Shape Control of Local Parameter Adjustable Five Rational Transect and Application

Jubao Qu\textsuperscript{a}, Sheng Liu\textsuperscript{a}, Shujuan Wang\textsuperscript{b} a*

\textsuperscript{a}(Department of Mathematics & Computer, Wuyi University, Wuyishan City, Fujian, 354300, China)

\textsuperscript{b}(College of Continuing education, Wuyi University, Wuyishan City, Fujian, 354300, China)

Abstract

In order to enlarge the image after low-resolution images can still be rich in detail, the effect of sharp edges, this article constructs a rational quintic spline interpolation, and applied to image interpolation, due to the interpolation function has five Local shape control performance parameters, making the image zoom in or out of, according to the need for control. Experimental results show that the interpolation functions are constructed five effective and robust.

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1. Introduction

Image interpolation, also known as image resolution enhancement technology, and its purpose is to more clearly see the details of the image, which is to increase the image resolution. A good image interpolation algorithm should not only ensure the smoothness of edges, but also to ensure image clarity. Traditional image interpolation methods are nearest neighbor interpolation, bilinear interpolation, bicubic interpolation, etc.
spline interpolation, etc.\cite{1-3}, these methods did not reflect the image and the edge of the particularity of local features, so there are local image blurry or jagged edges Phenomenon. How to determine the conditions in the case of interpolation, constrained interpolation curve flexibility to adapt to the shape of the actual needs of engineering. At present rational three, four interpolation both the structure and shape control has been more research\cite{4-5}. the rational Five interpolation due to its structure too much computation it takes to be overlooked. In fact, in some cases, rational quintic interpolation has its unique application of results. As a rational interpolation function expression argument, so you can change the case of interpolation conditions on the parameters chosen by the curve of the local Changes to the shape of the curve interpolation control convenience. Based on the structure of the rational quintic spline interpolation, interpolation curve of this constraint in the two problems given curve and its approximation properties, and amplification of the image Has been applied.

2. Interpolation function construction

To meet the $C^2$ construct a smooth continuous five interpolation spline $R(x)$, $R(x)$ has the following properties:

1). in each sub-interval $[x_i, x_{i+1}]$, on, $R(x)$ is a molecule to five times a rational function;
2). $R(x) = f_i$, $i = 0, 1, ..., n+1$

As a result, the five interpolation spline $R(x)$ expressed as

$$R(x) = \frac{p_i(x)}{q_i(x)}, i=0,1,...,n$$

Which $q_i(x) = (1-\theta)^2 \alpha_i + (1-\theta)\theta \alpha_i + \theta^2 \beta_i$

$$p_i(x) = (1-\theta)^3 \alpha_i f_{i-1} + \theta(1-\theta)^4 U_i + \theta^2 (1-\theta)^3 V_i + \theta^3 (1-\theta)^2 W_i + \theta^4 (1-\theta) T_i + \theta^5 \beta_i f_i$$

$U_i = (4\alpha_i + \beta_i) f_i + \alpha_i h_i d_i$, $V_i = \alpha_i (4 f_{i+1} - h_i d_{i+1}) + \beta_i (4 f_i + h_i d_i)$,

$T_i = (\alpha_i + 4 \beta_i) f_{i+1} - \beta_i h_i d_{i+1}$

$W_i$ is the adjustable parameter, its value is arbitrary. $\alpha_i > 0$, $\beta_i > 0$.

Easy to prove, given the data and selected parameters $\alpha_i, \beta_i$, as defined above five interpolation function is a rational existence, and $R(x)$ satisfy the $C^2$ continuity constraint.

When adjusting the parameters $W_i = 4(\alpha_i f_{i+1} + \beta_i f_i) + h_i (\beta_i d_i - \alpha_i d_{i+1})$ time, $R(x)$ becomes the standard five Hermite interpolation.

If so $R'(x_i+)$ = $R'(x_i-)$, $i = 0, I, ..., n-1$ when you can get $R(x)$ satisfy the continuity constraint $C^2$.

3. Curve constraints and error estimates

Let $f(x)$ is the interpolating function, so that $R(x)$ is $x$ is $[x_0, x_n]$ on by (1) defined the five interpolation spline function, $k(x)$ is $x$ belongs to $[x_0, x_n]$ defined the three functions, or $x_0 < x_i < ... < x_{n-1} < x_n$ for the points of the piecewise cubic function, $k_i$, $k'_i$, respectively, $k(x)$ in the $x_i$ function value and derivative at Value. At this time in $[x_{i-1}, x_i]$ on $k(x)$ can be expressed as

$$k(x) = (1-\theta)^3 k_{i-1} + (1-\theta)^2 (2k_{i-1} + k'_i h_i) + (1-\theta) \theta^2 (3k_i + k'_i h_i) + \theta^3 k_i$$

So be bound by the interpolation curve given the constraints between the three curve theorem:
Theorem 1 (Curve constraints theorem). For a given data \( \{ x_i, f_i, d_i, k_i, k_i', k_i'' \} \), \( i = 0,1, ..., n \), and \( k_i'' \leq f_i \leq k_i \). There are (1) defines the five interpolation spline function \( R(x) \) in \([x_{i-1}, x_i]\) on the cubic curve segment in \( k^*(x) \) above and in the cubic \( k(x) \) under Sufficient condition is: are parameters of \( \alpha_i, \beta_i \) satisfies the following inequalities.

