Physics Letters B 678 (2009) 491–496

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Localization of matters on thick branes

Jun Liang *, Yi-Shi Duan

Institute of Theoretical Physics, Lanzhou University, Lanzhou 730000, People's Republic of China

ARTICLE INFO

Article history:
Received 3 April 2009
Received in revised form 12 May 2009
Accepted 30 June 2009
Available online 4 July 2009
Editor: A. Ringwald

PACS:
11.10.Kk
04.50.-h
11.25.-w

Keywords:
Extra dimensions
Thick branes
Localization of fields
Zero modes

ABSTRACT

We study localization of various spin fields on flat thick branes, two different models are considered. For spin 0 scalar field, the massless zero mode is found to be normalizable on both the thick brane models. Spin 1 vector field cannot be normalizable on either of the two brane models. And for spin 1/2 field there is no bound zero mode for both the left and right chiral fermions. In order to localize the left or right chiral fermions on the thick brane models, the usual Yukawa scalar-fermion coupling is considered. It is shown that, the two models give different localization properties for fermions. Namely, whether there is a bound zero mode is related to the considered model.

1. Introduction

In recent years, brane world received a considerable attention from the physical community, as addressing the hierarchy problem [1–3], the supersymmetry breaking and the cosmological constant problem and so on [4,5]. In brane world scenarios, our world is a 3-brane embedded in a higher-dimensional space–time, and the Standard Model particles are assumed to live on the 3-brane, whereas the gravitational field is free to propagate in the whole space–time (the so-called bulk). Branes can be classified as thin and thickness ones [6]. Thin branes are constructed after introducing a tension term in the action, localized by a Dirac delta function. (See Ref. [7] and references therein.) The issue of fields and gravity localization in such branes is addressed with the help of Dirac delta functions, without any clear subjacent dynamics. On the other hand, thick branes are constructed in a dynamical way after one or more scalar fields coupled with gravity [5,6,8–10]. Thick branes are more natural in the sense that field and gravity localization can be studied with the introduction of smooth functions (instead of Dirac ones). Moreover, the thin brane solutions are recovered in certain limits [11,12].

In the brane world scenarios, an important issue is how gravity and different observable matter fields of the Standard Model of particle physics are localized on the brane. It has been shown that, in the Randall–Sundrum model in 5-dimensional space–time, graviton and spin 0 field can be localized on a brane with positive tension [13–15]. Spin 1 field cannot be localized either on a brane with positive tension or on a brane with negative tension [15]. (But spin 1 field can be localized on a string-like defect in high-dimensional space–time [16].) And moreover spin 1/2 and 3/2 can be localized on a negative-tension brane [15]. In order to achieve localization of fermions on a brane with positive tension, it seems that additional interactions except the gravitational interaction must be including in the bulk.

Of late, Bazeia, et al. investigate gravity localization on diverse thick brane models. The aim of the present Letter is to study localization of various spin fields on flat thick branes. Two thick brane models with analytic Schrödinger-like potential in Refs. [17,18] are considered. The organization of the Letter is as follows: First, in Section 2 we review the analytic models of Refs. [17,18]. In Section 3 we then study localization of various matters on thick branes. Finally, a summary and outlook are presented in Section 4.

2. Thick brane models

We start with [17]

* Corresponding author.
E-mail address: liangjunbeijing@yahoo.com.cn (J. Liang).

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doi:10.1016/j.physletb.2009.06.067

$$S=\int d^4x\,dy\,\sqrt{|g|}\left[-\frac{1}{4}R+\frac{1}{2}\partial_a\phi\partial^a\phi-V(\phi)\right], \tag{1}$$

where $g=\det(g_{ab})$ and the metric

$$ds^2=g_{ab}\,dx^a\,dx^b=e^{2A}\eta_{\mu\nu}\,dx^\mu\,dx^\nu-dy^2 \tag{2}$$

describes a background with 4-dimensional Poincaré symmetry with y as the extra dimension. Here $a,b=0,1,2,3,4$, and e^{2A} is the warp factor. We suppose that the scalar field and the warp factor only depend on the extra coordinate y. The action given by (1) leads to the following equations for the scalar field $\phi(y)$ and the function $A(y)$ from the warp factor:

$$\phi''+4A'\phi'=\frac{dV(\phi)}{d\phi}, \tag{3}$$

$$A''=-\frac{2}{3}\phi'^2, \tag{4}$$

$$A'^2=\frac{1}{6}\phi'^2-\frac{1}{3}V(\phi), \tag{5}$$

where the prime is used to represent derivative with respect to y.

