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Matter–antimatter asymmetry generated by loop quantum gravity

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Abstract

We show that loop quantum gravity provides new mechanisms through which observed matter–antimatter asymmetry in the Universe can naturally arise at temperatures less than GUT scale. This is enabled through the introduction of a new length scale $\mathcal{L}$, much greater than Planck length (l_{P}), to obtain semiclassical weave states in the theory. This scale which depends on the momentum of the particle modifies the dispersion relation for different helicities of fermions and leads to lepton asymmetry.

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1. Introduction

Many theories of quantum gravity are expected to bring non-trivial modifications to the underlying spacetime near Planck scale. Loop quantum gravity, which is one of the candidate theories of quantum gravity, predicts a discrete spectrum for geometrical operators [1]. However, inaccessibility of Planck scale in laboratories poses a challenge to test such predictions. It is hence desired that a contact be made with the classical world through some semi-classical techniques. This might also open a window to see the signatures of quantum gravity at the level of effective theories which may differ from conventional low energy theories.

Loop quantum gravity which is based on the quantization of spacetime itself, results in a polymer like structure of quantum spacetime. The classical spacetime is a coarse grained form of underlying discrete quantum spacetime and one of the important issues in loop quantum gravity is to understand the transition from discrete quantum spacetime to smooth classical spacetime. Though the low energy sector of loop quantum gravity and the transition to the classical spacetime is yet to be completely understood, there have been some attempts in this direction to obtain the semi-classical states in the theory which include construction of weave states which can approximate 3-metrics [2] and via coherent states which peak around classical trajectories [3]. For more discussion on related issues we refer the reader to a recent review [4].

The coherent state approach to understand low energy sector is based on finding quantum states which

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give minimum dispersion for the observables in the theory, whereas the weave state approach involves a new length scale $\mathcal{L} \gg l_P$ such that for distances $d \ll \mathcal{L}$ polymer structure of quantum spacetime becomes manifest and for $d \geqslant \mathcal{L}$ one recovers continuous flat classical geometry. This approach was extended to study the construction of weave states describing gravity coupled to massive spin-$1/2$ Majorana fields in a series of important papers by Alfaro, Morales-Técotl, and Urrutia (AMU) [5–7]. This new length scale modifies the dispersion relation for different helicities of fermions and breaks Lorentz invariance in the theory.

Apart from loop quantum gravity, deformations of the Lorentz invariance manifest by means of a slight deviation from the standard dispersion relations of particles propagating in the vacuum have been suggested in various ways, see, for example, [8,9]. The modifications to dispersion relation may arise if the underlying spacetime is non-commutative [10]. These theories which are characterized by a non-commutativity parameter of dimensions of square of length, may serve as a description for foamy structure of quantum spacetime. Similar modifications have also been studied in the framework of String theory [11, 12]. These approaches foresee a dispersion relation in vacuo of particles of the form (we shall use natural units $c = 1 = \hbar$)

$$E^2 \approx p^2 + m^2 + f(M, pl_P), \tag{1.1}$$

where $f(x)$ is a model dependent function, M fixes a characteristic scale not necessarily determined by Planck length $l_P \sim 10^{-19}$ GeV^{-1}, and $pl_P \ll 1$. As a consequence of Eq. (1.1), the *quantum gravitational medium* responds differently to the propagation of particles of different energies.

With the breakdown of the Lorentz invariance in the theory, *CPT* violation is expected and so new mechanisms to generate observed matter–antimatter asymmetry in the Universe. The matter–antimatter asymmetry in the Universe is conventionally understood, for example, through baryogenesis processes occurring at GUT or electroweak scales. However, most of these conventional mechanisms to generate this asymmetry in standard model or extensions of it run into one or another problem [13], like inflation would significantly dilute the asymmetry produced during GUT era. Any low temperature mechanism to generate this asymmetry is hence highly desirable. The origin of matter–antimatter asymmetry in the Universe thus remains one of the unsolved puzzles whose resolution may be possible through some new aspects of physics arising through a fundamental theory. There have been earlier proposals based on quantum gravity framework to generate matter–antimatter asymmetry, like from primordial spacetime foam [14], quantum gravity deformed uncertainty relations [15] and string based scenarios [16]. In this Letter, we would like to present another interesting scenario arising out from quantum gravity through weave states. We would show that weave states of spin-$1/2$ fields provide a natural mechanism to generate matter–antimatter asymmetry at temperatures of the order of reheating temperature of inflation, far below the GUT temperature.

