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DESIGN VERIFICATION THROUGH TOLERANCE STACK UP ANALYSIS OF MECHANICAL ASSEMBLY AND LEAST COST TOLERANCE ALLOCATION

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Abstract

Geometric dimensioning and Tolerancing (GDT) constitutes the dominant approach for design and manufacture of mechanical parts that control inevitable dimensional and geometrical deviations within appropriate limits. The stack up of tolerances and their redistribution without hampering the functionality is very important for cost optimization. This paper presents a methodology that aims towards the systematic solution of tolerance stack up problem involving geometric characteristics.Conventional tolerance stack up analysis is usually difficult as it involves numerous rules and conditions. The methodology presented i.e. generic capsule method is straightforward and easy to use for stack up of geometrical tolerances of components and their assembly using graphical approach. In the work presented in this paper, angularity tolerance has been considered for illustration of the methodology. Two approaches viz. Worst Case (WC) and Root Sum Square (RSS) have been used. An example of dovetail mounting mechanism has been taken for purpose of stack up of angularity. Based on the stacked tolerance, it can be verified with the design tolerance of the assembly. Based on the comparison, designer has to reassign the appropriate tolerances to fulfil the functionality if required. If the stacked tolerance is as per designer requirement, then reallocation of tolerances on individual components should be done. Costs versus tolerance data are available for each component. With optimization technique, the optimized cost has been calculated for the assembly.

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Keywords: GDT; Angularity; Stack up Analysis; Graphical Method; WC Approach; RSS Approach, Reallocation; Optimization Technique.

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1. Introduction

Tolerance is an essential part of design and manufacturing. Design and Tolerancing (DT) is used to specify the size, shape, form, orientation, and location of features on a part. Features toleranced with GDT reflect the actual relationship between mating parts. Drawings with properly applied geometrical tolerancing provide the best opportunity for uniform interpretation and cost effective assembly. GDT is used to ensure the proper assembly of mating parts, to improve quality and to reduce cost by proper selection of manufacturing process. Before designers can properly apply geometric tolerancing, they must carefully consider the fit and function of each feature of every part. Properly applied geometrical tolerancing ensures interchangeablity of the parts. Geometrical tolerancing allows the designers to specify the maximum available tolerance and consequently design the most economical parts. A properly toleranced drawing is not only a picture that communicates the size and shape of the part, but it also explains the tolerance relationships between features.

In this paper, angularity is taken for study. Tolerance stack ups of individual components and their assembly have been carried out using graphical approach. Based on the stacked tolerance, it can be verified with the design tolerance of the assembly. Based on the comparison, designer has to reassign the appropriate tolerances to fulfil the functionality if required. If the stacked tolerance is as per designer requirement, then reallocation of tolerances on individual components has been done. Quantitative estimates of thecost of components with respect to tolerances along with simultaneous selection of processes are carried out, which permits the selection of component tolerances in mechanical assemblies for minimum cost of production.