The potential is supposed to have the form

$$V(\phi)=\frac{1}{8}\left(\frac{dW}{d\phi}\right)^2-\frac{1}{3}W^2, \tag{6}$$

where $W(\phi)$ is in principle an arbitrary function of the field ϕ – in the supersymmetric context W is named superpotential. The particular relation between V and W in (6) leads to a description in terms of a set of first-order differential equations, which are given by Refs. [8–12,19–26]

$$\phi'=\frac{1}{2}\frac{\partial W}{\partial\phi}, \tag{7}$$

$$A'=-\frac{1}{3}W. \tag{8}$$

2.1. Model 1: $W_1(\phi)=3a\sinh(b\phi)$

For this model, we have

$$A(y)=-\frac{1}{3b^2}\ln\left[\sec^2\left(\frac{3}{2}ab^2y\right)\right]. \tag{9}$$

We introduce a new variable z that turns the metric into a conformal one. The new conformal coordinate z is defined by

$$dz=e^{-A(y)}\,dy. \tag{10}$$

In general, this model doesn't give an analytic relation for $z(y)$. However, for $b^2=\frac{1}{3}$ we get

$$z=\int e^{-A(y)}\,dy=\frac{2}{a}\tan\frac{ay}{2}. \tag{11}$$

Inverting this expression and substituting in the expression for $A(y)$, we further get

$$A(z)=-\ln\left(1+\frac{a^2z^2}{4}\right), \tag{12}$$

where a is a parameter. Differentiating (9) with respect to y and using (7) and (8) we can obtain

$$\phi(y)=\sqrt{3}\ln\left(\sec\frac{ay}{2}+\tan\frac{ay}{2}\right). \tag{13}$$

We turn back to z coordinate, (13) becomes

$$\phi(z)=\sqrt{3}\ln\left[\sqrt{1+\left(\frac{a}{2}z\right)^2}+\frac{a}{2}z\right]. \tag{14}$$

2.2. Model 2: $W_2(\phi)=3\arcsin(b\phi)$

For this model, we have

$$A(y)=-\frac{2}{3b^2}\ln\left[q\cosh\left(\frac{3}{2}ab^2y\right)\right]. \tag{15}$$

For $b^2=\frac{2}{3}$ we can obtain an analytic expression for $z(b)$

$$z=\frac{q}{a}\sinh(ay) \tag{16}$$

with

$$A(z)=-\ln\left(q\sqrt{\frac{a^2z^2}{q^2}+1}\right) \tag{17}$$

and

$$\phi(z)=\sqrt{\frac{3}{2}}\arcsin\left[\tanh\left(\operatorname{arcsinh}\frac{az}{q}\right)\right]. \tag{18}$$

3. Localization of various matters

In this section, we study whether various bulk fields with spin ranging from 0 to 1 can be localized on thick branes by means of only the gravitational interaction.

3.1. Spin 0 scalar field

In this subsection we study localization of a real scalar field on thick branes described in previous section. Let us consider the action of a massless real scalar coupled to gravity:

$$S_0=\frac{1}{2}\int d^5x\,\sqrt{-g}g^{MN}\partial_M\Phi\partial_N\Phi, \tag{19}$$

from which the equation of motion can be derived:

$$\frac{1}{\sqrt{-g}}\partial_M\left(\sqrt{-g}g^{MN}\partial_N\Phi\right)=0. \tag{20}$$

By considering (2) the equation of motion (20) becomes

$$\left[\partial_z^2+3(\partial_zA)\partial_z+\eta^{\mu\nu}\partial_\mu\partial_\nu\right]\Phi=0. \tag{21}$$

The separation of variable is taken as

$$\Phi(x,z)=\sum_n\phi_n(x)\chi_n(z), \tag{22}$$

and demanding $\phi_n(x)$ satisfies the 4-dimensional massive Klein–Gordon equation

$$\eta^{\mu\nu}\partial_\mu\partial_\nu\phi_n(x)=-m_n^2\phi_n(x), \tag{23}$$

we obtain the equation for $\chi_n(z)$

$$\left[\partial_z^2+3(\partial_zA)\partial_z+m_n^2\right]\chi_n(z)=0. \tag{24}$$