2. Lepton asymmetry in AMU formalism

In the weave state approach, the goal is to find a loop state which approximates a classical geometry at a scale much larger than l_P. A semi-classical weave state corresponding to Majorana fermions is characterized by a scale length $\mathcal{L}$ such that $l_P \ll \mathcal{L} \leqslant \lambda_D = 1/p$, where λ_D is the de Broglie wavelength of the fermion and p its corresponding momentum. For the Dirac equation with quantum corrections to be properly defined on a continuous flat spacetime arising through weave state construction, it is required that the scale length $\mathcal{L} \leqslant 1/p$ [5]. Such a scale is known as mobile scale [7] which is different for different fermionic species and the upper bound on $\mathcal{L}$ corresponding to the weave state of a particular fermion is set by the momentum of that fermion. One may also treat this scale as a universal scale and obtain bounds on it by observations [17–19], however, in this Letter we would restrict to the case of $\mathcal{L}$ as a mobile scale.

The introduction of scale $\mathcal{L}$ leads to modifications in dispersion relation which have been studied in various interesting contexts [20–22]. Similar phenomena have also been studied for photons [6,23,24]. The dispersion relation with leading order terms in l_P and $\mathcal{L}$ can be written as [18]

$$E_\pm^2 = (1 + 2\alpha)p^2 + \eta p^4 \pm 2\lambda p + m^2 \tag{2.1}$$

with

$$\alpha = \kappa_1 \left(\frac{l_{\mathrm{P}}}{\mathcal{L}}\right)^2, \qquad \eta = \kappa_3 l_{\mathrm{P}}^2, \qquad \lambda = \kappa_5 \left(\frac{l_{\mathrm{P}}}{2\mathcal{L}^2}\right), \tag{2.2}$$

where κ_1, κ_3 and κ_5 are of the order of unity. Here '$+$' and '$-$' refer to two helicity states of the fermion. We would specialize to the limiting case of Majorana fermions with vanishingly small mass, $m \to 0$. In this way we can treat the fermions as Weyl particles. We would further neglect l_{P}^2 terms in comparison to other dominating terms in the above dispersion relation. Thus, the dispersion relation can be rewritten as

$$E_{\pm}^2 = p^2 \pm 2\lambda p. \tag{2.3}$$

It should be noted that though the above modification to the dispersion relation is of linear in momentum, the correction term effectively behaves as the one cubic in momentum. This is because $\mathcal{L}$ is a mobile scale and its upper bound scales as $1/p$. Thus, the above correction, which is similar to other cubic in momentum modifications to dispersion relation [4], dies out rapidly at low momenta. We would now discuss the implications of this dispersion relation for the case of neutrinos, where the helicity dispersion can be casted in terms of the difference between energy levels of particle and antiparticle states which leads to a net difference in their number densities and hence lepton asymmetry in this framework.

Matter–antimatter asymmetry is generally understood through Sakharov conditions who in his seminal paper [25], showed that to generate the non-zero baryonic number to entropy $\eta_{\mathrm{B}} \sim (2.6\text{–}6.2) \times 10^{-10}$ from a baryonic symmetric universe, the following requirements are necessary: (1) baryon number processes violating in particle interactions; (2) C and CP violation in order that processes generating B are more rapid with respect to $\bar{B}$; (3) out of the equilibrium: since $m_{\mathrm{B}} = m_{\bar{\mathrm{B}}}$, as follows from CPT symmetry, the equilibrium space phase density of particles and antiparticles are the same. To maintain the number of baryon and antibaryon different, i.e., $n_{\mathrm{B}} \neq n_{\bar{\mathrm{B}}}$, the reaction should freeze out before particles and antiparticles achieve the thermodynamical equilibrium.