2. Literature Review

A lot of work has been done in the field of conventional tolerancing. Conventional tolerancing methods do a good job for dimensioning and tolerancing of size features and are still used in good capacity. But these methods do not cater precisely for form, profile, runout, location and orientation features as discussed by Cogorno (2006), Meadows (2009) and Drake (1999). GDT is used extensively for location, profile, runout, form and orientation features. In more theoretical terms, there are two types of tolerancing schemes i.e. parametric and geometric. Parametric tolerancing consists of identifying a set of parameters and assigning limits to the parameters that define a range of values which has been discussed by Requicha (1993). Singh et al. (2009) reviewed different methods of tolerance allocation and found mean shift models and the combination of the basic approaches. Singh et al. (2009) reviewed tolerance synthesis approaches for tolerance stack up i.e. the worst case and the root sum square approach. Swift et al. (1999) introduced a knowledge based statistical approach to tolerance allocation. In this approach, a systematic analysis for estimating process capability levels at the design stage is used in conjunction with statistical methods for the optimization of tolerances in assembly stack up. Chase et al. (1990) demonstrated that the methods for tolerance allocation for minimum production cost can be extended to include process selection from a set of alternate processes. Ngoi et al. (2010) discussed the stack up of geometrical tolerances using generic capsule method. Ngoi et al. (1997) presented an elegant approach by using the ‘Quickie’ technique towards tolerance stack up analysis for geometrical tolerances. Ngoi et al. (1999) also presented a straightforward graphical approach known as the “Catena” method for tolerance stack up, involving geometric characteristics in form control – flatness, straightness, circularity and cylindricity. He and Gibson (1992) developed an extension of computerised trace method to determine the relationship between geometrical tolerances and manufacturing dimensions and tolerances. This method minimizes the cost of scrapas the objective function which is a function of manufacturing tolerances. Requirements of design sizes, geometrical tolerances (both form and position) and machining allowances are expressed mathematically as constraints for the optimization. Shivkumar et al. (2011) presented a general new methodology using intelligent algorithms for simultaneous optimal selection of design and manufacturing tolerances with alternative manufacturing process selection. Mansuy et al. (2011) presented an original method that enables to solve problems for the case of serial assembly (stacking) without clearances. This method is based on the use of influence coefficients to obtain the relationship between the functional tolerance and the tolerances associated with the geometry of the mechanism’s interface surfaces. Sahani et al. (2012) presented review of different techniques for stack up for flatness geometrical tolerances.

3. Methodology

Graphical Approach for Stack up Tolerances

A case is taken up for the stack up of angularity for components and their assembly. This assembly consists of two components i.e. 'Dovetail Female' and 'Dovetail Male' as shown in Fig. 2 &3 and their assembly 'Dovetail Assembly' is shown in Fig. 4.

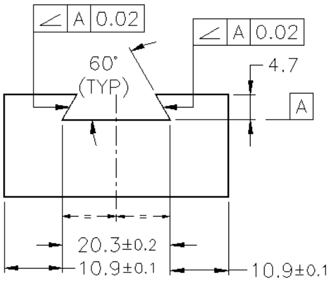

Fig. 1. Dovetail female.

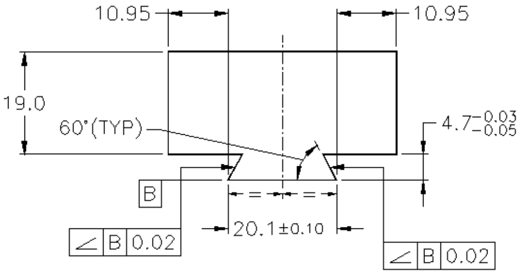

Fig. 2. Dovetail male.

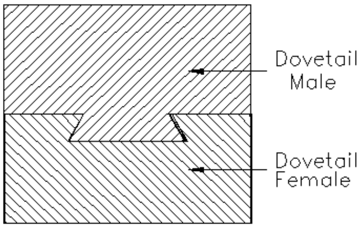

Fig. 3. Dovetail assembly.

The part number assigned for the 'Dovetail Female' component is 1 while the part number for the 'Dovetail Male' component is 2. The labelling of surfaces and vertices of 'Dovetail Female' and 'Dovetail Male' is shown in Fig. 5.

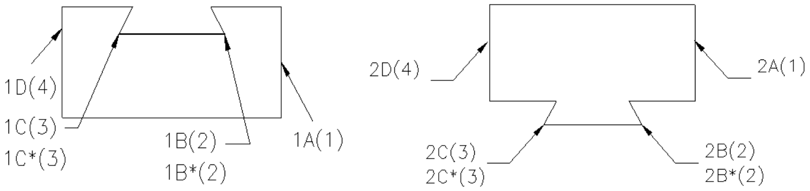

Fig. 4.Labelled dovetail.

The angularity tolerance is the distance between two lines or surfaces that are at an angle to the datum surface (AN) and encompass the line or surface is given at an angle (<ANB), which is transferred to the horizontal surface (AN). The angular tolerance transformation sketch is shown in Fig. 6.