The 5-dimensional action (19) reduces to the 4-dimensional action for massive scalars, when integrated over the extra dimension under (24) is satisfied and the following orthonormality condition is obeyed

$$\int_{-\infty}^{+\infty}dz\,e^{3A}\chi_m(z)\chi_n(z)=\delta_{mn}. \tag{25}$$

Define $\tilde{\chi}_n(z)=e^{\frac{3}{2}A}\chi_n(z)$, we get the Schrödinger-like equation

$$\left[-\partial_z^2+V(z)\right]\tilde{\chi}_n(z)=m_n^2\tilde{\chi}_n(z), \tag{26}$$

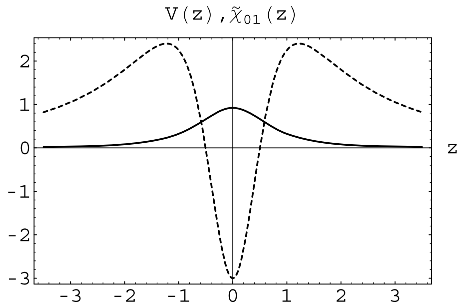

Fig. 1. The dashed line and solid one for the potential (28) and the zero mode (29), respectively. The parameter is set to $a=2$.

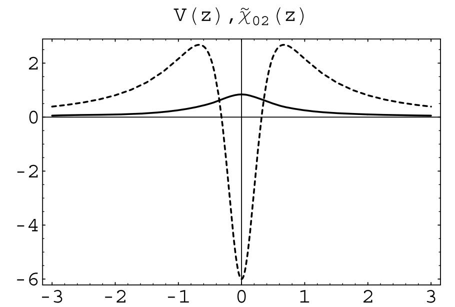

Fig. 2. The dashed line and solid one for the potential (30) and the zero mode (31), respectively. The parameters are set to $a=2$ and $q=1$.

where m_n is the mass of the Kaluza–Klein (KK) excitation and the potential is given by

$$V(z)=\frac{3}{2}\partial_z^2 A+\frac{9}{4}(\partial_z A)^2. \tag{27}$$

The potential only depends on the warp factor exponent A and has the same form as the case of graviton [17]. For Model 1, the Schrödinger-like potential is reduced to

$$V(z)=\frac{3a^2(a^2z^2-1)}{4(1+a^2z^2/4)^2}. \tag{28}$$

This potential has the asymptotic behavior: $V(z=\pm\infty)=0$ and $V(z=0)=-3a^2$. In fact this is a volcano type potential [27–31], this means that the potential provides no mass gap to separate the scalar zero mode from KK modes. The zero mode $m_0^2=0$ is determined analytically as

$$\tilde{\chi}_{01}(z)=\frac{N_{01}}{(1+a^2z^2/4)^{3/2}}, \tag{29}$$

where $N_{01}=(8/3\pi)^{1/2}(a/2)^{1/2}$. This function represents the lowest energy eingenfunction of the Schrödinger-like equation (26). In fact, (26) can be written as $H\tilde{\chi}=m_n^2\tilde{\chi}$ [5,8,9,32], where the Hamiltonian operator is given by $H=Q^\dagger Q$ with $Q=-\partial_z+\frac{3}{2}\partial_z A$. Since the operator H is positive definite, there are no normalizable modes with negative m_n^2, namely, there is no tachyonic scalar mode, the scalar zero mode is the lowest mode in the spectrum. In addition to the massless mode, the volcano type potential (28) suggests that there exists a continuum gapless spectrum of KK mode with positive $m_n^2>0$, which is delocalized KK massive scalars [5,8,9,32,33]. The shapes of the potential (28) and the zero mode (29) are shown in Fig. 1.

For Model 2, we can get the following Schrödinger-like potential

$$V(z)=\frac{3a^2(-2q^2+5a^2z^2)}{4(q^2+a^2z^2)^2}, \tag{30}$$

and the normalizable zero mode

$$\tilde{\chi}_{02}(z)=\frac{N_{02}}{(q^2+a^2z^2)^{3/4}}, \tag{31}$$

where $N_{02}=\frac{1}{\sqrt{2}}(a)^{1/4}(q)^{5/4}$ is a normalization constant. The potential (30) and the corresponding zero mode (31) are shown in Fig. 2.