\[
\begin{align*}
\phi_i(4f_i, h_i, d_i, \exists k_i, k_i^*) \leq 0 \\
\phi_i(4f_i, h_i, d_i, 2k_i, \exists k_i, k_i^*) \leq 0 \\
\phi_i(4f_i, h_i, d_i, 2k_i, \exists k_i, k_i^*) \leq 0 \\
\phi_i(4f_i, h_i, d_i, 2k_i, \exists k_i, k_i^*) \leq 0 \\
\phi_i(4f_i, h_i, d_i, 2k_i, \exists k_i, k_i^*) \leq 0 \\
\phi_i(4f_i, h_i, d_i, 2k_i, \exists k_i, k_i^*) \leq 0
\end{align*}
\]

Proof: The following proof of \( R(x) \) in \([x_{i-1}, x_i]\) on the cubic curve segment in \( k^*(x) \) above sufficient condition. By formula (1) shows that \([x_{i-1}, x_i]\),

\[
q_i(x) = (1-\theta)^3 \alpha_i + (1-\theta) \theta \alpha_i + \theta^2 \beta_i > 0,
\]

\[
k^*(x) = (1-\theta)^3 k_i + (1-\theta)^2 \theta (2k_i - k_i') + (1-\theta)^2 \theta (3k_i + k_i'^* h_i) + \theta \beta_i
\]

So \( R(x) = \frac{p_i(x)}{q_i(x)} \geq k^*(x) \), Equivalent to \( p_i(x) - q_i(x)k^*(x) \geq 0 \).

Let \( \Phi_i(x) = p_i(x) - q_i(x)k^*(x) \geq 0 \), There

\[
\Phi_i(x) = (1-\theta)^3 \alpha_i f_i - \theta (1-\theta)^4 U_i + \theta^2 (1-\theta)^3 V_i + \theta^3 (1-\theta)^2 W_i + \theta^4 (1-\theta) T_i + \theta^5 \beta_i f_i
\]

\[
-[(1-\theta)^3 k_i + (1-\theta)^2 \theta (2k_i - k_i') + (1-\theta)^2 \theta (3k_i + k_i'^* h_i) + \theta \beta_i]]
\]

\[
\times [(1-\theta)^2 \alpha_i + (1-\theta) \theta \alpha_i + \theta^2 \beta_i]
\]

\[
= (1-\theta)^3 \alpha_i (f_i - k_i') + (1-\theta)^4 \theta [U_i - \alpha_i (2k_i - k_i'^* h_i) - \alpha_i k_i] + (1-\theta)^3 \theta^2 [V_i - \alpha_i (2k_i - k_i'^* h_i) + 3k_i + k_i'^* h_i - \beta_i k_i] + (1-\theta)^2 \theta^3 [W_i - \alpha_i k_i - \beta_i (3k_i + k_i'^* h_i) - \alpha_i (3k_i + k_i'^* h_i)] + (1-\theta) \theta^4 [T_i - \alpha_i k_i - \beta_i (3k_i + k_i'^* h_i)] + \theta \beta_i (f_i - k_i)
\]

\[
= (1-\theta)^3 \alpha_i (f_i - k_i') + (1-\theta)^4 \theta \Psi_i + (1-\theta)^3 \theta^2 \Omega_i
\]

\[
+ (1-\theta)^2 \theta^3 \Xi_i + (1-\theta) \theta^4 \Theta_i + \theta \beta_i (f_i - k_i)
\]

Which: \( \Psi_i = U_i - \alpha_i (2k_i - k_i'^* h_i) - \alpha_i k_i \)

\[
\Omega_i = V_i - \alpha_i (2k_i - k_i'^* h_i) + 3k_i + k_i'^* h_i - \beta_i k_i
\]

\[
\Xi_i = W_i - \alpha_i k_i - \beta_i (3k_i + k_i'^* h_i)
\]

\[
\Theta_i = T_i - \alpha_i k_i - \beta_i (3k_i + k_i'^* h_i)
\]
\[
\begin{align*}
\Xi_i &= W_i - \alpha_i k_i - \beta_i (2k_i - k''_i h_i) - \alpha_i (3k_i + k''_i h_i) \\
&= 4(\alpha_i f_{i+1} + \beta_i f_i) + h_i (\beta_i d_i - \alpha_i d_{i-1}) - \beta_i (2k_i - k''_i h_i) - \alpha_i (3k_i + k''_i h_i) \\
&= 4(\alpha_i f_{i+1} - h_i d_{i+1} - 4k_i - k''_i h_i) + \beta_i (4f_i + h_i d_i - 2k_{i-1} - k''_i h_i) \\
\Theta_i &= T_i - \alpha_i k_i - \beta_i (3k_i + k''_i h_i) = \alpha_i (f_{i+1} - k_i) - \beta_i (4f_{i+1} + h_i d_{i+1} + 3k_i + k''_i h_i)
\end{align*}
\]

Because \( f_{i-1} \geq k_{i-1}, f_i \geq k_i \), \( \Pi_i \geq 0, \Omega_i \geq 0, \Xi_i \geq 0, \Theta_i \geq 0, \) then for all \( x \in [x_{i-1}, x_i] \) have \( \Phi_i(x) \geq 0 \) established that \( R(x) \) is a curve segment \( k^*(x) \) above. Similarly, to prove \( R(x) \) in \( [x_{i-1}, x_i] \) is located on the third curve segment \( k(x) \) under the sufficient condition.