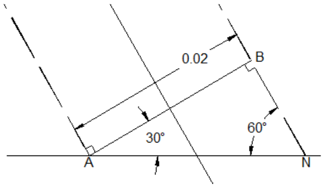

Fig. 5.Angle transformation.

Now from the Fig. 6,

$$\sin 60^\circ = AB / AN$$
$$\text{So, } AN = AB / \sin 60^\circ$$
$$= 0.02 / \sin 60^\circ$$
$$= 0.0231$$

Having completed the labelling phase, the graphical model is then constructed for 'Dovetail Female', 'Dovetail Male' and 'Dovetail Assembly' as shown in Fig. 7, 8 &9.

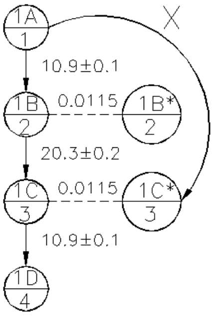

Fig. 6.GDT model for dovetail female.

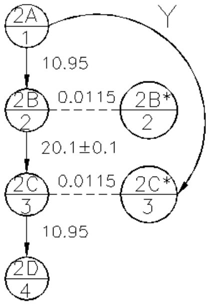

Fig. 7.GDT model for dovetail male.

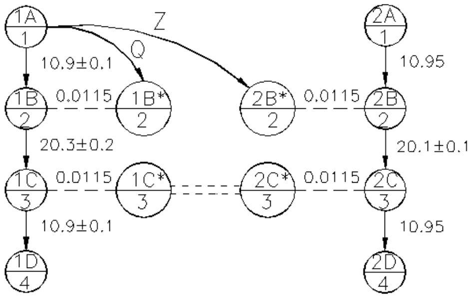

Fig. 8.GDT model for dovetail assembly.

To determine the distance between surface A to edge C for ‘Dovetail Female’ component, the stack up path is identified for calculation. It will follow the loop 1A - 1C* - 1C - 1B - 1A. The expression derived from the stack path is then

1A1C* - 1C*1C -1C1B-1B1A = 0

Substituting the values,

$X - (\pm 0.0115) - (20.3 \pm 0.2) - (10.9 \pm 0.1) = 0$

Worst Case (WC) Approach:

The total tolerance stack up can be written as

$$\Delta Y = \sum_{i=1}^{n} \delta i$$

Where,
n = Number of constituent dimensions
δi = Tolerance associated with dimension

$X - (\pm 0.0115) - (20.3 \pm 0.2) - (10.9 \pm 0.1) = 0$
$X - (31.2 \pm 0.3115) = 0$
$X = 31.2 \pm 0.3115$
Maximum and minimum values of X are

$X_{max} = 31.5115$
$X_{min} = 30.8885$

Root Sum Square (RSS) Approach:

Total tolerance of assembly can be written as

$$\Delta Y = \sqrt{\sum_{i=1}^{n} \delta_i^2}$$

Where,
n = Number of constituent dimensions
δi = Tolerance associated with dimension

$X = 31.2 \pm \sqrt{(0.0115^2 + 0.2^2 + 0.1^2)}$
$X = 31.2 \pm 0.2239$

Maximum and minimum values of X are

$X_{max} = 31.4239$
$X_{min} = 30.9761$

To determine the distance between surface A to edge C for ‘Dovetail Male’ component, the stack up path is identified for calculation. It will follow the loop 2A - 2C* - 2C - 2B - 2A. The expression derived from the stack path is then

2A2C* - 2C*2C -2C2B-2B2A = 0

Putting the values,

$Y - (\pm 0.0115) - (20.1 \pm 0.1) - (10.95) = 0$

Worst Case (WC) Approach:

$Y - (\pm 0.0115) - (20.1 \pm 0.1) - (10.95) = 0$
$Y = 31.05 \pm 0.1115$

Maximum and minimum values of Y are

$Y_{max} = 31.1615$
$Y_{min} = 30.9385$

Root Sum Square (RSS) Approach:

$Y=31.05 \pm \sqrt{(0.0115^2+0.1^2)}$
$Y = 31.05 \pm 0.1007$

Maximum and minimum values of Y are
$Y_{max} = 31.1507$
$Y_{min} = 30.9493$

In the case of an assembly, the graphical model is constructed part by part i.e. one model for the ‘Dovetail Female’ and another model for the ‘Dovetail Male’. The two part models are then linked together by double dashed line that represents contact. The labelling of surfaces and vertices of Dovetail Assembly is shown in Fig.10.

