Turbo Codes with Modified Code-matched Interleaver over AWGN and Rayleigh Fading Channel

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Abstract: A novel Code-Matched interleaver is proposed which decreases the number of the low weight codewords to improve the performance of the Turbo code. The modified design can adapt more kinds of Turbo codes determined by the generator matrix, while it doesn't decrease the bit error rate performance of Turbo codes at moderate to high signal to noise ratio. At the same time, in Rayleigh fading channel, the new Code-Matched interleaver can also debase the error floor.

Key words: Turbo; interleaver; weight distribution; Code-Matched; distance spectrum

AWGN 和衰落信道下具有改进型 Code matched 交织器的 Turbo 码. 罗骥, 张曦林, 袁东风. 中国航空学报(英文版), 2005, 18(2): 147-152.

摘 要: 提出了一种改进型的 Code-Matched 交织器, 它能减少低重量码的数量, 从而提高 Turbo 码的性能。这种改进型的 Code-Matched 交织器可以适用于多种不同生成矩阵产生的 Turbo 码, 而且不会影响 Turbo 码在中高信噪比处的性能。同时, 在 Rayleigh 信道下, 这种交织器能降低错误平台。

关键词: Turbo; 交织器; 重量分布; Code-Matched; 距离谱

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Turbo codes was firstly proposed in 1993 by C. Berrou[1], which achieves almost reliable data communication at signal-to-noise ratios near to Shannon-limits. The interleaver design is a key role in determining the bit error rate (BER) performance of Turbo codes. Turbo codes have two parallel component encoders separated by an interleaver which permutes the input information sequence randomly so that the output of the encoder has the character of long and random codewords and this is a basic operation inherent in the interleaver. The aim of the permutation process under some special rule is to make the output sequence have some peculiarity, and correspondently, the Turbo codes BER performance is improved by the interleaver.

Some works on the interleaver whose design is based on the weigh distribution of Turbo codes have been done in Refs. [2, 3]. A Code-Matched interleaver design criteria derived from the analysis of distance spectrum of a particular Turbo codes were formulated in Ref. [3]. However, the design criteria focus on the special Turbo codes. In this paper some innovations on the interleaver have been made in order to change the codewords weight distribution and decrease the low weight codewords number effectively. Therefore, the Turbo codes BER performance from moderate to high signal-to-noise ratio (SNR) is improved and the error floor is debased.

1 Turbo Codes Distance Spectrum Analysis

Turbo codes can be represented by an equivalent block code if the component encoders are forced to the all-zero state at the end of the each block. So the weight distribution of equivalent block code is considered and applied in the calculation of the Turbo code bit error probability bounds over AWGN channel[4].

Given an $[N \quad K]$ linear block code, its

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weight distribution can be expressed by the code weight enumerating function (WEF), the WEF of a code[5] is

$$A(X) = \sum_{i=1}^{N} A_i X^i \quad (1)$$

where A_i is the number of codewords of Hamming weight i and X is a dummy variable. The set

$$\{A_{d_{\min}}, A_{d_{\min}+1}, \ldots A_i, \ldots A_n\} \quad (2)$$

is called the weight distribution or the weight spectrum of the code.

For systematic block codes, the codeword weights can be sperated into input information weight and parity check information weight, and the input redundancy enumerating function (IRWEF) of a code is defined as

$$A(W, Z) = \sum_{\omega=0}^{K} \sum_{z=0}^{N-K} A_{\omega, z} W^{\omega} Z^{z} \quad (3)$$

where $A_{\omega, z}$ is the number of codewords of the block code with input information weight ω and parity check information weight z. The overall Hamming codeword weight of a codeword is $d = \omega + z$. Obviously, it can be seen that

$$A_i = \sum_{\omega+z=j} A_{\omega, z} = \sum_{\omega=0}^{i} A_{\omega, i-\omega} \quad (4)$$

Furthermore, the IRWEF can be decomposed according to the contributions of distinct input information weights ω, as

$$A(W, Z) = \sum_{\omega=0}^{K} A_{\omega}(Z) W^{\omega} \quad (5)$$

where $A_{\omega}(Z)$ is called the conditional weight enumerating function. The conditional WEF describes the parity check weights of the codewords generated by input information of weight ω. It is given by[6]

$$A_{\omega}(Z) = \sum_{z=0}^{N-K} A_{\omega, z} Z^{z} \quad (6)$$

Then it can be seen that the relationship between the IRWEF and the conditional WEF is as below

$$A_{\omega}(Z) = \frac{1}{\omega!} \cdot \left. \frac{\partial^{\omega} A(W, Z)}{\partial W^{\omega}} \right|_{W=0} \quad (7)$$

