

# The investigation of ship maneuvering with hydrodynamic effects between ships in curved narrow channel

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## Abstract

The hydrodynamic interaction between two large vessels can't be neglected when two large vessels are closed to each other in restricted waterways such as in a harbor or narrow channel. This paper is mainly concerned with the ship maneuvering motion based on the hydrodynamic interaction effects between two large vessels moving each other in curved narrow channel. In this research, the characteristic features of the hydrodynamic interaction forces between two large vessels are described and illustrated, and the effects of velocity ratio and the spacing between two vessels are summarized and discussed. Also, the Incheon outer harbor area through the PALMI island channel in Korea was selected, and the ship maneuvering simulation was carried out to propose an appropriate safe speed and distance between two ships, which is required to avoid sea accident in confined waters. From the inspection of this investigation, it indicates the following result. Under the condition of  $S_{P12} \leq 0.5L$ , it may encounter a dangerous tendency of grounding or collision due to the combined effect of the interaction between ships and external forces. Also considering the interaction and wind effect as a parameter, an overtaken and overtaking vessel in narrow channel can navigate while keeping its own original course under the following conditions; the lateral separation between two ships is about kept at 0.6 times of ship length and 15 degrees of range in maximum rudder angle. On the other hand, two ships while overtaking in *curved* narrow channel such as Incheon outer harbor in Korea should be navigated under the following conditions;  $S_{P12}$  is about kept at 1.0 times of ship length and the wind velocity should not be stronger than 10 m/s.

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**Keywords:** Ship maneuvering; Hydrodynamic force; Narrow channel; Safe speed and distance; Wind

## 1. Introduction

The increasing number of large vessels because of the rapid development of shipping leads to a high density of vessels in confined waters, so that the lateral and longitudinal spacing between vessels become smaller and the unpredictable interaction effect between them happens in restricted waterways such as in a harbor or narrow channel. When two large vessels navigating closely, the asymmetric flow around a vessel induced by the vicinity of other vessel causes pressure

differences between port and starboard sides, thus two vessels may suffer an attractive force or a repulsive force and bow inward moment or bow outward moment. So, the hydrodynamic forces and moments acting on a vessel in confined waters are more complicated than those in unrestricted waterways, and it will become difficult to steer the vessel because of the hydrodynamic interaction effects between two vessels. Specially, proximal navigation of large vessels including overtaking, and congested vessel traffic in curved narrow channel is potentially hazardous. Therefore, it is of great significance to study the hydrodynamic interaction forces and moments between two vessels to ensure a safe navigation. For this to be possible, the hydrodynamic forces and moments between two vessels in confined waters should be properly

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understood, and the work of hydrodynamic interaction effects between vessels have been made by researchers. Newman (1965, 1972) reported the force and moment on a slender body of revolution moving near a wall and some theory for ship maneuvering. Yeung and Tan (1980) analyzed hydrodynamic interactions of a slow-moving vessel with a coastline or an obstacle in shallow water using slender-body theory. In this paper, the assumptions of the theory are that the fluid is inviscid and the flow irrotational except for a thin vortex sheet behind the vessel. Similar works were reported by Beck et al. (1975), Beck (1977), Cohen et al. (1983), Davis (1986), Landweber et al. (1991). Also, Kijima et al. (1991) studied on the interaction effects between two ships in the proximity of a bank wall, and Korsmeyer et al. (1993) analyzed the theory and computation for the interaction forces among multiple ships or bodies which are operating near to each other. Also, Yasukawa (2002) studied on the maneuvering motions between two ships navigating in the proximity. Despite the past investigations, the hydrodynamic interaction force and moment between two vessels in narrow channel still needs to be considered from the viewpoint of safe maneuvering. Also, in the perspective of safe maneuvering, this paper will consider the safe spacing and velocity between two vessels for the sake of reducing sea accidents in curved narrow channel, and create a base for traffic safety system in restricted waterways. Therefore in this paper, the Inchon outer harbor area through the PALMI island channel in Korea was selected, and the ship maneuvering simulation was carried out to propose an appropriate safe speed and distance between two large vessels, which is required to avoid sea accident from the viewpoint of marine safety in confined waters.

## 2. Formulation of the problem

The coordinate system fixed on each vessel is shown by  $o_i - x_i y_i (i = 1, 2)$  in Fig. 1. Consider two vessels designated as ship 1 and ship 2 moving at speed  $U_i (i = 1, 2)$  in an inviscid fluid of depth  $h$ . In this case, each vessel is assumed to move at each other in a straight line through calm water. In Fig. 1,  $S_{P12}$  and  $S_{T12}$  are transverse and longitudinal distance between two vessels. Also,  $V_w$  and  $v$  mean the wind velocity and wind direction.