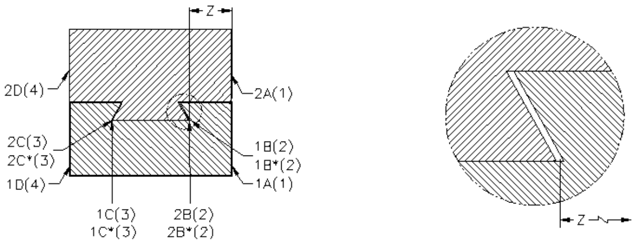

Fig. 9. Labelled dovetail assembly.

Here the mating edges are 1C of first part and 2C of second part. Unknown parameter is the distance between surface A of part 1 to edge B of part 2. Upon completion of the model, the stack path is identified. Since the requirement is to find the minimum value of Z, the correct stack path should pass through the double dashed line that connects between 1C* and 2C*. The expression derived from the stack path is then

1A2B* + 2B*2B + 2B2C + 2C2C* + 2C*1C* + 1C*1C – 1C1B – 1B1A= 0

Upon substitution,

$Z \pm 0.0115 + (20.1 \pm 0.1) \pm 0.0115 \pm 0.0 \pm 0.0115 - (20.3 \pm 0.2) - (10.9 \pm 0.1) = 0$

Worst Case (WC) Approach:

$Z - (11.1 \pm 0.4) \pm 0.0345 = 0$
$Z - (11.1 \pm 0.4345) = 0$
$Z = 11.1 \pm 0.4345$

Maximum and minimum values of Z are

$Z_{max} = 11.5345$
$Z_{min} = 10.6655$

Root Sum Square (RSS) Approach:

$Z = 11.1 \pm \sqrt{(0.1^2 + 0.2^2 + 0.1^2 + 0.0115^2 + 0.0115^2 + 0.0115^2)}$
$Z = 11.1 \pm 0.2458$

Maximum and minimum values of Z are

$Z_{max} = 11.3458$
$Z_{min} = 10.8542$

Following the same procedure, the expression for the distance 1A1B* is obtained as

1A1B* + 1B*1B - 1B1A = 0

Upon substitution,

$Q \pm 0.0115 - (10.9 \pm 0.1) = 0$

Worst Case (WC) Approach:

$Q \pm 0.0115 - (10.9 \pm 0.1) = 0$
$Q - (10.9 \pm 0.1115) = 0$
$Q = 10.9 \pm 0.1115$

Maximum and minimum values of Q are

$Q_{max} = 11.0115$
$Q_{min} = 10.7885$

Root Sum Square (RSS) Approach:

$Q = 10.9 \pm \sqrt{(0.0115^2 + 0.1^2)}$
$Q = 10.9 \pm 0.1007$

Maximum and minimum values of Q are
$Q_{max} = 11.0007$
$Q_{min} = 10.7993$

Calculation of Clearance (P):

Worst Case approach gives

Maximum Clearance, $P_{max} = Z_{max} - Q_{min}$
$= 11.5345 - 10.7885$
$= 0.746$

Minimum Clearance, $P_{min} = Z_{min} - Q_{max}$
$= 10.6655 - 11.0115$
$= -0.346$

Root Sum Square approach gives

Maximum Clearance, $P_{max} = Z_{max} - Q_{min}$
$= 11.3458 - 10.7993$
$= 0.5465$

Minimum Clearance, $P_{min} = Z_{min} - Q_{max}$
$= 10.8542 - 11.0007$
$= -0.1465$

Tolerance Allocation:

Critical tolerances in mechanical devices are generally the result of tolerance stack-up, ortolerance accumulation in assemblies of parts. The variation in the resultant clearances,interference fits, lubrication paths, end play, etc. depends on the variations in each of thecomponent parts in the assembly. The assembly tolerance is generally specified based onperformance requirements, while the component tolerances are closely related to the capabilitiesof the production processes. The most common tolerance specification problem encountered byengineering designers is tolerance allocation, which is the distribution of the specified assemblytolerance among the components of the assembly.