GUT theories offer an ideal setting for Sakharov's conditions to be satisfied [26]. Baryon number violation occurs in these theories since gauge bosons mediate interactions that transform quarks into leptons and antiquarks. C is maximally violated in the electroweak sector, and CP violation follows by making the coupling constants of lepto-quark gauge bosons complex. Finally, out of equilibrium condition is achieved by the expansion of the universe when the reaction rates become lower than the Hubble expansion rates at some *freeze-out* temperature. Such a temperature is characterized by the decoupling temperature T_d, which, in the GUT baryogenesis scenario, is given by $T_d \sim 10^{16}$ GeV. However, GUT baryogenesis runs into problems because inflation occurring at similar temperature dilutes the baryon asymmetry. For baryons to be produced after inflation it is necessary to reheat the Universe to the scale of M_{GUT} which is unrealistic in inflationary scenarios. In fact, bounds on gravitino production give the reheating temperature T_{R} of the order of 10^8–10^{10} GeV [27], whereas in SUSY inflation models this may be raised to 10^{12} GeV [28]. Similarly, processes like electroweak baryogenesis and leptogenesis suffer from problems like very small region of parameter space which can yield asymmetry and lack of direct measurement of relevant parameters [13].

It is worth to quote some alternative mechanisms proposed in literature which are not based on quantum gravity. As observed in Refs. [29,30], if the CPT symmetry and the baryon number is violated, a baryon asymmetry could arise in thermal equilibrium. This mechanism to generate baryon asymmetry has been applied in different contexts: the spontaneous breaking of CPT induced by the coupling of baryon number current with a scalar field [29]; baryogenesis asymmetry generated from primordial tensor perturbation [31] and matter–antimatter asymmetry through interaction between gravitational curvature and fermionic spin [32]. For other mechanisms related to the lepton asymmetry, see Ref. [33] and reference therein, as well as Ref. [34].

In loop quantum gravity, the different dispersion relations of particles having different helicity determines a deviation from thermal equilibrium between neutrinos and antineutrinos, $n(\nu) \neq n(\bar{\nu})$, where $n(\nu)$ and

$n(\bar{\nu})$ are the number density of positive helicity neutrinos and negative helicity antineutrinos, respectively. We would further assume that there are no additional mechanisms which give rise to neutrino asymmetry. In such a case, the deviation from the chemical equilibrium, which generates the baryon asymmetry, occurs only due to loop quantum gravity effects and the expansion of the universe. If neutrinos are produced with energy E, then the dispersion relation (2.3) gives

$$E^2 = p^2 + 2\lambda p \implies p = \sqrt{E^2+\lambda^2} - \lambda, \tag{2.4}$$

for neutrinos, and

$$E^2 = p^2 - 2\lambda p \implies p = \sqrt{E^2+\lambda^2} + \lambda, \tag{2.5}$$

for antineutrinos. Note that energy dispersion relation *forbids* antineutrinos to have $p \in [0, 2\lambda]$, whereas no such restriction arises for neutrinos. This is purely a loop quantum gravity effect induced through quantum structure of spacetime which seemingly favors one helicity over another.

The number density of neutrinos at the equilibrium for a given temperature T is (for $k_B = 1$)

$$n(\nu) = \frac{gT^3}{2\pi^2}\int_0^{\chi} dx\, \frac{x(\sqrt{x^2+z}-\sqrt{z})^2}{\sqrt{x^2+z}}\frac{1}{e^x+1}, \tag{2.6}$$

where $\chi = 1/(\mathcal{L}T)$, $x = E/T$, $z = (\lambda/T)^2$, T satisfies the relation $\dot{T} = -HT$, and $H = \dot{a}/a$, being $a(t)$ the scale factor of the universe, [30]. The dot stands for the derivative with respect to the cosmic time. The departure from the chemical equilibrium caused by the expansion of the universe is encoded in the quantity $\mathcal{F} = 3Hn + \dot{n}$ [30], which turns out to be

$$\mathcal{F}(\nu) = 2\left(\frac{\lambda}{T}\right)^2 HT^3 \times \frac{d}{dz}\left[\frac{g}{2\pi^2}\int_0^{\chi} dx\, \frac{x(\sqrt{x^2+z}-\sqrt{z})^2}{\sqrt{x^2+z}}\frac{1}{e^x+1}\right].$$