3.2. Spin 1 vector field

Let us start with the action of U(1) vector field:

$$S_1=-\frac{1}{4}\int d^5x\sqrt{-g}g^{MN}g^{RS}F_{MR}F_{NS}, \tag{32}$$

where $F_{MN}=\partial_M A_N-\partial_N A_M$ as usual. From this action the equation motion is given by

$$\frac{1}{\sqrt{-g}}\partial_M\left(\sqrt{-g}g^{MN}g^{RS}F_{NS}\right)=0. \tag{33}$$

In the background metric (2), this equation of motion (33) is reduced to

$$\eta^{\mu\nu}\partial_\mu F_{\nu 4}=0, \tag{34}$$

$$\partial^\mu F_{\mu\nu}+(\partial_z+\partial_z A)F_{\nu 4}=0. \tag{35}$$

We assume that A_μ are Z_2-even and that A_4 is Z_2-odd with respect to the extra dimension z, which results in that A_4 has no zero mode in the effective 4-dimensional theory. Furthermore, in order to consistent with the gauge invariant equation $\oint dz\,A_4=0$, we use gauge freedom to choose $A_4=0$ [31]. After an integration by parts, (32) yields

$$S_1=-\frac{1}{4}\int d^4x\,dz\left[e^A\eta^{\nu\lambda}\eta^{\nu\rho}F_{\mu\nu}F_{\lambda\rho}+2\eta^{\mu\nu}A_\mu\partial_z\left(e^A\partial_z A_\nu\right)\right]. \tag{36}$$

Let us decompose the vector field as follows

$$A_\mu(x,z)=\Sigma_n a_\mu^{(n)}\rho_n(z), \tag{37}$$

and imposing orthonormality condition

$$\int_{-\infty}^{+\infty} dz\,e^A\rho_m(z)\rho_n(z)=\delta_{mn}. \tag{38}$$

The action (36) reduces to

$$S_1=\sum_n\int d^4x\left(-\frac{1}{4}\eta^{\mu\lambda}\eta^{\nu\rho}f_{\mu\nu}^{(n)}f_{\lambda\rho}^{(n)}+\frac{1}{2}m_n^2\eta^{\mu\nu}a_\mu^{(n)}a_\nu^{(n)}\right), \tag{39}$$

where $f_{\mu\nu}^{(n)}=\partial_\mu a_\nu^{(n)}-\partial_\nu a_\mu^{(n)}$ is the 4-dimensional field strength tensor, and $\rho_n(z)$ has been required to satisfy the equation

$$\left[\partial_z^2+(\partial_z A)\partial_z+m_n^2\right]\rho_n(z)=0. \tag{40}$$

By defining $\tilde{\rho}_n=e^{A/2}\rho_n$, we get the Schrödinger-like equation for the vector field

$$\left[-\partial_z^2+V(z)\right]\tilde{\rho}_n(z)=m_n^2\tilde{\rho}_n(z), \tag{41}$$

where the potential $V(z)$ is given by

$$V(z)=\frac{1}{2}\partial_z^2 A+\frac{1}{4}(\partial_z A)^2. \tag{42}$$

For Model 1, the potential (42) is reduced to

$$V(z)=\frac{a^2(a^2z^2/2-1)}{4(1+a^2z^2/4)^2}. \tag{43}$$

The vector zero mode is determined analytically as

$$\tilde{\rho}_{01}(z) = \frac{N_{01}}{(1+a^2z^2/4)^{1/2}}, \tag{44}$$

where N_{01} is a normalization constant. As the integral of $\tilde{\rho}_{01}^2(z)$ does not convergent on $(-\infty, +\infty)$, the vector zero mode is non-normalized.

For Model 2, the potential (42) is reduced to

$$V(z) = \frac{a^2(-2q^2+3a^2z^2)}{4(q^2+a^2z^2)^2}, \tag{45}$$

and the zero mode is

$$\tilde{\rho}_{02}(z) = \frac{N_{02}}{(q^2+a^2z^2)^{1/4}}. \tag{46}$$

As the integration $\int_{-\infty}^{+\infty}[(q^2+a^2z^2)^{-1/2}]\,dz$ is not convergent, we cannot still obtain normalizable vector zero mode on the brane.