The component tolerances could be distributed equally among all of the parts in an assembly.However, each component tolerance may have a different manufacturing cost associated with itdue to part complexity or process differences. By defining a cost vs. tolerance function for eachcomponent dimension, the component tolerances may be allocated to minimize cost ofproduction. A substantial amount of research has been carried out regarding optimal tolerance allocation usingcost vs. tolerance functions.

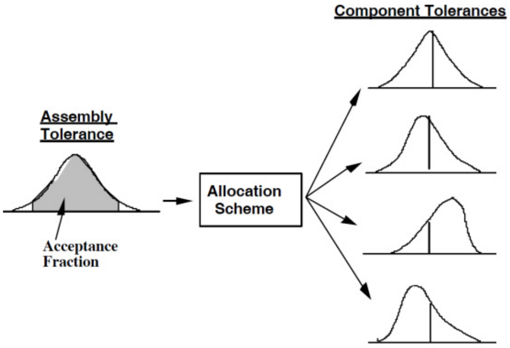

Fig. 10.Tolerance Reallocation.

If permissible assembly tolerance is 50 μ for the dovetail assembly, it will be divided in these two parts. The minimum limit of tolerance achieved by the manufacturing process is 10 μ. So, the limits of tolerance for components will vary from 10 to 40 μ. Out of different cost models, reciprocal and reciprocal squared models for component number 1; linear and reciprocal models for component number 2 has been chosen. Here constant coefficient A represents the fixed cost, such as tooling, setup, prior operation etc. and the term B represent the cost of producing a single component dimension to a specified tolerance T in μ.

Table 1. Values of constant parameter A and variable parameter B

Part. No.	**Process**	**Cost Model**	**A**	**B**
Part 1	Reciprocal	A+B/T	1000	5000
	Reciprocal Squared	$A+B/T^2$	1000	60000
Part 2	Liner	A-BT	30000	600
	Reciprocal	A+B/T	12000	50000

Calculation for Cost:

Taking the values of tolerance for component number 1 as t1 and for 2 is t2.

For t1=10 and t2=40
Cost of manufacturing Part 1 by process1 (C11) =1000+5000/10= 1000+500= **1500**
Cost of manufacturing Part 1 by process2 (C12) = 1000+60000/100= 1000+600=1600

Cost of manufacturing Part 2 by process1 (C21) =30000-600*40= 30000-24000= **6000**
Cost of manufacturing Part 2 by process2 (C22) = 6000+50000/40= 6000+1000=7000

Minimum Cost of Assembly= 1500+6000=7500

For t1=20 and t2=30
Cost of manufacturing Part 1 by process1 (C11) =1000+5000/20= 1000+250= 1250
Cost of manufacturing Part 1 by process2 (C12) = 1000+60000/400= 1000+150=**1150**

Cost of manufacturing Part 2 by process1 (C21) =30000-600*30= 30000-18000= 12000
Cost of manufacturing Part 2 by process2 (C22) = 6000+50000/30= 6000+1333=**7333**

Minimum Cost of Assembly= 1150+7333=8483

For t1=30 and t2=20
Cost of manufacturing Part 1 by process1 (C11) =1000+5000/30= 1000+167= 1167
Cost of manufacturing Part 1 by process2 (C12) = 1000+60000/900= 1000+67=**1067**

Cost of manufacturing Part 2 by process1 (C21) =30000-600*20= 30000-12000= 18000
Cost of manufacturing Part 2 by process2 (C22) = 6000+50000/20= 6000+2000=**8000**

Minimum Cost of Assembly= 1167+8000=9167

For t1=40 and t2=10
Cost of manufacturing Part 1 by process1 (C11) =1000+5000/40= 1000+125= 1125
Cost of manufacturing Part 1 by process2 (C12) = 1000+60000/1600= 1000+38=**1038**