It vanishes as $\lambda = 0$.[1] Similar results hold for antineutrinos:

$$n(\bar{\nu}) = \frac{gT^3}{2\pi^2}\int_{2\sqrt{z}}^{\chi} dx\, \frac{x(\sqrt{x^2+z}+\sqrt{z})^2}{\sqrt{x^2+z}}\frac{1}{e^x+1},$$

$$\mathcal{F}(\bar{\nu}) = 2\left(\frac{\lambda}{T}\right)^2 HT^3 \times \frac{d}{dz}\left[\frac{g}{2\pi^2}\int_{2\sqrt{z}}^{\chi} dx\, \frac{x(\sqrt{x^2+z}+\sqrt{z})^2}{\sqrt{x^2+z}}\frac{1}{e^x+1}\right]. \tag{2.8}$$

Thus the net neutrino asymmetry generated via loop quantum gravity effects would become

$$\begin{aligned}\Delta n &= |n(\nu) - n(\bar{\nu})| \\ &= \frac{2g\lambda T^2}{\pi^2}\int_0^{\chi} dx\, \frac{x}{e^x+1} + \frac{gT^3}{2\pi^2} I(z) \\ &\approx \frac{2g\lambda T^2}{\pi^2}\left[\frac{\pi^2}{12} + 12\sum_{n=1}^{\infty}\frac{(-\exp(\chi))^n}{n^2} + \frac{12}{\mathcal{L}T}\ln(1+e^{\chi}) - \frac{6}{(\mathcal{L}T)^2}\right],\end{aligned} \tag{2.9}$$

where

$$I(z) \equiv \int_0^{2\sqrt{z}} dx\, \frac{x(\sqrt{x^2+z}-\sqrt{z})^2}{\sqrt{x^2+z}\,(e^x+1)}. \tag{2.10}$$

In evaluating (2.9) we have neglected the contribution coming from the $I(z)$-term since at low temperatures with respect to Planck's one it is expected to be very small compared to other terms. If we note that $\mathcal{L} \leqslant \lambda_D$

[1] In absence of loop quantum gravity corrections, the deviation from the chemical equilibrium occurs only if particles are massive. In fact, being

$$n = \frac{gT^3}{2\pi^2}\int_0^{\infty}\frac{x^2\,dx}{e^{\sqrt{x^2+(m/T)^2}}+1},$$

the function $\mathcal{F}$ becomes [30]

$$\mathcal{F} = 2\left(\frac{m}{T}\right)^2 HT^3 \frac{d}{dy}\left[\frac{g}{2\pi^2}\int_0^{\infty}\frac{x^2\,dx}{e^{\sqrt{x^2+y}}+1}\right], \tag{2.7}$$

where $y = (m/T)^2$ and it vanishes as $m = 0$ [30,35].

and use the upper bound $\mathcal{L} \sim 1/\bar{p} \sim T$, where $\bar{p}$ is the de Broglie momenta of neutrinos at a particular temperature, we can estimate the neutrino asymmetry arising at that temperature.

Near GUT temperatures $T \sim 10^{16}$ GeV, the ratio of neutrino asymmetry to entropy density, $\Delta n/s$ ($s \sim 0.44 g_* T^3$, with $g_* \sim 10^2$ [26]), turns out to be of the order of 10^{-5} which would, however, be washed out by inflation. Interesting temperatures would be near reheating temperatures[2] of the order of 10^{10}–10^{11} GeV where this ratio would become of the order of 10^{-10}. At lower temperatures the amount of asymmetry generated would keep on decreasing till it becomes negligible, though the asymmetry generated at reheating temperature would hold till the neutrinos finally decouple. This lepton asymmetry would lead to the baryon asymmetry through various GUT and electroweak processes and thus contribute to the existing mechanisms to produce matter–antimatter asymmetry in the Universe.

3. Conclusion

An intriguing prediction of modern approaches to quantum gravity is a slight departure from Lorentz's invariance, which manifests in a deformation of the dispersion relations of photons and fermions. Such results have been indeed suggested in loop quantum gravity [5,6,24], string theory [11,12] and non-commutative geometry [10]. The former is endowed with a scale length characterizing the scale on which new effects are non-trivial, thus to wonder if there exist different scenarios where these effects become testable (see [17,36]) is certainly of current interest.