The bit error probability over an AWGN channel can be upper-bounded by a union bound as

$$P_r(e) \leqslant \sum_{d=d_{\min}} B_d Q \sqrt{2 d_0 R E_b / N_0} \quad (8)$$

where B_d is the error coefficient and can be obtained from the code IRWEF

$$B_d = \sum_{d=\omega+z} \frac{\omega}{K} A_{\omega, z} \quad (9)$$

B_d determines the contribution of the codewords with the same weight d to the bit error probability. The set of all pairs of (d, B_d) denoted by

$$\{(d_{\min}, B_{d_{\min}}), (d_{\min}+1, B_{d_{\min}+1}), \ldots, (d_i, B_{d_i}), \ldots\}$$

is called the code distance spectrum.

The computer simulation proves that for a Turbo code with a random interleaver, the impact of error coefficient B_d on the code error performance is significant at low SNR's and the code effective free distance d_f mainly determines the bit error probability at high SNR's.

Here $d_f = 2 + 2Z_{\min}$, and $Z_{\min}$ denotes the lowest weight of the parity check sequence generated by an information sequence with weight-2.

Since the code effective free distance determines the performance at high SNR's, the bit error probability can be expressed by the dominant term

$$P_r(e) \approx B_f Q \sqrt{2 d_f R E_b / N_0} \quad (10)$$

where B_f is the error coefficient related to the code effective free distance d_f. So it can be seen that optimizing the bit error probability implies the generating of the codewords with distance equal to or below d_f to be prevented.

2 Code-Matched Interleaver Application Condition Analysis

In fact, a Code-Matched interleaver is first a S-random interleaver. The S-random interleaver can spread low weight input patterns to generate higher weight codewords. According to Ref. [7] weight-3 input patterns are not considered because this pattern can be broken by the interleaver's S-constraint. Therefore, breaking weight-2, and weight-4 input patterns are especially considered.

2.1 Input patterns with weight $\omega = 2$

Table 1 lists the parameters which are needed when designing the Code-Matched interleaver combined with different generator matrixes, The sym-

Table 1 The parameters for several generator matrixes

m	$g_0(D)$	$g_1(D)$	μ	Z_{min}
2	7	5	3	4
3	15	17	7	6
4	37	21	5	4
4	31	33	15	10
4	31	27	15	10

bol "m" is the memory order of the two component encodes. In this paper, the two component encodes of Turbo codes have the same generator matrix $\boldsymbol{G} = [1 \quad g_1(D)/g_0(D)]$, where $g_0(D)$ is the feedback polynomial, $g_1(D)$ is the forward polynomial and "μ" is the distance between two "1" for an input pattern with weight $\omega = 2$, and the minimum weight of the parity check sequence generated by an weight-2 input pattern is denoted by Z_{min}. In order to explain the significance of Table 1 for the design of Code-Matched interleaver, the generator matrix [1 21/37] is taken as example.

The weight-2 input pattern c_2 generates the lowest weight of the parity check sequence $\tilde{c}_2$

$$c_2 = (0, 0, \ldots 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, \ldots, 0, 0) \qquad (11)$$

where the distance between the two "1" is $\mu = 5$. The parity check

$$\tilde{c}_2 = (0, 0, \ldots 0, 0, 1, 1, 0, 0, 1, 1, 0, 0, \ldots, 0, 0) \qquad (12)$$

and it can be seen here that $Z_{min} = 4$.

