and determined Chern–Simons like action is generated as a one-loop perturbative correction via an appropriate Lorentz-breaking coupling of the gauge field with the spinor field. Unlike the known schemes of calculations, within this scheme this term is found to be regularization independent.

The possibility of violation of the Lorentz and CPT symmetries in the nature has been intensively discussed in recent years [1–17]. Several theoretical investigations have pointed out that these symmetries can be broken. In these studies it was mostly suggested that this violation can be implemented in QED via adding the Chern–Simons-like term $\mathcal{L}_{\rm CS} = \frac{1}{2}k_\mu \epsilon^{\mu\alpha\beta\gamma} F_{\alpha\beta} A_\gamma$, with k_μ being a constant four-vector characterizing the preferred direction of the space–time, to the photon sector, with, at the same time, another CPT-odd term, i.e., $\bar\psi \not{b}\gamma_5\psi$, is added to the fermion sector, with the b_μ is a constant four-vector introducing CPT symmetry breaking. It is well known that this extension of the QED does not break the gauge symmetry of the action and equations of motion but it modifies the dispersion relations for different polarization of photons and Dirac spinors. The Chern–Simons-like term is known to have some important implications, such as birefringence of light in the vacuum [9]. Many interesting investigations in the context of Lorentz-CPT violation have appeared recently in the literature. For instance, several issues were addressed, such as Cherenkov-type mechanism called "vacuum Cherenkov radiation" to test the Lorentz symmetry [18], changing of gravitational redshifts for differently polarized Maxwell–Chern–Simons photons [19], evidence for the Lorentz-CPT violation from the measurement of CMB polarization [20], supersymmetric extensions [21], breaking of the Lorentz group down to the little group associated with k_μ [22] and magnetic monopoles inducing electric current [23]. Among these developments one of the most interesting and controversial results is the dynamical origin of the Lorentz-breaking parameter k_μ which arises due to integration over the fermion fields in the modified Dirac action involving the b_μ vector. The result is the induction of the Chern–Simons-like term via radiative corrections which may lead to a relation between the parameters k_μ and b_μ.

The induction of the Chern–Simons-like Lorentz-CPT violating term, $\mathcal{L}_{\rm CS}$, is one of the most important results in the study of the Lorentz symmetry violation [3,4,8]. This term naturally emerges as a perturbative correction in the theory suggested in [4] as a possible extension of QED by an axial-vector term

$$\mathcal{L} = \bar\psi(i\not{\partial} - m)\psi - \bar\psi\not{b}\gamma_5\psi - e\bar\psi\not{A}\psi. \tag{1}$$

After carrying out the integration over fermions, one can obtain the relation between the coefficients k_μ and b_μ in terms of some loop integrals with some of them being divergent. Therefore one has to implement some regularization to calculate these integrals, thus, the constant C relating the coefficients as $k_\mu = Cb_\mu$ turns out to be dependent on the regularization scheme used [24]. The ambiguity of the results manifested in the dependence on the regularization scheme has been intensively discussed in the literature, and several studies have shown that C can be found to be finite but undetermined [25–29]. Astrophysical observations impose very stringent experimental bounds on k_μ (see [30] for different estimations of the Lorentz-breaking coefficients). Since the coefficient k_μ of the radiatively induced Chern–Simons term depends on b_μ it is natural to expect that the constant b_μ can also suffer an experimental bound in this framework. However, if ambiguities are present there is no way to know the experimental bounds for the constant b_μ, because C is simply undetermined. In other words, we cannot define the fate of the constant that is responsible for the Lorentz and CPT violation in the fermion sector by straightforward measuring k_μ. In the sequel we are going to extend the well studied Lagrangian (1) in attempt to shed some light on the issue of inducing Chern–Simons term in the ambiguity-free manner.

In this Letter we propose an extension of the usual theory through introduction of new chiral couplings which can eliminate such ambiguities. So, let us extend the usual Lagrangian (1) as follows:

$$\mathcal{L} = \bar\psi(i\not{\partial} - m)\psi - \bar\psi\not{b}(1+\gamma_5)\psi - e\bar\psi\not{A}(1-\gamma_5)\psi. \tag{2}$$

* Corresponding author.
E-mail address: petrov@fisica.ufpb.br (A.Yu. Petrov).