3.3. Spin 1 spinor field

Now we are ready to consider spin 1/2 fermion. We introduce the vielbein $e_M^{\bar{M}}$ (and its inverse $e_{\bar{M}}^{M}$) through the usual definition $g_{MN} = e_M^{\bar{M}} e_N^{\bar{N}} \eta_{\bar{M}\bar{N}}$, where $\bar{M}, \bar{N}, \ldots,$ denote the local Lorentz indices. From the formula $\Gamma^M = e_{\bar{M}}^M \Gamma^{\bar{M}}$ with Γ^M and $\Gamma^{\bar{M}}$ being the curved gamma matrices and the flat gamma ones, respectively. We have $\Gamma^M = (e^{-A}\gamma^\mu, -ie^{-A}\gamma^5)$. Our starting action is the Dirac action of a massless spin 1/2 fermion coupled to gravity and scalar [16,34–38]

$$S_{1/2} = \int d^5x\sqrt{-g}\left[\bar{\Psi} i\Gamma^M D_M \Psi - \eta\bar{\Psi}F(\phi)\Psi\right], \tag{47}$$

from which the equation of motion is given by

$$\left[i\Gamma^M D_M - \eta F(\phi)\right]\Psi = 0, \tag{48}$$

where $D_M = (\partial_M + \omega_M) = \partial_M + \frac{1}{4}\omega_M^{\bar{M}\bar{N}}\Gamma_{\bar{M}}\Gamma_{\bar{N}}$ is the covariant derivative, where the spin connection $\omega_M^{\bar{M}\bar{N}}$ is defined as

$$\omega_M^{\bar{M}\bar{N}} = \frac{1}{2}e^{N\bar{M}}\left(\partial_M e_N^{\bar{N}} - \partial_N e_M^{\bar{N}}\right) - \frac{1}{2}e^{N\bar{N}}\left(\partial_M e_N^{\bar{M}} - \partial_N e_M^{\bar{M}}\right) - \frac{1}{2}e^{P\bar{M}}e^{Q\bar{N}}(\partial_P e_{Q\bar{R}} - \partial_Q e_{P\bar{R}})e_M^{\bar{R}}. \tag{49}$$

The non-vanishing components of ω_M are

$$\omega_\mu = -\frac{i}{2}(\partial_z A)\gamma_\mu\gamma_5. \tag{50}$$

And the Dirac equation (48) becomes

$$\left[i\gamma^\mu\partial_\mu + \gamma^5(\partial_z + 2\partial_z A) - \eta e^A F(\phi)\right]\Psi = 0. \tag{51}$$

The full 5-dimensional spinor can be split in the general chiral decomposition

$$\Psi(x,z) = \sum_n \psi_{Ln}(x)\alpha_{Ln}(z) + \sum_n \psi_{Rn}(x)\alpha_{Rn}(z), \tag{52}$$

with $\gamma^5\psi_{Ln}(x) = -\psi_{Ln}(x)$ and $\gamma^5\psi_{Rn}(x) = \psi_{Rn}(x)$, where $\psi_{Ln}(x)$ and $\psi_{Rn}(x)$ are left-hand and right-hand components of a 4-dimensional Dirac field, respectively. Let us assume that $\psi_{Ln,Rn}$ satisfy 4-dimensional massive Dirac equations $i\gamma^\mu\partial_\mu\psi_{Ln}(x) = m_n\psi_{Rn}(x)$ and $i\gamma^\mu\partial_\mu\psi_{Rn}(x) = m_n\psi_{Ln}(x)$. We obtain the following coupled eigenvalue equations:

$$\left[\partial_z + 2\partial_z A + \eta e^A F(\phi)\right]\alpha_{Ln} = m_n\alpha_{Rn}, \tag{53}$$

$$\left[\partial_z + 2\partial_z A - \eta e^A F(\phi)\right]\alpha_{Rn} = -m_n\alpha_{Ln}. \tag{54}$$

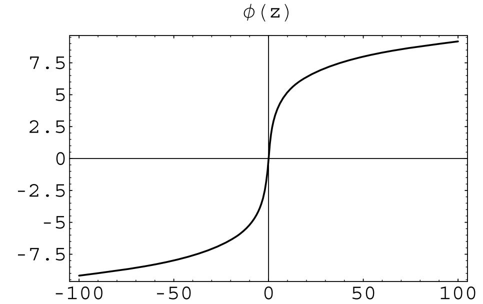

Fig. 3. Plot of the scalar field (58).