If the interleaver maps the input sequence to a sequence with the same weight, the resulting overall codeword weight is

$$d = 2 + 2Z_{min} \qquad (13)$$

A weight-2 input sequence which can generate finite weight parity check sequence can be represented by

$$C_2(D) = (1 + D^{\mu k_1}) D^{\tau_1} \qquad (14)$$

where $k_1 = 1, 2, 3\ldots$, and τ_1 is the time delay. The parity check weight of Eq.(14) is given by

$$d = k_1(Z_{min} - 2) + 2 \qquad (15)$$

If the interleaver maps Eq.(14) to $\tilde{C}_2(D) = (1 + D^{\mu k_2}) D^{\tau_2}$, the overall weight of the generated codeword is given by

$$d = \omega + \omega(y_1) + \omega(y_2) \qquad (16)$$

where ω is the input weight, and $\omega(y_1)$ and $\omega(y_2)$ denote the first and second encoder output weights respectively. Then the overall weight of the codeword generated by the weight-2 input sequence is

$$d = 6 + (k_1 + k_2)(Z_{min} - 2) \qquad (17)$$

Let d^{ω}_{max} denote the maximum weight of the codewords generated by the weight ω input patterns. In Ref.[3], the Code-Matched interleaver design only focuses on breaking the input patterns that generate codewords with no larger than $d^2_{max} = 20$, then

$$6 + (k_1 + k_2)(Z_{min} - 2) \leqslant d^2_{max} \qquad (18)$$

which is equivalent to

$$k_1 + k_2 \leqslant \frac{d^2_{max} - 6}{Z_{min} - 2} \qquad (19)$$

and then, $k_1 + k_2 \leqslant 7$.

It can be found that for generator matrix [1 5/7], [1 17/15] and [1 21/37], Z_{min} is 4, 6, 4 respectively. The input weight-2 sequence and the two encoders outputs can have several combinations to satisfy function (18). For example, for [1 21/37],

(2, 4, 14) $k_1 = 1$, $k_2 = 6$;

(2, 6, 12) $k_1 = 2$, $k_2 = 5$;

(2, 8, 10) $k_1 = 3$, $k_2 = 4$ when $k_1 + k_2 = 7$

But for [1 33/31] and [1 27/31], $2 + 2Z_{min} = 22$, even the combination (2, 10, 10), in which $k_1 = 1$, $k_2 = 1$, is beyond $d^2_{max} = 20$ given in Ref.[3]. So for generator matrixes [1 33/31] and [1 27/31], the Code-Matched interleaver design conditions need to be changed. For [1 33/31] and [1 27/31], μ is large, so generally the S-constraint of the interleaver can spread the input sequence to generate high weight codewords. For the reason above, the Code-Matched interleaver proposed in Ref.[3] is not always the best selection as compared with other kinds of interleavers when synthesizing the complexity and gain.

2.2 Input patterns with weight $\omega = 4$

The weight-4 input sequences can be considered as the compound of two single weight-2 sequences

$$c_4(D) = (1 + D^{\mu k_1'}) D^{\tau_1} + (1 + D^{\mu k_2'}) D^{\tau_2}$$

and the input to the second encoder,

$$\tilde{c}_4(D) = (1 + D^{\mu k_3'})D^{\tau_3} + (1 + D^{\mu k_4'})D^{\tau_4}$$

where $\tau_2 > \tau_1 + \mu k_1'$, $\tau_4 > \tau_3 + \mu k_3'$ and the overall weight of the generated codeword can be calculated from Eq. (16),

$$d = 4 + y_1(4) + y_2(4) = 12 + (k_1' + k_2' + k_3' + k_4')(Z_{\min} - 2) \quad (20)$$

From all above analyses, the mapping set Φ including the conditions can be concluded as follows:

(1) $|\pi(i_1) - \pi(i_2)| \bmod \mu \neq 0$ whenever $|i_1 - i_2| \bmod \mu = 0$ and $k_1 + k_2 \leqslant (d^2_{\max} - 6)/(Z_{\min} - 2)$;

(2) $|\pi(i_1) - \pi(i_3)| \bmod \mu \neq 0$ and $|\pi(i_2) - \pi(i_4)| \bmod \mu \neq 0$ whenever $|i_1 - i_2| \bmod \mu = 0$ and $|i_3 - i_4| \bmod \mu = 0$, $k_1' + k_2' + k_3' + k_4' \leqslant (d^4_{\min} - 12)/(Z_{\min} - 2)$; or $|\pi(i_1) - \pi(i_4)| \bmod \mu \neq 0$ and $|\pi(i_2) - \pi(i_3)| \bmod \mu \neq 0$ whenever $|i_1 - i_2| \bmod \mu = 0$ and $|i_3 - i_4| \bmod \mu = 0$ and $k_1' + k_2' + k_3' + k_4' \leqslant (d^4_{\min} - 12)/(Z_{\min} - 2)$;

(3) $S < \sqrt{\frac{N}{2}}$, where N is the interleaversize.