Cost of manufacturing Part 2 by process1 (C21) =30000-600*10= 30000-6000= 24000
Cost of manufacturing Part 2 by process2 (C22) = 6000+50000/10= 6000+4000=1**0000**

Minimum Cost of Assembly= 1125+10000=11125

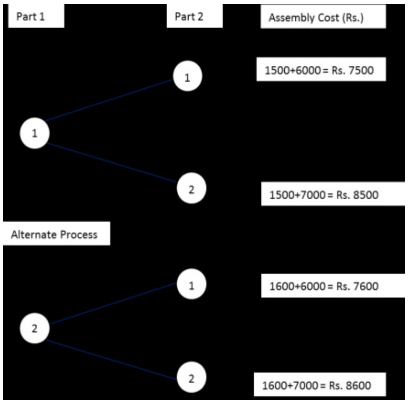

Fig. 11. Labelled dovetail assembly.

One method of finding the minimum cost tolerance allocation is to perform an exhaustive search over all possible combinations of processes. For the example problem, we would first choose process 1 of Part 1, process 1 for Part 2. Next, we would choose process 1 of Part 1, process 2 of part 2 and calculate the cost of fabrication. This procedure would be repeated until all possible combinations had been tried. By comparing the cost of assembly, the minimum cost specifications will be selected. Figure12 illustrates the procedure as a tree in which one process is selected from each column as you move from left to right.

The number of combinations increases geometrically as the number of parts and the number of processes increases. For the example problem, the number of combinations will be 4(2 x 2). In general, if there are N parts in an assembly and each part having n_i alternative processes, the number of combinations may be calculated from: n_1 x n_2 x n_3 x ... x n_N. Thus, a larger assembly of 10 parts, having just two processes per part, would have 2^{10} or 1024 combinations to check minimum assembly cost.

4. Results

Results obtained by Worst Case (WC) and Root Sum Square (RSS) approach for individual parts and their assembly are shown in Table 2. The results of stack up analysis show that the maximum clearance in assembly comes out to be positive value whereas minimum clearance takes negative value. This indicates a situation where the assembly of parts can't be done. Hence, reallocation of tolerances by the designer is required to remove the possibility of negative clearance. Based on the functional assembly tolerances, reallocation of tolerances has been done. A number of combinations have been taken to find out the cost of the assembly with simultaneous selection of manufacturing processes. By comparing the cost of assembly, it has been found that the minimum cost of assembly is Rs. 7500. Corresponding to this minimum cost, the process 2 is selected for part1 and process 2 is selected for part 2 with corresponding tolerance t1 as 10 μ and t2 as 40 μ.

Table 2.Results.

Approach / Part Name		WC		RSS	
		Max	Min	Max	Min
Dovetail Female *X*		31.5115	30.8885	31.4239	30.9761
Dovetail Male *Y*		31.1615	30.9385	31.1507	30.9493
Dovetail Assembly	*Z*	11.5345	10.6655	11.3458	10.8542
	Q	11.0115	10.7885	11.0007	10.7993
	P	0.746	-0.346	0.5465	-0.1465

5. Conclusion

The present paper explains an efficient and effective graphical method that aims towards the systematic solution of tolerance stack up problems. The method is straightforward and easy to use for stack up of tolerances of components and their assembly using graphical approach. Based on the results of stack up analysis, the maximum clearance in assembly comes out to be positive value whereas minimum clearance takes negative value. This indicates a situation where the assembly of parts can’t be done. Based on this conclusion, designer has to reassign the appropriate tolerances to fulfil the functionality if required. Once the functionality is fulfilled, the processes for various components will be selected based on minimum cost of assembly by reallocating the tolerances on individual components. Here in this paper, very limited components used in assembly, if number of components in an assembly is more, huge amountof mathematical calculations is required. Moreover, limited number of combination of reallocated tolerances has been taken, out of which the minimum cost is being selected. For this are allocation, an automatic program can be developed by using suitable algorithm. It can be further done with using some optimization technique like as genetic algorithm univariate search etc.

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