Assuming small Froude number, the free surface is assumed to be rigid wall, which implies that the effects of

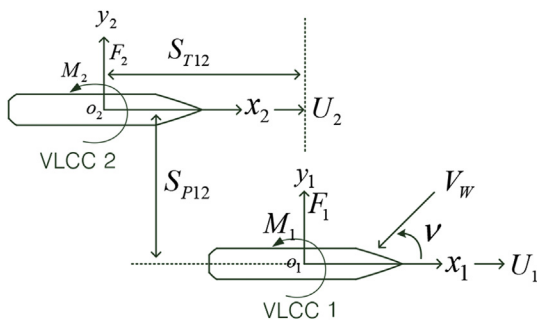


Fig. 1. Coordinate system.

waves are neglected. Then, double body models of the two vessels can be considered. The velocity potential  $\phi(x, y, z; t)$  which expresses the disturbance generated by the motion of the vessels should satisfy the following conditions:

$$\nabla^2 \phi(x, y, z; t) = 0 \tag{1}$$

$$\left. \frac{\partial \phi}{\partial z} \right|_{z=\pm h} = 0 \tag{2}$$

$$\left. \frac{\partial \phi}{\partial n_i} \right|_{B_i} = U_i(t)(n_x)_i \tag{3}$$

$$\phi \rightarrow 0 \text{ at } \sqrt{x_i^2 + y_i^2 + z_i^2} \rightarrow \infty \tag{4}$$

Where,  $B_i$  is the body surface of vessel  $i$ .  $(n_x)_i$  is the  $x_i$  component of the unit normal  $\vec{n}$  interior to  $B_i$ . A following assumptions of slenderness parameter  $\epsilon$  are made to simplify the problem.

$$L_i = O(1), B_i = O(\epsilon), d_i = O(\epsilon) (i = 1, 2), h = O(\epsilon), S_{P12} = O(1)$$

Under these assumptions, the problem can be treated as two-dimensional in the inner and outer region.

### 2.1. Inner and outer solution

The velocity potential  $\Phi_i (i = 1, 2)$  in the inner region can be replaced by the velocity potential representing two-dimensional problems of a vessel cross section between parallel walls representing the bottom and its mirror image above the water surface. Then,  $\Phi_i$  can be expressed as follows (Kijima et al., 1991):

$$\Phi_i(y_i, z_i; x_i; t) = U_i(t)\Phi_i^{(1)}(y_i, z_i) + V_i^*(x_i, t)\Phi_i^{(2)}(y_i, z_i) + f_i(x_i, t) \tag{5}$$

where,  $\Phi_i^{(1)}$  and  $\Phi_i^{(2)}$  are unit velocity potentials for longitudinal and lateral motion,  $V_i^*$  represents the cross-flow velocity at  $\sum_i(x_i)$ , and  $f_i$  is a term being constant in each cross-section plane, which is necessary to match the inner and outer region. In the meantime, the velocity potential  $\phi_i$  in the outer region is represented by distributing sources and vortices along the body axis (Kijima et al., 1991):

$$\phi_i(x, y; t) = \sum_{j=1}^2 \frac{1}{2\pi} \left\{ \int_{L_j} \sigma_j(s_j, t) \log \sqrt{(x - \xi)^2 + (y - \eta)^2} ds_j + \int_{L_{wj}} \gamma_j(s_j, t) \tan^{-1} \left( \frac{y - \eta}{x - \xi} \right) ds_j \right\} \tag{6}$$

where  $\sigma_j(s_j, t)$  and  $\gamma_j(s_j, t)$  are the source and vortex strengths, respectively.  $L_j$  and  $w_j$  denote the integration along vessel  $j$  and

vortex wake shed behind the vessel  $j$ , respectively.  $\xi$  and  $\eta$  represent the source and vortex point.

2.2. Matching and hydrodynamic force and moment

By matching terms of  $\Phi_i$  and  $\phi_i$  that have similar nature, the following integral equation for  $\gamma_i$  can be obtained as follows (Kijima et al., 1991):

$$\begin{aligned} & \frac{1}{2C_i(x_i)} \int_{x_i}^{\frac{L_i}{2}} \gamma_i(\xi_i, t) d\xi_i - \frac{1}{2\pi} \int_{L_j w_j} \gamma_j(s_j, t) \left\{ \frac{1}{x_i - \xi_i} \right\} ds_j \\ & - \sum_{j=1, j \neq i}^2 \frac{1}{2\pi} \int_{L_j w_j} \gamma_j(s_j, t) \frac{\partial G_j^{(\gamma)}}{\partial y_i}(x_0, y_0; \xi, \eta) ds_j \\ & = \sum_{j=1, j \neq i}^2 \frac{1}{2\pi} \int_{L_j} \sigma_j(s_j, t) \frac{\partial G_j^{(\sigma)}}{\partial y_i}(x_0, y_0; \xi, \eta) ds_j \end{aligned} \quad (7)$$