In this Letter we have shown that such modifications induced by loop quantum gravity might help to put some light on unsolved problems like matter–antimatter asymmetry in standard model. Application of weave states for Majorana fermions naturally leads to difference in energies for different chiralities which may be interpreted as difference in particle and antiparticle energies for the case of massless neutrinos. This leads to asymmetry between matter and antimatter species and yields the observed value at around reheating temperatures. Our proposal introduces a way for generation of matter–antimatter asymmetry via loop quantum gravity, whose complete analysis would require relaxing the massless limit and secondly taking into account various standard model interactions in unison with loop quantum gravity. Then we shall be able to know how the above mechanism to generate matter–antimatter asymmetry contributes relative to other processes. This opens up a new arena to make phenomenological studies in loop quantum gravity in future.

It is a remarkable phenomena that quantum structure of spacetime itself may generate matter–antimatter asymmetry in the universe. In fact, this might be a generic feature of theories of quantum gravity. It reflects that quantum gravity may lead to effects occurring at lower energy scales, specially in the desert between electroweak and Planck scale, which may provide natural answers to some unsolved problems.

[2] In this case $\lambda/T \sim l_{\mathrm{P}}/\mathcal{L}$ and the corrections to the neutrino asymmetry due to $I(z)$ term would go as $(l_{\mathrm{P}}/\mathcal{L})^3$ which for the temperature range of 10^{11} GeV would be of the order of 10^{-24}. Hence, our approximation in Eq. (2.9) is justified.

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References

[1] C. Rovelli, Living Reviews, Vol. 1, Loop Quantum Gravity, at http://www.livingreviews.org/articles;
T. Thiemann, gr-qc/0110034;
T. Thiemann, gr-qc/0210094;
R. Gambini, J. Pullin, Loops, Knots, Gauge Theories and Quantum Gravity, Cambridge Univ. Press, Cambridge, UK, 1996;
J. Pullin, hep-th/9301028;
C. Rovelli, Class. Quantum Grav. 8 (1991) 1613;
A. Ashtekar, Lectures on New Perturbative Canonical Gravity, World Scientific, Singapore, 1991.

[2] A. Ashtekar, C. Rovelli, L. Smolin, Phys. Rev. Lett. 69 (1992) 237.

[3] T. Thiemann, gr-qc/0110034;
M. Varadarajan, J.A. Zapata, Class. Quantum Grav. 17 (2000) 4085;
H. Sahlmann, T. Thiemann, O. Winkler, Nucl. Phys. B 606 (2001) 401;
T. Thiemann, gr-qc/0206037.