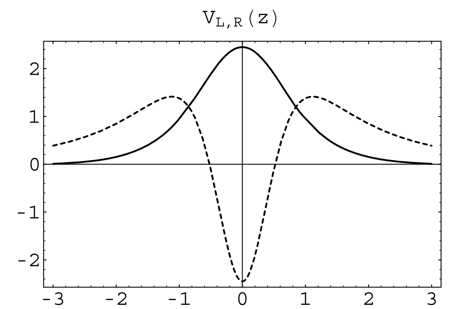

Fig. 4. The dashed line corresponds to the potential (59) for left chiral fermions whereas the solid one represents the potential (60) for right chiral fermions. The parameters are set to $a=2$ and $\eta=1$.

The full 5-dimensional action then reduces to the standard 4-dimensional action for the massive chiral fermions (with mass m_n), provided the above (53) and (54) are satisfied by the bulk fermions and the following orthonormality conditions are obeyed:

$$\int_{-\infty}^{+\infty} e^{4A}\alpha_{Ln}\alpha_{Rn}\,dz = \delta_{Lm}\delta_{Rn}. \tag{55}$$

By defining $\tilde{\alpha}_{Ln} = e^{2A}\alpha_{Ln}$, we get the Schrödinger-like equation for the left chiral fermions

$$\left[-\partial_z^2 + V_L(z)\right]\tilde{\alpha}_{Ln} = m_n^2\tilde{\alpha}_{Ln}, \tag{56}$$

where the potential $V_L(z)$ is given by

$$V_L(z) = e^{2A}\eta^2F^2(\phi) - e^A\eta\partial_z F(\phi) - \partial_z A e^A \eta F(\phi). \tag{57}$$

For the right chiral fermions, the corresponding potential can be written out easily by replacing $\eta \to -\eta$ from the above potential (57). It can be seen clearly that, for localization of the left (right) fermions, there must be some kind of Yukawa coupling since both the potentials for left and right chiral fermions are vanish in the case of no coupling ($\eta = 0$). A simple choice for $F(\phi)$ is $F(\phi) = \phi$. In the following, we will discuss for Model 1 and Model 2, respectively.

For Model 1, without loss generality we set $a = 2$, then (14) becomes

$$\phi(z) = \sqrt{3}\ln\left(\sqrt{1+z^2} + z\right). \tag{58}$$

The shape of the scalar field (58) is plotted in Fig. 3. And the potential (57) is reduced to

$$V_L(z) = \frac{6\eta^2[\ln(\sqrt{1+z^2}+z)]^2}{(1+z^2)^2} + \frac{2\eta z\ln(\sqrt{1+z^2}+z)\sqrt{6}}{(1+z^2)^2} - \frac{\sqrt{6}\eta}{(1+z^2)^{3/2}}. \tag{59}$$

For the right chiral fermions, we have

$$V_R(z) = V_L(z)\big|_{\eta\to-\eta} = \frac{6\eta^2[\ln(\sqrt{1+z^2}+z)]^2}{(1+z^2)^2} - \frac{2\eta z \ln\sqrt{1+z^2}+z)\sqrt{6}}{(1+z^2)^2} + \frac{\sqrt{6}\eta}{(1+z^2)^{3/2}}. \tag{60}$$

Noticing that both the potentials for the left and right fermions have the asymptotic behavior: $V_{L,R}(z=\pm\infty)=\infty$; the value of the potential (59) at $z=0$ is given by $V_L(0)=-\sqrt{6}\eta$ and the value of the potential $V_R(z)$ at $z=0$ is given by $V_R(0)=\sqrt{6}\eta$. They are shown in Fig. 4. As the potential $V_L(z)$ for left fermions has a negative value at the location of brane for only positive η, it could trap the left chiral fermion zero mode solved in (53) by setting $m_0=0$:

$$\tilde{\alpha}_{L0}(z) = e^{2A}\alpha_{L0}(z) \propto \exp\left[-\eta\int^z dz' e^{A(z')}\phi(z')\right] \quad (\eta>0). \tag{61}$$

In order to check the normalization condition (55) for the zero mode (61), we need to check whether the integral

$$\int_{-\infty}^{+\infty} dz \exp\left[-2\eta\int^z dz' e^{A(z')}\phi(z')\right] \tag{62}$$

is convergent. This integral can be written as

$$\int_{-\infty}^{+\infty} \exp\left(-2\eta\int^z dz'\, e^{A(z')}\phi(z')\right)dz = \int_{-\infty}^{+\infty} \exp\left[-2\eta\int^z dz'\,\frac{\sqrt{3}\ln(\sqrt{1+z'^2}+z')}{1+z'^2}\right]dz. \tag{63}$$