The hydrodynamic forces acting on vessels can be obtained by solving this integral equation for  $\gamma_i$ . The solution  $\gamma_i$  of equation (7) should satisfy the additional conditions:

$$\begin{aligned} \gamma_i(x_i, t) &= \gamma_i(x_i) \text{ for } x_i < -\frac{L_i}{2}, \int_{-\infty}^{\frac{L_i}{2}} \gamma_i(\xi_i, t) d\xi_i = 0, \gamma_i \left( x_i \right. \\ &= \left. -\frac{L_i}{2}, t \right) = -\frac{1}{U_i} \frac{d\Gamma_i}{dt} \end{aligned} \quad (8)$$

where  $\Gamma_i$  is the bound circulation of vessel  $i$ . The lateral force and yawing moment acting on vessel  $i$  can be obtained as follows:

$$\begin{aligned} F_i(t) &= -h_i \int_{-\frac{L_i}{2}}^{\frac{L_i}{2}} \Delta P(x_i, t) dx_i \\ M_i(t) &= -h_i \int_{-\frac{L_i}{2}}^{\frac{L_i}{2}} x_i \Delta P(x_i, t) dx_i \end{aligned} \quad (9)$$

where  $\Delta p$  is the difference of linearized pressure about  $x_i$ -axis and non-dimensional expression for the lateral force,  $C_{Fi}$ , and yawing moment,  $C_{Mi}$ , affecting upon two vessels is given by

$$C_{Fi} = \frac{F_i}{\frac{1}{2} \rho L_i d_i U_i^2}, \quad C_{Mi} = \frac{M_i}{\frac{1}{2} \rho L_i^2 d_i U_i^2} \quad (10)$$

where  $L_i$  is the length of vessel  $i$  and  $d_i$  is the draft of vessel  $i$ .  $\rho$  is the water density.

3. Prediction of interaction effects between two vessels

In this section, the hydrodynamic interaction effects acting on two vessels while overtaking in shallow waters have been examined. A parametric study on the numerical calculations has been conducted on vlcc as shown in Table 1. Provided that the speed of VLCC 1 (denoted as  $U_1$ ) is maintained at 10 kt, the velocity of overtaking VLCC 2 (denoted as  $U_2$ ) was selected as 12 kt, respectively.

Figs. 2 and 3 display the hydrodynamic interaction forces between two vessels in  $U_2/U_1 = 1.2$ . Calculations in these Figures were made by changing horizontal direction between two vessels from 0.2 times to 1.0 times of ship length. Observing the characteristics of hydrodynamic interaction forces between two vessels in Figs. 2 and 3, its qualitative characteristics are similar in considering different distances, but its quantitative characteristics differ; when the spacing between two vessels is less than about 0.4 times of ship length, hydrodynamic interaction effect dramatically increases, and when spacing between two vessels is more than about 0.6 times of ship length, hydrodynamic interaction effect reversely decreases. Also it can be inferred from the calculation result when the spacing between two vessels is more than about 1.0 times of ship length, hydrodynamic interaction effect almost diminishes.

Figs. 4 and 5 show the hydrodynamic interaction moments between two vessels in  $U_2/U_1 = 1.2$ . Calculations in these figures were made by changing horizontal direction between two vessels from 0.2 times to 1.0 times of ship length. Observing the characteristics of hydrodynamic interaction moments between two vessels in Figs. 4 and 5, almost identical to Figs. 2 and 3, its qualitative characteristics are similar in considering different distances, but its quantitative characteristics differ; when the spacing between two vessels is less than about 0.4 times of ship length, hydrodynamic interaction effect dramatically increases, and when spacing between two vessels is more than about 0.6 times of ship length, hydrodynamic interaction effect dramatically decreases. Also it can be inferred from the calculation result when the spacing between two vessels is more than about 1.0 times of ship length, hydrodynamic interaction effect almost diminishes. In conclusion inferred from Figs. 2–4, 5, when the spacing between two vessels is about 1.0 times of ship length regardless of velocity ratio, the hydrodynamic interaction force and moment acted on overtaking and overtaken vessel almost diminishes. Also, the hydrodynamic interaction effect acting on the overtaken vessel is bigger than the one of the overtaking vessel. Furthermore, the maximum repulsive force value is

Table 1  
Principal particulars.

	VLCC 1	VLCC 2
$L$ (m)	320 m	320 m
$B$ (m)	58 m	58 m
$d$ (m)	19.3 m	19.3 m
$C_B$	0.8018	0.8018

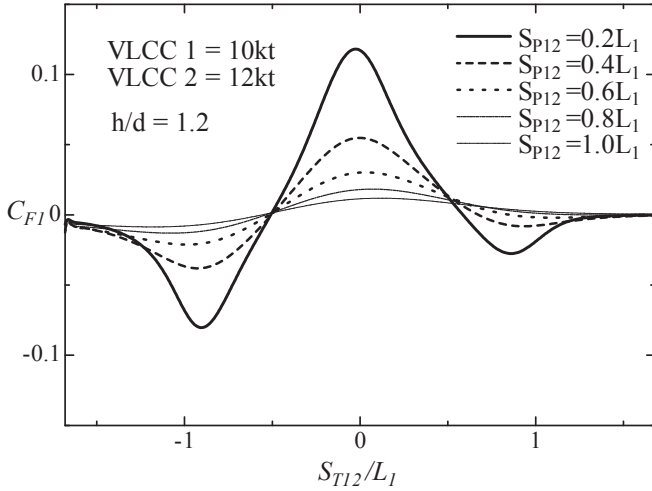


Fig. 2. Computed hydrodynamic forces acting on overtaken VLCC 1.