Let i_1, i_2, i_3, i_4 denote the positions of 1s in the weithg-4 input sequence, while $d^2_{\max} = d^4_{\max} = 20$ for [1 21/37], [1 17/15] and [1 5/7]; $d^2_{\max} = d^4_{\max} = 54$ for [1 33/31] and [1 27/31].

3 The Modified Code-Matched Interleaver Design

In this section a modified algorithm for the Code-Matched interleaver is presented.

Step 1: Generating a random sequence with length N, N = interleaver size. What is accompanied with this random sequence is set $A = \{1, 2, \ldots, N\}$;

Step 2: Sorting this random sequence. According to the new positions of the members of the sequences, the positions of the members of set A are relocated. For example, a random sequence {0.7320, 0.3120, 0.5216, 0.7153}, A = {1, 2, 3, 4}, after sorting the random sequence, there will be {0.3120, 0.5216, 0.7153, 0.7320}, and then A = {2, 3, 4, 1}. From Step 1 and Step 2, a random interleaver is obtained. A is the reading order of the input sequence.

Step 3: $i = 2$, checking if a_i in the A satisfies the mapping function s. If yes, $i = i + 1$; and if not, $s = a_i$ from i to N, $a_i = a_{i+1}$, $a_N = s$.

Step 4: Repeat Step 3. If i equals to a certain value and from this value to N, a_i can not satisfy s, then move the segment from the value to N to the head of the whole sequences of A.

Step 5: Go to Step 3. After $T(< S)$ iteration from Step 3 to Step 5, for S-random interleaver, the N interleaver outputs can always be obtained if $S = \sqrt{\frac{N}{2}}$; but for the Code-Matched interleaver, it may be impossible to have all N outputs, then reduce S by 1 and go to Step 3.

Compared with the method proposed by Ref. [3], this method's complexity is decreased greatly.

4 Simulation and Analysis

In this section some simulation results are provided in Figs. 1-5. The modified Code-Matched interleaver performance is compared with random and S-random interleavers and the input data are all random sequences.

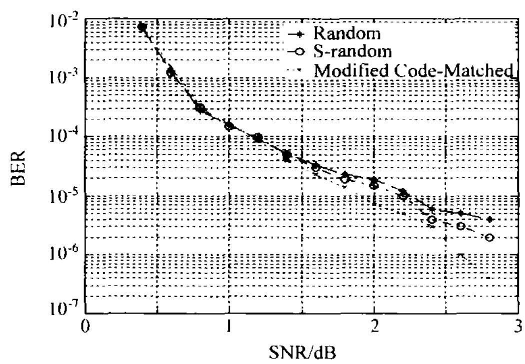

Fig. 1 BER performances of random, S-random and modified Code-Matched interleavers with interleaver size of 1024, 16 state Generator matrix= [1 21/37]

For S-random interleaver, S is chosen to be 22 and 42 respectively for interleaver size N = 1024 and N = 4096. For the modified Code-Matched interleavers, S is 17 and 35 respectively for N = 1024 and N = 4096. The number of iterations in the decoder is selected to be 8 for N = 1024 and 18

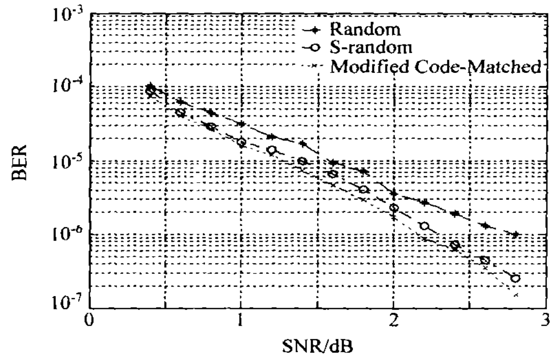

Fig. 2 BER performance of random, S-random and modified Code-Matched interleavers with interleaver size of 4096. 16 state Generator matrix [1 21/37]