[4] L. Smolin, hep-th/0303185.
[5] J. Alfaro, H.A. Morales-Téctol, L.F. Urrutia, Phys. Rev. Lett. 84 (2000) 2318.
[6] J. Alfaro, H. Morales-Técolt, L.F. Urrutia, Phys. Rev. D 65 (2002) 103509;
J. Alfaro, H. Morales-Técolt, L.F. Urrutia, in: R.T. Jantzen, V. Gurzadyan, R. Ruffini (Eds.), Proceedings of the Ninth Marcel Grossmann Meeting on General Relativity, World Scientific, Singapore, 2002.
[7] J. Alfaro, H. Morales-Téctol, L.F. Urrutia, Phys. Rev. D 66 (2002) 124006.
[8] G. Amelino-Camelia, J. Ellis, N.E. Mavromatos, D.V. Nanopoulos, S. Sarkar, Nature 393 (1998) 763;
For a review, see: G. Amelino-Camelia, Lect. Notes Phys. 541 (2000) 1.
[9] D.V. Ahluwalia, hep-ph/0212222.
[10] G. Amelino-Camelia, S. Majid, Int. J. Mod. Phys. A 15 (2000) 4301;
A. Matusis, L. Susskind, N. Toumbas, JHEP 0012 (2000) 02;
G. Amelino-Camelia, Int. J. Mod. Phys. D 11 (2002) 35.
[11] V.A. Kostelecky, S. Samuel, Phys. Rev. D 39 (1989) 683;
D. Colladay, V.A. Kostelecky, Phys. Rev. D 58 (1998) 116002;
D. Colladay, V.A. Kostelecky, Phys. Rev. D 55 (1997) 6760;
V.A. Kostelecky, R. Lehnert, Phys. Rev. D 63 (2001) 065008;
O. Bertolami, C.S. Carvalho, Phys. Rev. D 61 (2000) 103002.
[12] J. Ellis, K. Farakos, N.E. Mavromatos, V.A. Mitsou, D.V. Nanopoulos, Astrophys. J. 535 (2000) 139;
J. Ellis, N.E. Mavromatos, D.V. Nanopoulos, G. Volkov, Gen. Relativ. Gravit. 32 (2000) 1777;
J. Ellis, N.E. Mavromatos, D.V. Nanopoulos, Phys. Rev. D 61 (2000) 027503.
[13] M. Dine, A. Kusenko, hep-ph/0303065.
[14] D.V. Ahluwalia, M. Kirchbach, Int. J. Mod. Phys. D 10 (2001) 811.
[15] G. Amelino-Camelia, Mod. Phys. Lett. A 12 (1997) 1387.
[16] O. Bertolami, D. Colladay, A.V. Kostelecky, R. Potting, Phys. Lett. B 395 (1997) 178.
[17] J. Alfaro, G. Palma, Phys. Rev. D 65 (2002) 103516.
[18] J. Alfaro, G. Palma, hep-th/0208193.
[19] G. Lambiase, Mod. Phys. Lett. A 18 (2003) 23;
G. Lambiase, gr-qc/0302053;
The general formalism can be found in: G. Lambiase, Phys. Lett. B 560 (2003) 1.
[20] F.W. Stecker, S.L. Glashow, Astropart. Phys. 16 (2002) 97;
O.C. De Jager, F.W. Stecker, Astrophys. J. 566 (2002) 738.
[21] D. Sudarsky, L.F. Urrutia, H. Vucetich, Phys. Rev. Lett. 89 (2002) 231301.
[22] T.J. Konopka, S.A. Major, New J. Phys. 4 (2002) 57.
[23] R.J. Gleiser, C.N. Kazameh, Phys. Rev. D 64 (2001) 083007.
[24] R. Gambini, J. Pullin, Phys. Rev. D 59 (1999) 124021.
[25] A.D. Sakharov, JETP Lett. 5 (1967) 24.
[26] E.W. Kolb, M.S. Turner, The Early Universe, Addison–Wesley, Reading, MA, 1989.
[27] S. Sarkar, Rep. Prog. Phys. 59 (1996) 1493.
[28] J. McDonald, Phys. Rev. D 61 (2000) 083513.
[29] A. Cohen, D. Kaplan, Phys. Lett. B 199 (1987) 251;
A. Cohen, D. Kaplan, Nucl. Phys. B 308 (1988) 913.
[30] A.D. Dolgov, Ya.B. Zeldovic, Rev. Mod. Phys. 53 (1981) 1.
[31] S. Mohanty, B. Mukhopadhyay, A.R. Prasanna, hep-ph/0204257.
[32] P. Singh, B. Mukhopadhyay, Mod. Phys. Lett. A 18 (2003) 779;
B. Mukhopadhyay, P. Singh, gr-qc/0301002;
B. Mukhopadhyay, P. Singh, gr-qc/0303053.
[33] A.D. Dolgov, Phys. Rep. 370 (2002) 333.
[34] M. Claudon, L.J. Hall, I. Hinchliffe, Nucl. Phys. B 241 (1984) 309;
K. Yamamoto, Phys. Lett. B 168 (1986) 341;
O. Bertolami, G.G. Ross, Phys. Lett. B 183 (1987) 163;
A. Linde, Phys. Lett. B 70 (1977) 306;
V.A. Kuzmin, V.A. Rubakov, M.E. Shaposhnikov, Phys. Lett. B 191 (1987) 171.
[35] D. Toussaint, S.B. Treiman, F. Wilczek, A. Zee, Phys. Rev. D 19 (1979) 1036.
[36] L.F. Urrutia, Mod. Phys. Lett. A 17 (2002) 943;
J. Ellis, N.E. Mavromatos, D.V. Nanopoulos, Phys. Rev. D 63 (2001) 124025;
J. Ellis, E. Gravanis, N.E. Mavromatos, D.V. Nanopoulos, gr-qc/0209108;
T. Jacobson, S. Liberati, D. Mattingly, Phys. Rev. D 66 (2002) 081302;
T. Jacobson, S. Liberati, D. Mattingly, hep-ph/0209264;
G. Lambiase, Gen. Relativ. Gravit. 33 (2001) 2151;
S. Sarkar, Mod. Phys. Lett. A 17 (2002) 1025.