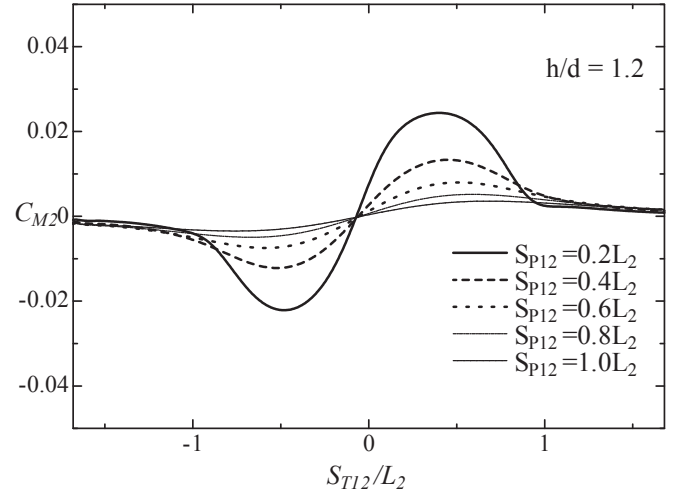


Fig. 5. Computed hydrodynamic moments acting on overtaking VLCC 2.

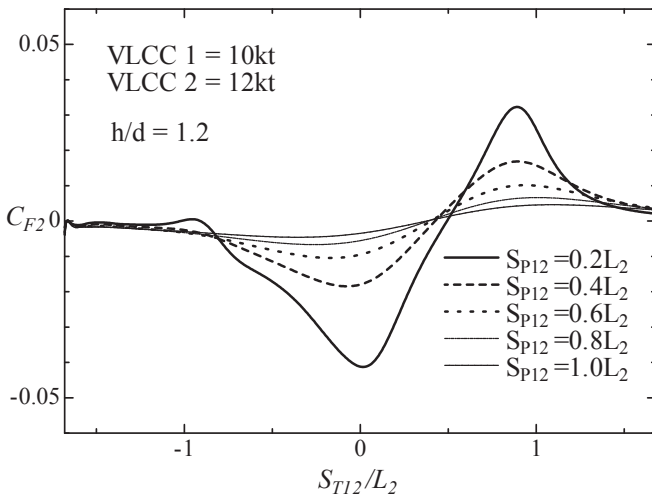


Fig. 3. Computed hydrodynamic forces acting on overtaking VLCC 2.

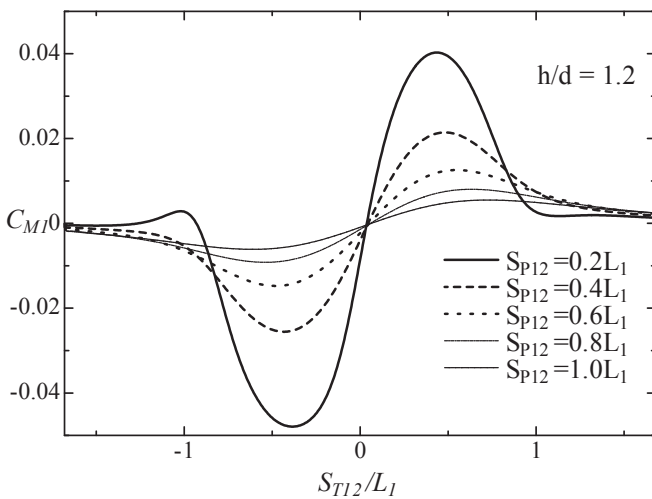


Fig. 4. Computed hydrodynamic moments acting on overtaken VLCC 1.

achieved when the mid-ship of overtaking vessel passes the one of overtaken vessel. Then the sway force reverses to attain

the steady motion due to the sufficient longitudinal distance between two ships.

### 3.1. Simulation of ship maneuvering motion under the influence of interaction effects between two vessels

In this section, the ship maneuvering motions are simulated numerically using the predicted hydrodynamic interaction effects acting on two vessels. In the meantime, the mathematical model of ship maneuvering motion under the influence of interaction effects between two vessels can be expressed as follows (Kijima et al., 1990):

$$\begin{aligned} & (m'_i + m'_{xi}) \left( \frac{L_i}{U_i} \right) \left( \frac{\dot{U}_i}{U_i} \cos \beta_i - \dot{\beta}_i \sin \beta_i \right) + (m'_i + m'_{yi}) r'_i \sin \beta_i \\ & = X'_{Hi} + X'_{Pi} + X'_{Ri} + X'_{Wi} \end{aligned} \quad (11)$$

$$\begin{aligned} & - (m'_i + m'_{yi}) \left( \frac{L_i}{U_i} \right) \left( \frac{\dot{U}_i}{U_i} \sin \beta_i + \dot{\beta}_i \cos \beta_i \right) + (m'_i + m'_{xi}) r'_i \cos \beta_i \\ & = Y'_{Hi} + Y'_{Ri} + Y'_{Li} + Y'_{Wi} \end{aligned} \quad (12)$$

$$(I'_{zzi} + I'_{zzi}) \left( \frac{L_i}{U_i} \right)^2 \left( \frac{\dot{U}_i}{L_i} r'_i + \frac{U_i}{L_i} \dot{r}'_i \right) = N'_{Hi} + N'_{Ri} + N'_{Li} + N'_{Wi} \quad (13)$$