Proof end.

**Theorem 2 (Error estimates theorem).** Let \( f(x) \in C^2(a,b), \Delta: a = x_0 < x_1 < \cdots < x_n = b \) is the interval \( [a, b] \) on a partition of, \( R(x) \) is type (l) defined by the five interpolation spline function, then there

\[
\|f(x) - R(x)\| \leq \frac{h_i^2 c_i}{4} \|f''\| (3)
\]

Called the optimal error coefficient \( c_i \) is a positive bounded rational. It can be seen that the approximation error of order \( O(h^2) \).

4. Application of numerical analysis and interpolation

Interpolation data set \( (0,0), (0.5,0.808), (1,1), (1.5,0.808), (2,0), (2.5,-0.808), (3,-1), (3.5,-0.808), (4,0), \) that is \( h=0.5 \), where \( R(x) \) is in \( [0,4] \) on the (l)-style defined by five rational spline curve. to take on the constraint curve \( k(x) \) and lower bound curves \( k^*(x) \) respectively

\[
k(x) = \begin{cases} 
-x^2 + 2x + 0.08, & 0 \leq x < 2 \\
x^2 - 6x + 8.08, & 2 \leq x \leq 4 
\end{cases}, \quad k^*(x) = \begin{cases} 
-x^2 + 2x - 0.08, & 0 \leq x < 2 \\
x^2 - 6x + 7.98, & 2 \leq x \leq 4 
\end{cases}
\]

Just select to meet the inequalities (2) of the positive parameter \( \alpha_i, \beta_i \), can make \( R(x) \) constraint on the two curves \( k(x) \) and \( k^*(x) \) between. As figure 1.

![Image](image_url)

**Fig.1** \( \alpha_3 = 4, \alpha_5 = 9, \beta_2 = 0.18, \beta_4 = 2.36 \), Other \( \alpha_i = 7, \beta_i = 0.46 \) constraints image

Image interpolation technology does not affect the sensory effects in the premise of the image processing. Image interpolation is the use of known low magnification image resolution interpolated high resolution image, in the aerospace, medical imaging, infrared imaging and many other areas Wide range of applications. To illustrate this algorithm for image interpolation results, use the standard Lena image as subjects, is 5 times magnification, Figure 2 is under a variety of interpolation algorithms in the Matlab...
simulation of the effect of treatment comparison. Seen from the figure, bilinear interpolation and cubic spline interpolation of the local images are obvious mosaic and edge blur, but the algorithm can be based on local features of the control parameters automatically adjust the amplification to achieve better results.

![Simulated images](image)

Fig. 2. (a) Original whole image; (b) Bilinear local image; (c) Cubic spline interpolation of the local image; (d) Our Algorithm

The following different algorithms for different interpolation image to enlarge, and by PSNR (peak signal noise ratio) and the correlation coefficient (NC) were analyzed. Can be found, the algorithm has a good correlation between signal to noise ratio and robustness.

Table 1. The effect of different image interpolation algorithm performance comparison

<table>
<thead>
<tr>
<th>Image</th>
<th>Our Algorithm</th>
<th>Nearest neighbor interpolation</th>
<th>Bilinear interpolation</th>
<th>Cubic spline interpolation</th>
</tr>
</thead>
<tbody>
<tr>
<td>PSNR</td>
<td>NC</td>
<td>PSNR</td>
<td>NC</td>
<td>PSNR</td>
</tr>
<tr>
<td>Lena</td>
<td>44.37</td>
<td>0.923</td>
<td>41.84</td>
<td>0.884</td>
</tr>
<tr>
<td>Cameraman</td>
<td>45.25</td>
<td>0.954</td>
<td>38.93</td>
<td>0.895</td>
</tr>
<tr>
<td>Baboon</td>
<td>44.51</td>
<td>0.943</td>
<td>39.73</td>
<td>0.865</td>
</tr>
<tr>
<td>Couple</td>
<td>47.83</td>
<td>0.893</td>
<td>42.38</td>
<td>0.906</td>
</tr>
<tr>
<td>Toy</td>
<td>44.85</td>
<td>0.943</td>
<td>40.65</td>
<td>0.908</td>
</tr>
</tbody>
</table>

5. Conclusion

This structure of the rational interpolation function five times, not only has the simple mathematical expression, but also in the case of interpolation conditions remain unchanged by adjusting the parameters of the selection curve of the local modification, both for applications, and easy to theory. Between the three curves are given sufficient conditions for theorems, and through a numerical example, according to map and image interpolation application, we prove the adaptability and superiority of the algorithm.

References