Erratum


Sándor Jenei

Institute for Discrete Mathematics and Geometry, Technical University of Vienna, Wiedner Hauptstrasse 8–10, A-1040 Vienna, Austria
Institute of Mathematics and Informatics, University of Pécs, Ifjúság u. 6, H-7624 Pécs, Hungary

A R T I C L E   I N F O

Article history:
Received 31 May 2010
Accepted 31 May 2010
Available online 4 July 2010
Communicated by U. Kohlenbach

In Section 2 replace the definition of $\bullet_Q$ in Definition 1 by $x \bullet_Q y = \inf\{u \bullet v \mid u > x, v > y\}$. It is defined only if the infimum exists. Proposition 1 remains unchanged.

**Theorem 0.** Let $(X, \bullet, \rightarrow, \leq)$ be a commutative residuated semigroup on a complete chain equipped with the order topology. Let $a, b, c \in X$ be such that $a = b \rightarrow c$. Let $(x, y) \in X \times X$ be such that

1. neither $x$ nor $y$ equals the top element of the chain (if any) and we have $x \bullet y = x \bullet_Q y$,
2. $x$ is $a$-closed, and $y$ is $b$-closed,
3. either we have

$$
\sup\{t \rightarrow_c | t > x \bullet y\} = x \bullet y \rightarrow_c \tag{1}
$$

or we have

$$
\sup\{t \rightarrow_a | t > (x \rightarrow_a a) \bullet (y \rightarrow_b b)\} = (x \rightarrow_a a) \bullet (y \rightarrow_b b) \rightarrow c. \tag{0}
$$

Then we have

$$(x \rightarrow_a a) \bullet (y \rightarrow_b b) = x \bullet y \rightarrow_c. \tag{1}
$$

**Proof.** First assume (1) holds. Note that $\bullet_Q$ is an operation on $X$ since the chain is complete. We have

$$
[x \bullet y] \bullet [(x \rightarrow_a a) \bullet (y \rightarrow_b b)] = [y \bullet (y \rightarrow_b b)] \bullet [x \bullet (x \rightarrow_a a)] \leq b \bullet a = b \bullet (b \rightarrow c) \leq c,
$$

and hence $(x \rightarrow_a a) \bullet (y \rightarrow_b b) \leq x \bullet y \rightarrow_c$. In order to prove $(x \rightarrow_a a) \bullet (y \rightarrow_b b) \geq x \bullet y \rightarrow_c$ it suffices to show that

$$(x \rightarrow_a a) \bullet (y \rightarrow_b b) > x_1 \bullet y_1 \rightarrow c \quad \text{for } x_1 > x, y_1 > y.
$$

Indeed, if it holds, then we have $(x \rightarrow_a a) \bullet (y \rightarrow_b b) > \sup\{x_1 \bullet y_1 \rightarrow c | x_1 > x, y_1 > y\}$. We have $\inf\{x_1 \bullet y_1 | x_1 > x, y_1 > y\} = x_1 \bullet y = x \bullet y$ by definition of $\bullet_Q$ and condition 1 and hence, by (1) we have $\sup\{x_1 \bullet y_1 \rightarrow c | x_1 > x, y_1 > y\} = x \bullet y \rightarrow c$, as stated.

To this end, it suffices to verify $[x_1 \bullet y_1] \bullet [(x \rightarrow_a a) \bullet (y \rightarrow_b b)] > c$ for $x_1 > x, y_1 > y$ since $X$ is a chain. Since $x$ is $a$-closed we have $x \rightarrow_a a > x_1 \rightarrow_a a$ by Proposition 1/2. Analogously we obtain $y \rightarrow_b b > y_1 \rightarrow_b b$. Therefore $x_1 \bullet (x \rightarrow_a a) > a$
and \( y_1 \ast (y \rightarrow \ast b) > b \) since \( X \) is a chain. Hence, referring to \( a = b \rightarrow \ast c \), we obtain \([x_1 \ast y_1] \ast [(x \rightarrow \ast a) \ast (y \rightarrow \ast b)] \geq b \ast [x_1 \ast (x \rightarrow \ast a)] > c\), as stated.

Next assume (0) holds. Observe that \( x \) and \( y \) are interchangeable with \( x \rightarrow \ast a \) and \( y \rightarrow \ast b \), respectively, in (1) since \( x \) is \( a \)-closed, \( y \) is \( b \)-closed. That is, we may substitute \( x \rightarrow \ast a \) in place of \( x \) (and hence \( (x \rightarrow \ast a) \rightarrow \ast a = a \) in place of \( x \rightarrow \ast a \)), and \( y \rightarrow \ast b \) in place of \( y \) in (1) and apply the already proven case. \( \Box \)

Replace Theorem 1 and its proof by the following (Fig. 1 remains unchanged):

**Theorem 1.** Let \( (X, \ast, \rightarrow \ast, \leq) \) be a commutative residuated semigroup on a complete, dense chain equipped with the order topology. Let \( a, b, c \in X \) be such that \( a = b \rightarrow \ast c \). Let \((x, y) \in X \times X \) be

1. a continuity point of \( \ast \) such that neither \( x \) nor \( y \) equals the top element of the chain (if any);
2. such that \( x \) is \( a \)-closed, and \( y \) is \( b \)-closed;
3. such that \( t \rightarrow \ast c \) is right-continuous either at \( t = x \ast y \) or at \( t = (x \rightarrow \ast a) \ast (y \rightarrow \ast b) \).

Then (1) holds.

**Proof.** For a chain which is dense in the order topology, conditions 1 and 3 follow from conditions 1 and 3 of Theorem 0, respectively. \( \Box \)

Corollary 1 and Theorem 2 remain the same. In the proof of Theorem 2, in the displayed equations (right under \( \limsup \) and \( \liminf \)), replace \( p \leq u, q \leq v, (p, q) \neq (u, v) \) (resp. \( x \geq u \rightarrow \ast a, y \geq v \rightarrow \ast b, (x, y) \neq (u \rightarrow \ast a, v \rightarrow \ast b) \)) by \( p < u, q < u \) (resp. \( x > u \rightarrow \ast a, y > v \rightarrow \ast b \)).

The text after the proof of Theorem 2 remains unchanged until Corollary 2, which has to be replaced by:

**Corollary 2.** Let \( \ast \) be a left-continuous \( t \)-norm, with \( a, b, c \in [0, 1] \) such that \( a = b \rightarrow \ast c \). Suppose \([p, q] \subset [a, 1] \) and \([r, s] \subset [b, 1] \). Assume that every element in \([p, q] \) is \( a \)-closed, every element in \([r, s] \) is \( b \)-closed, and that the \( c \)-negation function is continuous either on \([p \ast r, q \ast s] \) or on \([(q \rightarrow \ast a) \ast (s \rightarrow \ast b), (p \rightarrow \ast a) \ast (r \rightarrow \ast b)] \).

Then the values of \( \ast \) on \([p, q] \times [r, s] \) uniquely determine, via (2), the values of \( \ast \) on \([q \rightarrow \ast a, p \rightarrow \ast a] \times [s \rightarrow \ast b, r \rightarrow \ast b] \) and vice versa. In addition, the values of \( \ast \) on any subset of \([p, q] \times [r, s] \) determine the values of \( \ast \) on the corresponding subset of \([q \rightarrow \ast a, p \rightarrow \ast a] \times [s \rightarrow \ast b, r \rightarrow \ast b] \) via (2). (See the two subfigures on the left in Fig. 2.) \( \Box \)

The rest of Section 2 remains unchanged.