0370-2693/$ – see front matter
doi:10.1016/j.physletb.2008.05.015

In order to show that the above Lagrangian preserves gauge invariance we rewrite it in terms of new gauge fields, a vector field and an axial field, in the form [31]

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - m - \mathcal{V}\!\!\!/ - \mathcal{A}\!\!\!/\gamma_5)\psi, \tag{3}$$

where the vector and axial gauge fields are defined as $\mathcal{V}\!\!\!/ = b\!\!\!/ + eA\!\!\!/$ and $\mathcal{A}\!\!\!/ = b\!\!\!/ - eA\!\!\!/$, respectively. In this sense one can understand b_μ and A_μ as R- and L-handed external fields. Hence, this Lagrangian is invariant under the local vector $U_V(1)$ gauge transformation

$$\begin{aligned} &\psi \to \exp[i\alpha(x)]\psi, \\ &\bar{\psi} \to \bar{\psi}\exp[-i\alpha(x)], \\ &\mathcal{V}_\mu \to \mathcal{V}_\mu - \partial_\mu\alpha(x). \end{aligned} \tag{4}$$

The couplings we propose in the theory imply in a special gauge invariant and Lorentz-CPT violating theory which we call "extended chiral QED", where divergences among loop integrals are canceled. The extension is based on the observation that one can extend and impose some restrictions on the gauge invariant and CPT-Lorentz violating Lagrangian (1) by replacing $b\!\!\!/$ and $A\!\!\!/$ according to the transformations

$$\begin{aligned} &b\!\!\!/\gamma_5 \to b\!\!\!/(1+\gamma_5), \\ &A\!\!\!/ \to A\!\!\!/(1-\gamma_5). \end{aligned} \tag{5}$$

One can also verify that other combinations of signs in the interacting terms above produce either only divergent integrals or mixture of divergent and finite integrals. Note that both b_μ and A_μ manifest themselves as external fields with opposite chiralities coupled to fermion fields. As we will show below, in this model the divergences are canceled and no regularization scheme is required. As we just anticipated, by combining other signs of γ_5 in (2), one could also construct interactions with same chirality, but in this case the divergences would persist.

The one-loop effective action $S_{\text{eff}}[b, A]$ of the gauge field A_μ in this theory can be expressed in the form of the following functional trace

$$S_{\text{eff}}[b, A] = -i\,\mathrm{Tr}\ln[p\!\!\!/ - m - b\!\!\!/(1+\gamma_5) - eA\!\!\!/(1-\gamma_5)]. \tag{6}$$

Notice that in this expression, the Tr symbol stands for the trace over Dirac matrices, trace over internal space, as well as for the integration in momentum and coordinate spaces. Hence, in the case of Eq. (6) the calculations are complicated since because the electromagnetic field A_μ is coordinate dependent, hence it does not commute with functions of momentum. Therefore it is not easy to separate out the momentum and space dependent quantities and carry out the integrations in respective spaces. To solve this problem, we will use the method of derivative expansion [32] (see also [24]), and proceed as follows.

The functional trace in (6) can be represented as

$$S_{\text{eff}}[b, A] = S_{\text{eff}}[b] + S'_{\text{eff}}[b, A], \tag{7}$$

where the first term is $S_{\text{eff}}[b] = -i\,\mathrm{Tr}\ln[p\!\!\!/ - m - b\!\!\!/(1+\gamma_5)]$, which does not depend on the gauge field, and the only nontrivial dynamics is concentrated in the second term $S'_{\text{eff}}[b, A]$ given by the following power series

$$S'_{\text{eff}}[b, A] = i\,\mathrm{Tr}\sum_{n=1}^{\infty}\frac{1}{n}\left[\frac{1}{p\!\!\!/ - m - b\!\!\!/(1+\gamma_5)}eA\!\!\!/(1-\gamma_5)\right]^n. \tag{8}$$