Where,  $m'_i$  represents non-dimensionalized mass of ship  $i$ ,  $m'_{xi}$  and  $m'_{yi}$  represent  $x, y$  axis components of non-dimensionalized added mass of ship  $i$ ,  $\beta_i$  means drift angle of ship  $i$ , respectively. The subscripts  $H, P, R, I$  and  $W$  mean ship hull, propeller, rudder, component of the hydrodynamic interaction effect between two vessels and wind.  $X, Y$  and  $N$  represent the external force of  $x, y$  axis and yaw moment about the center of

gravity of the ship. A rudder angle is controlled to keep course as follows:

$$\delta_i = \delta_{0i} - K_1(\psi_i - \psi_{0i}) - K_2 r'_i - K_3(S'_{pi} - S'_{p0i}) \quad (14)$$

where,  $\delta_i, \psi_i, r'_i$  represent rudder angle, heading, non-dimensional angular velocity of ship  $i$ , and  $S'_{pi}$  is non-dimensionalized predicted course of ship  $i$ . Subscript '0' indicates initial values. Also,  $K_1, K_2$  and  $K_3$  mean the control gain constants.

#### 4. Results and discussion

This section presents calculation results of ship maneuvering motion using the predicted hydrodynamic interaction effects between two vessels while overtaking in shallow waters under the influence of wind force based on the Fujiwara et al. (1998) method. Also, the Incheon outer harbor area through the PALMI island channel in Korea was selected, and the ship maneuvering simulation was carried out to propose an appropriate safe speed and distance between two vessels, which is required to avoid sea accident in confined waters. The simulation is carried out based on the estimated interaction forces between two vessels by assuming to move each other in a straight line. Comparison of ship trajectories between interaction effect and combined interaction and wind effect for the case of  $\nu = 120^\circ$  with various control gain constants is shown in Fig. 6. In this case, the wind velocity was taken as 10 m/s and the lateral distance between two vessels was taken as 0.3 times of ship length. Also, the control gain constants used in this numerical simulation are  $K_1 = K_2 = 5.0, K_3 = -1.0$  and maximum rudder angle,  $\delta_{max} = 10^\circ$ . As expected, the simulation result of ship maneuvering motion regarding the only interaction effect shows the little difference,

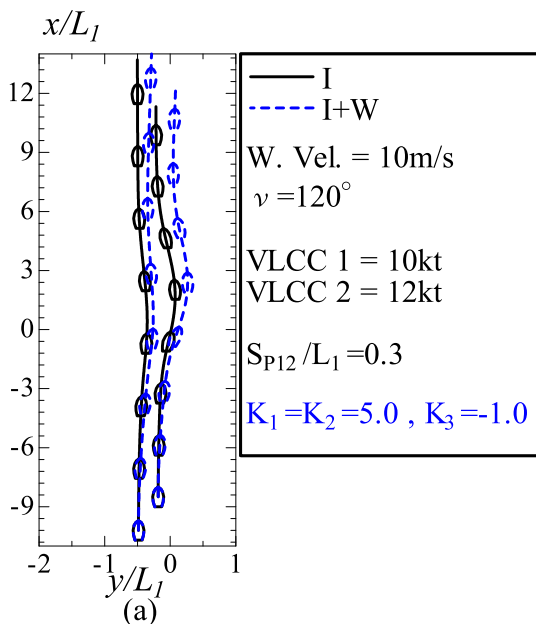


Fig. 6. Comparison of ship trajectories with function of interaction effect and combined interaction and wind effect.

compared to the case of interaction effect combined with wind force.

The ship trajectory of maneuvering simulation for a VLCC with function of rudder angle in Incheon outer harbor area through the PALMI island channel in Korea is shown in Fig. 7. As shown in Fig. 7 with no consideration of external forces such as interaction, wind and current effect, the course of a vessel did not deviate from its intended path with ranges of less than  $15^\circ$  in maximum rudder angle.

Fig. 8 shows the result of ship maneuvering simulation with function of the lateral distance between two vessels and rudder angle. In this Figure, the wind and current effect was not taken into account, and it is a simulation result regarding only interaction effect between two vessels. As shown in Fig. 8, an overtaken VLCC 2 with ranges of  $15^\circ$  in maximum rudder angle under the condition of 0.3 times of ship length is too much deviated from the intended course and approaches to the boundary of waterways, which is mainly attributed to lengthening the mutual effects on their relative position between two vessels. In this case, high-caution for the marine safety is required.

Fig. 9 shows the result of ship maneuvering simulation with function of the lateral distance between two vessels under the conditions, that the steady wind velocity and rudder angle were taken as 15 m/s and  $15^\circ$ , respectively. In this Figure, the current effect was not taken into account. As shown in Fig. 9, the deviation for the overtaken VLCC 2 under the condition of  $V_w = 15$  m/s was comparatively larger from its intended course. In this case, high-caution for the safety is required. However, as shown in Fig. 10, an overtaking and overtaken VLCC with maximum rudder angle of  $15^\circ$  under the condition of  $V_w = 10$  m/s can navigate while keeping its intended course even though the lateral distance between two vessels is about 0.5 times of ship length.