The shape of the function $\frac{\sqrt{3}\ln(\sqrt{1+z'^2}+z')}{1+z'^2}$ is shown in Fig. 5. As $\lim_{z'\to\pm\infty}\frac{\sqrt{3}\ln(\sqrt{1+z'^2}+z')}{1+z'^2}\to 0$, the integral (62) is divergent, that is to say, the left chiral fermions cannot be localized on the brane. For potential (60), by analogous analysis, it is found that the right fermions cannot be localized on the brane either.

Let us now turn to Model 2. For this model, $\phi(z)$ is expressed by (18), it's shape is plotted in Fig. 6. And the potential is reduced to

$$V_L(z) = \frac{\frac{3}{2}\eta^2\{\operatorname{arcsinh}[\tanh(\operatorname{arcsinh}\frac{az}{q})]\}^2}{q^2(\frac{a^2z^2}{q^2}+1)} - \frac{\sqrt{\frac{3}{2}}\eta\frac{a}{q}}{q(\frac{a^2z^2}{q^2}+1)\cosh(\operatorname{arcsinh}\frac{az}{q})} + \frac{\sqrt{\frac{3}{2}}\eta\frac{a^2}{q^2}z\{\arcsin[\tanh(\operatorname{arcsinh}\frac{az}{q})]\}}{q^2(\frac{a^2z^2}{q^2}+1)^{\frac{3}{2}}}. \tag{64}$$

For the right chiral fermions, the corresponding potential is given by $V_R(z)=V_L(z)|_{\eta\to-\eta}$.

For the case $\eta>0$, only the potential for the left chiral fermions have negative value at the location of the brane, which could trap the left chiral fermionic zero mode (61). It is the same as Model 1, we need to check whether (62) is convergent in order to check the normalization condition (55). It is convenient to analyze the problem in y coordinate for this case. In y coordinate (62) becomes

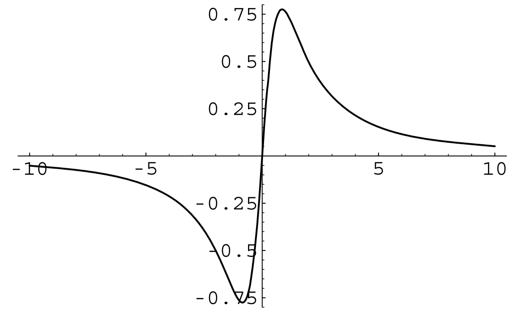

Fig. 5. Plot of the function $\frac{\sqrt{3}\ln(\sqrt{1+z'^2}+z')}{1+z'^2}$.

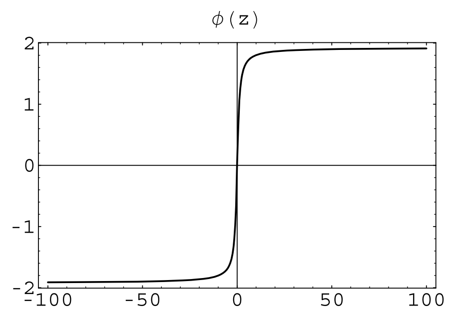

Fig. 6. Plot of the scalar field (18). The parameters are set to $a=1$ and $q=1$.

$$\int_{-\infty}^{+\infty} \exp\left[-A(y) - 2\eta\int^y dy'\phi(y')\right]dy. \tag{65}$$

Substituting (15), (16) and (18) into (62), and noting $b^2=\frac{2}{3}$, (65) is changed into

$$q\int_{-\infty}^{+\infty} \exp\left\{\ln\left[\cosh(ay)\right] - 2\eta\int^y dy' \arcsin\left[\tanh(ay')\right]\right\}dy. \tag{66}$$