To obtain the Chern–Simons-like term we should expand this expression up to the second order in the gauge field

$$S'_{\text{eff}}[b, A] = S^{(2)}_{\text{eff}}[b, A] + \cdots. \tag{9}$$

The dots in (9) stand for the terms of higher orders in the gauge field. Here

$$S^{(2)}_{\text{eff}}[b, A] = -\frac{ie^2}{2}\mathrm{Tr}[S_b(p)A\!\!\!/(1-\gamma_5)S_b(p)A\!\!\!/(1-\gamma_5)], \tag{10}$$

where $S_b(p)$ is the b^μ dependent propagator of the theory defined as

$$S_b(p) = \frac{i}{p\!\!\!/ - m - b\!\!\!/(1+\gamma_5)}. \tag{11}$$

Now, we can apply the key identity of the derivative expansion method [32], that is,

$$A_\mu(x)S_b(p) = S_b(p - i\partial)A_\mu(x), \tag{12}$$

with the propagator $S_b(p - i\partial)$ is expanded up to the first order in derivatives as

$$S_b(p - i\partial) = S_b(p) + S_b(p)\partial\!\!\!/ S_b(p) + \cdots. \tag{13}$$

Substituting this expression into Eq. (10), we obtain

$$S^{(2)}_{\text{eff}}[b, A] = \int d^4x\,\Pi^{\lambda\mu\nu}A_\mu\partial_\nu A_\lambda, \tag{14}$$

with the one-loop self-energy tensor is given by

$$\begin{aligned} \Pi^{\lambda\mu\nu} = -\frac{ie^2}{2}\int\frac{d^4p}{(2\pi)^4}\mathrm{tr}\big[&S_b(p)\gamma^\mu(1-\gamma_5) \\ &\times S_b(p)\gamma^\lambda S_b(p)\gamma^\nu(1-\gamma_5)\big], \end{aligned} \tag{15}$$

where the symbol tr denotes the trace of the product of the gamma matrices. Using a perturbative method for fermion propagator, we can expand $S_b(p)$ in the following series in b^μ

$$S_b(p) = S(p) + S(p)\big(-ib\!\!\!/(1+\gamma_5)\big)S(p) + \cdots, \tag{16}$$

with $S(p)$ being the usual fermion propagator. We find

$$\begin{aligned} \Pi^{\lambda\mu\nu} = -\frac{ie^2}{2}\int\frac{d^4p}{(2\pi)^4}\mathrm{tr}\big[&S(p)\big(-ib\!\!\!/(1+\gamma_5)\big)S(p)\gamma^\mu(1-\gamma_5) \\ &\times S(p)\gamma^\lambda S(p)\gamma^\nu(1-\gamma_5) + S(p)\gamma^\mu(1-\gamma_5) \\ &\times S(p)\big(-ib\!\!\!/(1+\gamma_5)\big)S(p)\gamma^\lambda S(p)\gamma^\nu(1-\gamma_5) \\ &+ S(p)\gamma^\mu(1-\gamma_5)S(p)\gamma^\lambda S(p)\big(-ib\!\!\!/(1+\gamma_5)\big) \\ &\times S(p)\gamma^\nu(1-\gamma_5)\big]. \end{aligned} \tag{17}$$

Thus, taking into account the fact that $\{\gamma_5, \gamma^\mu\} = 0$ and $(\gamma_5)^2 = 1$ and applying the following relation for trace

$$\mathrm{tr}(\gamma^\lambda\gamma^\mu\gamma^\nu\gamma^\rho\gamma_5) = 4i\epsilon^{\lambda\mu\nu\rho}, \tag{18}$$

we can write down the simple expression for self-energy tensor $\Pi^{\lambda\mu\nu}$ as

$$\Pi^{\mu\nu\lambda} = 8ie^2m^2b_\rho\int\frac{d^4p}{(2\pi)^4}\frac{N^{\mu\nu\rho\lambda}}{(p^2 - m^2)^4}, \tag{19}$$

where

$$N^{\mu\nu\rho\lambda} = 2\epsilon^{\mu\nu\rho\theta}p^\lambda p_\theta + \epsilon^{\mu\nu\rho\lambda}(p^2 - m^2). \tag{20}$$

The key property of this expression is that, unlike the results obtained earlier [8,24,33–35], this result is manifestly finite and does not require any regularization. The exact, regularization independent, value for $\Pi^{\lambda\mu\nu}$ is

$$\Pi^{\lambda\mu\nu} = \epsilon^{\lambda\mu\nu\rho}b_\rho\frac{e^2}{3\pi^2}. \tag{21}$$