#### 5. Conclusion

In this research, the hydrodynamic interaction effects for the sake of reducing sea accidents in confined waters were determined by estimating interaction effects between two large vessels, which is navigating through limited sea area such as narrow waterways. Also, the maneuvering motions of two vessels while overtaking in curved narrow channel such as Incheon outer harbor area in Korea are simulated numerically to investigate the minimum safe distance between two vessels, and to contemplate the appropriate safe speed between two vessels required to avoid sea accident. By calculating and analyzing its result, following conclusions were obtained.

In case of hydrodynamic interaction effects between two vessels, the hydrodynamic force dramatically increases as the spacing between two vessels decreases when the lateral distance between two vessels is less than about 0.4 times of the ship length, and when lateral distance between two vessels is more than about 0.6 times of the ship length, hydrodynamic interaction effect dramatically decreases as the lateral distance increases. Also when the lateral distance between two vessels is about 1.0 times of ship length, interaction effect almost diminishes. If the

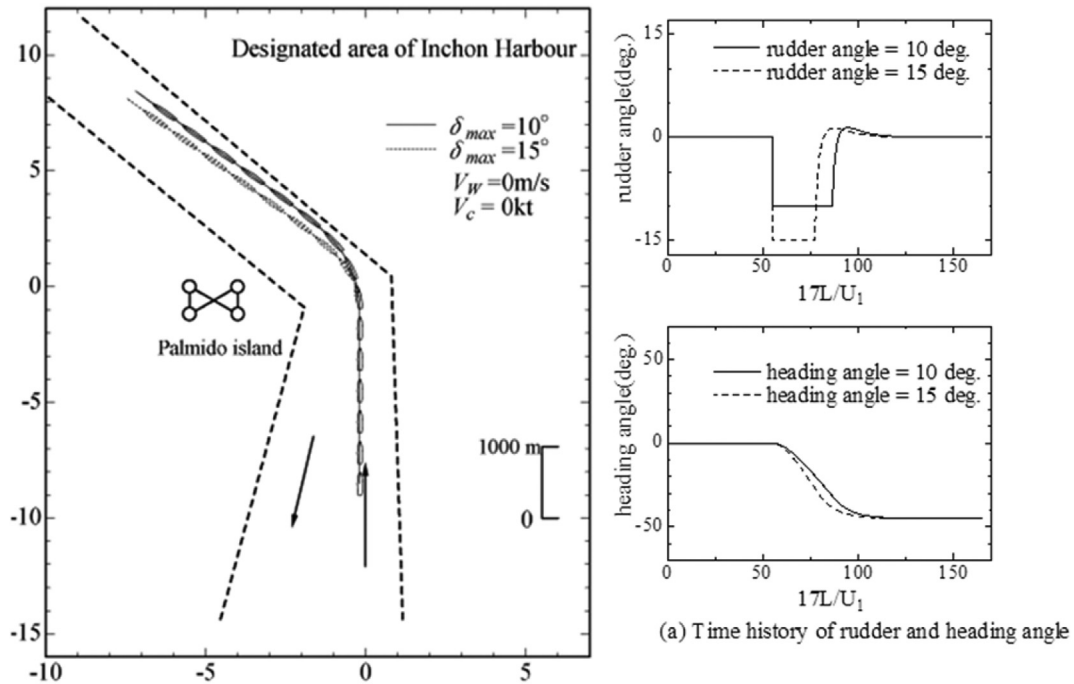


Fig. 7. Result of simulation in curved narrow channel ( $V_w = V_c = 0.0$ ).