For simplicity, we set $a=1$. The functions $\ln[\cosh(y)]$ and $\arcsin[\tanh(y)]$ are plotted in Fig. 7 and Fig. 8, respectively. From these figures we see that, when $y\to\infty$, $\ln[\cosh(y)]\sim y$, and $\arcsin[\tanh(y)]\sim\frac{\pi}{2}$, and so, $\ln[\cosh(y)]-2\eta\int^y \arcsin[\tanh(y')]\,dy' \sim -(-1+\eta\pi)y$. Then the inequality $-1+\eta\pi>0$, namely, $1<\eta\pi$ should be satisfied if the integral (66) is finite. When $y\to-\infty$, we can get the same restriction condition for the integral (66) being finite. Therefore, the zero mode of the left chiral fermions can be localized on the brane under certain condition. In Fig. 9 we plot the left chiral fermion potential (64) and the corresponding zero mode, which is gotten by solved (56) numerically under boundary conditions $\tilde{\alpha}_{L0}(0)=1$ and $\tilde{\alpha}'_{L0}(0)=0$. The first condition is arbitrarily fixed, and is adjusted with the normalization condition (55). Since for positive η the potential for right chiral fermions is always positive, it cannot trap the right chiral zero mode.

For the case $\eta<0$, things are the opposite and only the right chiral zero mode can be trapped on the brane.

4. Summary and outlook

In this Letter, we investigate the possibility of localizing various matter field on flat thick branes. Two analytical thick brane models which localize the graviton are considered. We have the same result for spin 0 field as in the case of gravity [17], i.e., the

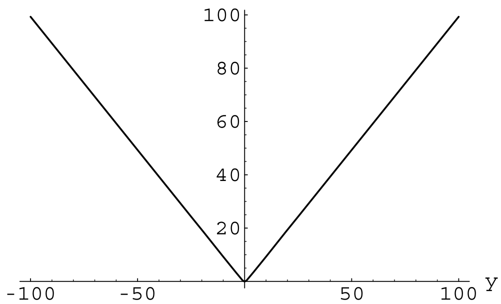

Fig. 7. Plot of the function $\ln[\cosh(y)]$.

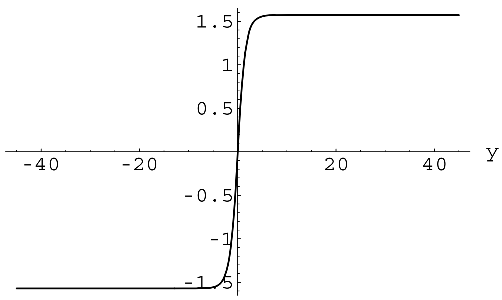

Fig. 8. Plot of the function $\arcsin[\tanh(y)]$.

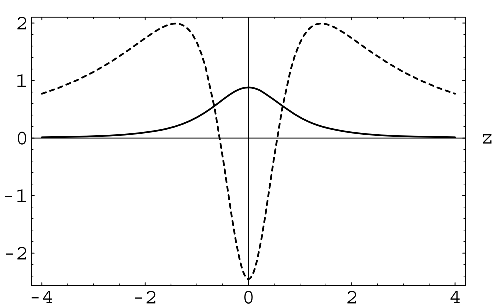

Fig. 9. The dashed line and solid one for the potential (64) and the corresponding zero mode, respectively. The parameters are set to $\eta = 2$, $a = 1$ and $q = 1$.

massless zero mode of spin 0 scalar field is found to be normalizable on both thick brane models. However, the zero mode for spin 1 vector field is non-normalized, in other words, the vector field cannot be localized on the thick brane models considered in this Letter. For spin 1/2 fermionic field, it is shown that, for the case of no Yukawa-type coupling, there is no existence of localized zero mode for both left and right chiral fermions. After including a Yukawa coupling to a scalar field, it is found that, fermions have different localization properties on the two thick brane models, whether there is a localized zero mode is decided by the considered model.

Finally, we want to mention that it has been pointed out that the matter fields with spin 1 and 1/2 can be confined on an AdS_4 brane in AdS_5 space–time [39]. We will also extend our analysis to curved thick brane. The crucial ingredient is the introduction of $W = W(\phi)$ and $Z = Z(\phi)$. In certain particular case, A and ϕ can be expressed by the extra coordinate z and the cosmological constant of the 4-dimensional space–time Λ. (See Refs. [18,40] for details.) Thus we can discuss localization of matter fields on the curved thick brane by the procedure used in this Letter. The work in this direction is now in progress, we wish to report it in the near future.

Acknowledgements

Jun Liang wish to thank Prof. Yu-Xiao Liu for helpful discussions. Thanks also go to the referee for comments and suggestions. This work was Supported by the Postdoctoral Education Fund of Lanzhou University, China Postdoctoral Science Foundation and the National Natural Science Foundation of China.

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