Thus, the effective action (14) acquires the familiar form

$$S = \frac{1}{2}\int d^4x\,\epsilon^{\lambda\mu\nu\rho}k_\rho A_\mu F_{\nu\lambda}, \tag{22}$$

with the following relation between the constant four-vectors k_ρ and b_ρ

$$k_\rho = \frac{e^2}{3\pi^2}b_\rho. \tag{23}$$

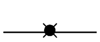
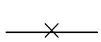
The numerical coefficient relation is finite and fixed, being regularization independent. We emphasize that this result is to be contrasted with other results obtained earlier being different from any other finite result previously found in the model (1), regardless of using regulators or not. For example, in [8] the coefficient $\frac{3}{16\pi^2}$ was found, in [10] the coefficient $-\frac{1}{16\pi^2}$ was found for the massless fermions and $\frac{3}{16\pi^2}$ for the massive ones, in [12]–$\frac{1}{4\pi^2}$, in [15,16]–$\frac{3}{8\pi^2}$, and in [24,35] there were found several results such as $\frac{1}{4\pi^2}$, $\frac{3}{16\pi^2}$ and $\frac{3}{8\pi^2}$.

Thus, our Lorentz-CPT violating "extended chiral QED" theory radiatively induces a Chern–Simons term which displays uniqueness of the result due to absence of the divergences achieved without imposing any regularization. A natural justification of our result can be based on the fact that the model considered in (2) represents a more complete description of the spinor-vector coupling than the usual model (1). We see that, within the derivative expansion method, for the model (1) only three contributions arise, whereas in our model (2) twelve contributions are present. This phenomenon can be related to the symmetry of the new chiral coupling terms in the Lagrangian (2). In this Lagrangian new vertices and new insertions into the fermion propagator are introduced. They can generate new one-loop contributions in comparison with the usual model (1), with they are responsible for divergence cancellations.

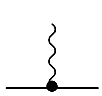
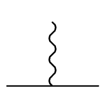
To explain how the cancellation of divergences and ambiguities occurs in the theory, let us consider the one-loop Feynman diagrams contributing to the two-point vertex function of the vector fields. The fermion propagator is

 $= \dfrac{i(\not p + m)}{p^2 - m^2}$

and the coefficients for CPT violation lead to insertions into the fermion propagator

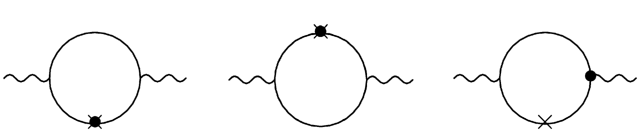
 $= -i\not b\gamma_5,$  $= -i\not b.$

The chiral and usual fermion–photon vertex are respectively

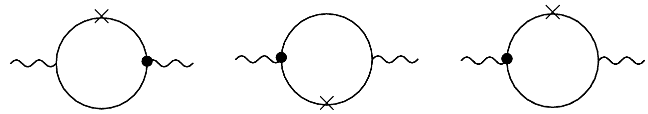
 $= ie\gamma^\mu\gamma_5,$  $= -ie\gamma^\mu,$

where e is the fermion charge and μ the space–time index on the photon line.

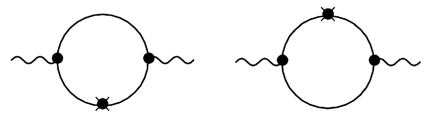
We find that the following diagrams contribute to the two-point function:



(a) (b) (c)



(d) (e) (f)



(g) (h)

The contributions to the one-loop self-energy tensor for these graphs look like:

$$\begin{aligned}\Pi^{\mu\nu}_{a,c,e,g}(q) = e^2\int\frac{d^4p}{(2\pi)^4}\,\mathrm{tr}\big[&\gamma^\mu S(p)\gamma^\nu S(p+q)\not b\gamma_5 S(p+q)\\ &-\gamma^\mu S(p)\gamma^\nu\gamma_5 S(p+q)\not b S(p+q)\\ &-\gamma^\mu\gamma_5 S(p)\gamma^\nu S(p+q)\not b S(p+q)\\ &+\gamma^\mu\gamma_5 S(p)\gamma^\nu\gamma_5 S(p+q)\not b\gamma_5 S(p+q)\big],\end{aligned} \tag{24}$$