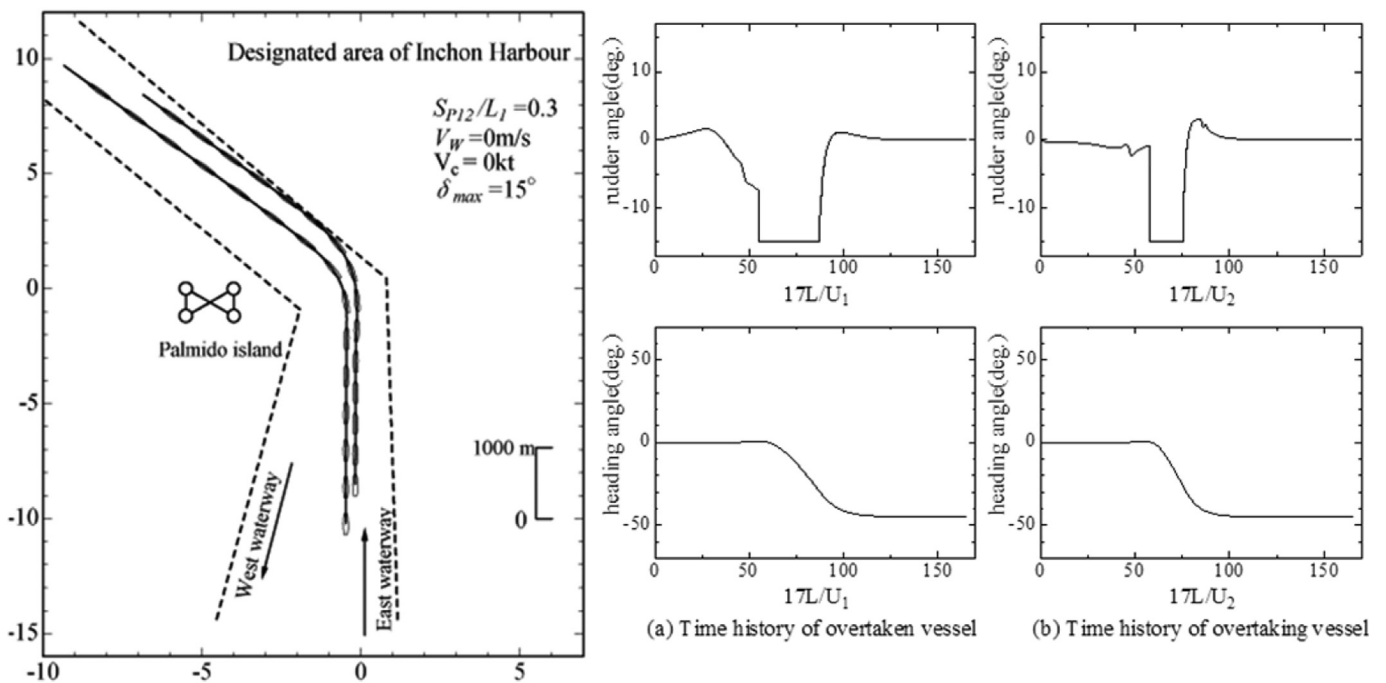


Fig. 8. Result of simulation under the influence of interaction effect in curved narrow channel ( $V_w = V_c = 0.0$ ,  $S_{P12} = 0.3L_1$ ).

wind was the only factor to be considered, an overtaken and overtaking vessel in narrow channel can navigate while keeping its own original course under the following conditions; the transverse distance between two vessels is more than about 0.6 times of ship length and maximum wind velocity should not be stronger than 10 m/s. Under the condition of  $S_{P12} \leq 1.0L$  in curved narrow channel, it may encounter a dangerous tendency

of collision due to the combined effect of the interaction between two vessels and wind force. Therefore, an overtaking and overtaken vessel while overtaking in curved narrow channel such as Incheon outer harbor in Korea should be navigated under the following conditions; the transverse distance between two vessels is more than about 1.0 times of ship length and maximum wind velocity should not be stronger than 10 m/s.