for the sum of the graphs a, c, e, g and

$$\begin{aligned}\Pi^{\mu\nu}_{b,d,f,h}(q) = e^2\int\frac{d^4p}{(2\pi)^4}\,\mathrm{tr}\big[&\gamma^\mu S(p)\not b\gamma_5 S(p)\gamma^\nu S(p+q)\\ &-\gamma^\mu S(p)\not b S(p)\gamma^\nu\gamma_5 S(p+q)\\ &-\gamma^\mu\gamma_5 S(p)\not b S(p)\gamma^\nu S(p+q)\\ &+\gamma^\mu\gamma_5 S(p)\not b\gamma_5 S(p)\gamma^\nu\gamma_5 S(p+q)\big],\end{aligned} \tag{25}$$

for the sum of the graphs b, d, f, h. Here $S(p)$ is the usual fermion propagator. Considering the self-energy tensors generated by each of these graphs, we can find finite and infinite contributions which must be regularized. Thus, separate summation of contributions of the graphs a, c, e, g or b, d, f, h can produce finite but undetermined results, whereas the complete summation of all contributions leads to a result that coincides with the finite and regularization independent (determined) Chern–Simons term coefficient (23) obtained within the derivative expansion method. Thus, in the framework of our extension we find that a theory is improved in such a way that quantum calculations give us a finite non-ambiguous result at the one-loop approximation.

We summarize our results as follows. Within our present analysis, we have shown that the presence of chiral couplings preserving a specific gauge invariance for the Lagrangian density, by using the derivative expansion method, allows to avoid ambiguities. These special symmetry transformations maintain the gauge invariance that takes place both for the Lagrangian density and for the action, which yields a new finite and determined result for the Chern–Simons coefficient. Differently of the usual theory studied earlier (see [24] and references therein), where the breaking of the gauge invariance occurs at the quantum level, in the present case the gauge invariance is maintained at both *classical and quantum levels* because of the absence of regulators, and the problem of divergences and ambiguities does not arise. The radiatively induced Chern–Simons term is not gauge invariant but its space–time integral is. Equivalently, it means that the Chern–Simons term is invariant only for zero momentum [25]. The regularization schemes such as Pauli–Villars regularization and gauge invariant dimensional regularization, commonly used to regularize divergent integrals in the usual theory, require gauge invariance at all energy scales which excludes a priori the possibility of inducing a Chern–Simons term. Since Chern–Simons terms are gauge invariant only for zero momentum, one has to use regularization schemes which also could involve gauge invariance only at zero momentum, in order to find a finite and determined Chern–Simons term coefficient k_μ. A natural way to solve this problem would consist in finding the possibility of computing the Chern–Simons coefficient without using any regularization scheme. Thus one possibility that we have chosen in this investigation was to search for a theory where the divergences in one-loop graphs could be eliminated in such a way that the remaining integrals of the theory would be ambiguity-free. We were able to find a theory that really does the job. The theory we formulated invokes new chiral couplings which preserve a special gauge invariance whereas Lorentz and CPT symmetries are violated. The theory represents itself as a model where both b_μ, which is present in the axial-vector term, and the gauge vector potential A_μ manifest themselves as external fields with opposite chirality interacting with fermion fields that renders a complete cancelation of divergences and, at the same time, no regularization scheme is required.

Acknowledgements

This work was partially supported by Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq). The work by A.Yu.P. has been supported by CNPq-FAPESQ DCR program, CNPq project No. 350400/2005-9.