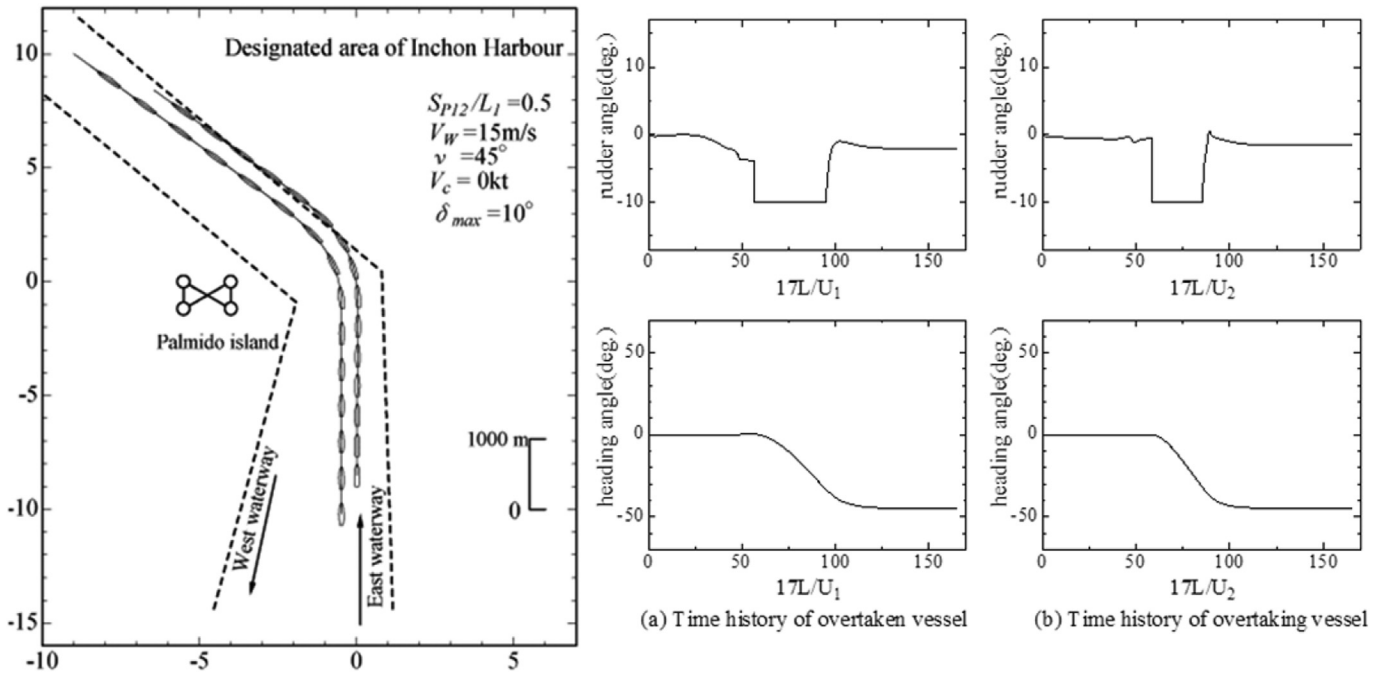


Fig. 9. Result of simulation under the influence of interaction effect in curved narrow channel ( $V_w = 15 \text{ m/s}$ ,  $V_c = 0.0$ ,  $S_{P12} = 0.5L_1$ ).

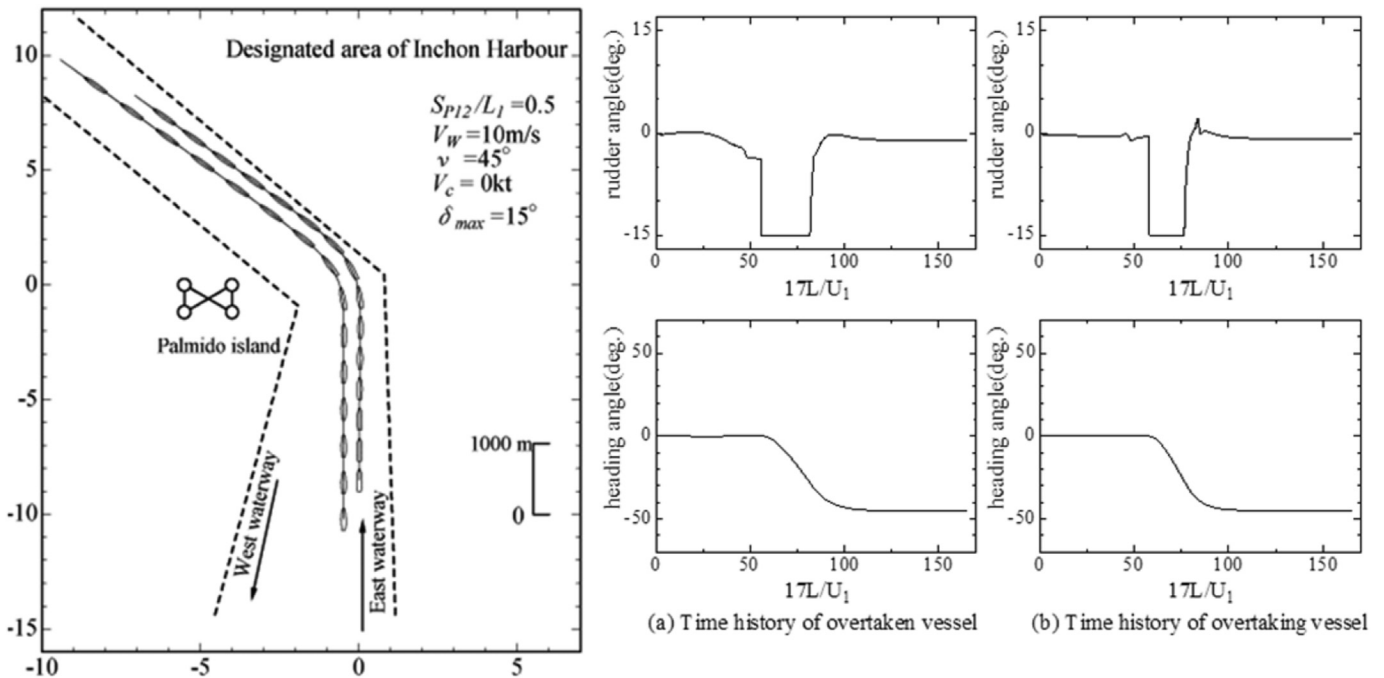


Fig. 10. Result of simulation under the influence of interaction effect in curved narrow channel ( $V_w = 10 \text{ m/s}$ ,  $V_c = 0.0$ ,  $S_{P12} = 0.5L_1$ ).

**Nomenclature**

- $B_i$  Breadth of ship  $i$
- $C_{Fi}, C_{Mi}$  Dimensionless hydrodynamic force and yaw moment of ship  $i$
- $d_i$  Draught of ship  $i$
- $\varepsilon$  Slenderness parameter
- $\Phi$  Velocity potential

- $h$  Water depth
- $L_i$  Length of ship  $i$
- $\Delta P$  Difference of linearized pressure about  $x_1$ -axis
- $\sigma, \gamma$  Source and vortex strength
- $\xi, \eta$  Source and vortex point
- $S_{P12}, S_{T12}$  Lateral and longitudinal distance between two ships
- $U_i$  Velocity of ship  $i$
- $V_w$  Wind velocity

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