References

[1] V.A. Kostelecky, S. Samuel, Phys. Rev. D 39 (1989) 683.
[2] S. Carroll, G. Field, R. Jackiw, Phys. Rev. D 41 (1990) 1231.
[3] M. Goldhaber, V. Trimble, J. Astrophys. Astron. 17 (1996) 17;
S. Carrol, G. Field, Phys. Rev. Lett. 79 (1997) 2394.
[4] D. Colladay, V.A. Kostelecky, Phys. Rev. D 55 (1997) 6760.
[5] S. Coleman, S.L. Glashow, Phys. Lett. B 405 (1997) 249;
S. Coleman, S.L. Glashow, Phys. Rev. D 59 (1998) 116008.
[6] D. Colladay, V.A. Kostelecky, Phys. Rev. D 58 (1998) 116002.
[7] J.M. Chung, P. Oh, Phys. Rev. D 60 (1999) 067702.
[8] R. Jackiw, V.A. Kostelecky, Phys. Rev. Lett. 82 (1999) 3572.
[9] R. Jackiw, Nucl. Phys. B (Proc. Suppl.) 108 (2002) 30.
[10] M. Perez-Victoria, Phys. Rev. Lett. 83 (1999) 2518.
[11] J.M. Chung, Phys. Rev. D 60 (1999) 127901;
J.M. Chung, Phys. Lett. B 461 (1999) 138.
[12] A.A. Andrianov, P. Giacconi, R. Soldati, JHEP 0202 (2002) 030.
[13] O. Bertolami, C.S. Carvalho, Phys. Rev. D 61 (2000) 103002.
[14] M. Chaichian, W.F. Chen, R. Gonzalez Felipe, Phys. Lett. B 503 (2001) 215.
[15] B. Altschul, Phys. Rev. D 69 (2004) 125009.
[16] B. Altschul, Phys. Rev. D 70 (2004) 101701.
[17] O. Bertolami, J.G. Rosa, Phys. Rev. D 71 (2005) 097901.
[18] R. Lehnert, R. Potting, Phys. Rev. Lett. 93 (2004) 110402.
[19] E. Kant, F.R. Klinkhamer, Nucl. Phys. B 731 (2005) 125.
[20] B. Feng, M. Li, J.-Q. Xia, X. Chen, X. Zhang, Phys. Rev. Lett. 96 (2006) 221302.
[21] H. Belich, J.L. Boldo, L.P. Colatto, J.A. Helayel-Neto, A.L.M.A. Nogueira, Phys. Rev. D 68 (2003) 065030.
[22] A.J. Hariton, R. Lehnert, Phys. Lett. A 367 (2007) 11.
[23] N.M. Barraz Jr., J.M. Fonseca, W.A. Moura-Melo, J.A. Helayel-Neto, Phys. Rev. D 76 (2007) 027701.
[24] F.A. Brito, J.R. Nascimento, E. Passos, A.Yu. Petrov, JHEP 0706 (2007) 016.
[25] R. Jackiw, Int. J. Mod. Phys. B 14 (2000) 2011;
J.M. Chung, Phys. Rev. D 60 (1999) 127901.
[26] G. Bonneau, Nucl. Phys. B 593 (2001) 398;
G. Bonneau, hep-th/0109105.
[27] W.F. Chen, hep-th/0106035.
[28] M. Perez-Victoria, JHEP 0104 (2001) 032.
[29] G. Bonneau, Nucl. Phys. B 764 (2007) 83.
[30] V.A. Kostelecky, N. Russell, arXiv: 0801.0287 [hep-ph].
[31] R.A. Bertlmann, Anomalies in Quantum Field Theory, Cambridge Univ. Press, 1996.
[32] I.J.R. Aitchison, C.M. Fraser, Phys. Lett. B 146 (1984) 63;
I.J.R. Aitchison, C.M. Fraser, Phys. Rev. D 31 (1985) 2605;
C.M. Fraser, Z. Phys. C 28 (1985) 101;
A.I. Vainshtein, V.I. Zakharov, V.A. Novikov, M.A. Shifman, Yad. Fiz. (Sov. J. Nucl. Phys.) 39 (1984) 77;
J.A. Zuk, Phys. Rev. D 32 (1985) 2653;
M.K. Gaillard, Nucl. Phys. B 268 (1986) 669;
A. Das, A. Karev, Phys. Rev. D 36 (1987) 623;
K.S. Babu, A. Das, P. Panigrahi, Phys. Rev. D 36 (1987) 3725.
[33] A.A. Andrianov, P. Giacconi, R. Soldati, JHEP 0202 (2002) 030.
[34] T. Mariz, J.R. Nascimento, E. Passos, R.F. Ribeiro, JHEP 0510 (2005) 019.
[35] M. Gomes, J.R. Nascimento, E. Passos, A.Yu. Petrov, A.J. da Silva, Phys. Rev. D 76 (2007) 047701.