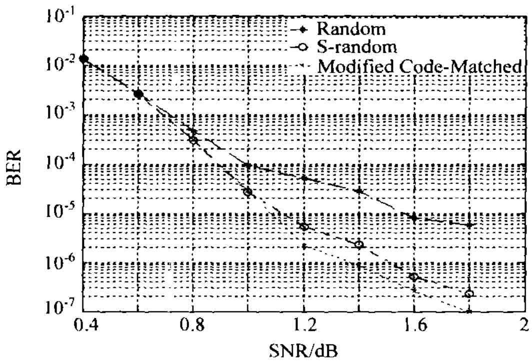

Fig. 3 BER performances of random, S-random and modified Code-Matched interleavers with interleaver size of 1024. 16 state Generator matrix G= [1 33/31]

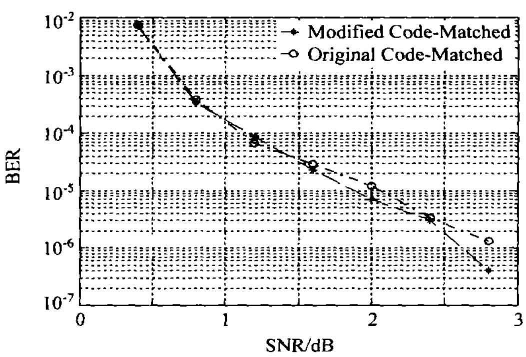

Fig. 4 BER performances of the modified Code-Matched interleaver and the original Code-Matched interleaver proposed in Ref. [3] with interleaver size of 1024. Generator matrix G= [1 21/37]

for $N = 4096$. The maximum a posteriori (MAP)[8] algorithm is used in the decoder, assuming an additive white Guassian noise (AWGN) channel.

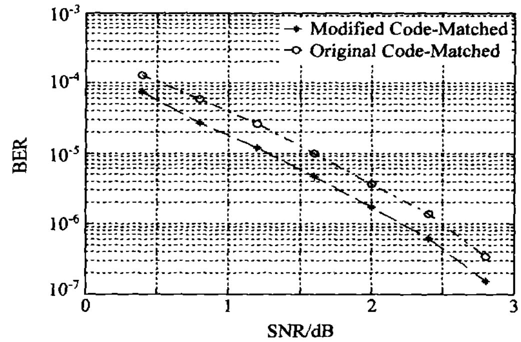

Fig. 5 BER performances of the modified Code-Matched interleaver and the original Code-Matched interleaver proposed in Ref. [3] with interleaver size of 4096. Generator matrix G= [1 21/37]

The curves in Fig. 6 and Fig. 7 are the BER performances of four main decoding algorithms with different interleavers over Rayleigh fading channel without channel state information (NSI).

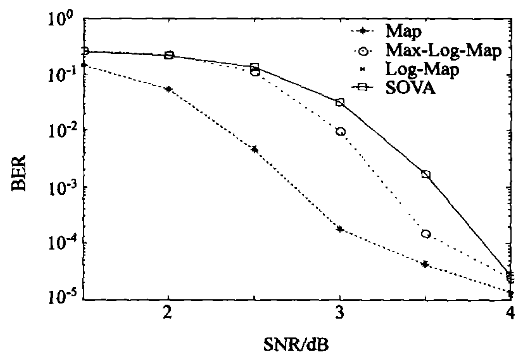

Fig. 6 BER performances of different decoding algorithm interleaver size 32 × 32, modified code-matched interleaver, 6 iterations, Rayleigh fading channel(NSI)

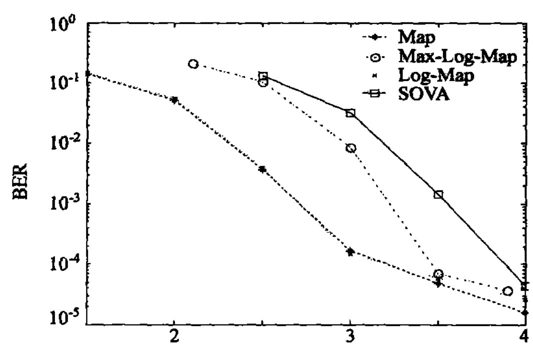

Fig. 7 BER performances of different decoding algorithm interleaver size 32 × 32, random interleaver, 6 iterations, Rayleigh fading channel (NSI)

Fig. 6 is the proposed Code-Matched interleaver with 6 iterations, and Fig. 7 is the random interleaver with 6 iterations. Comparing between Fig. 6 and Fig. 7, it can be seen that when the proposed interleaver is applied in the encode scheme the Turbo codes error floor is decreased. This can be seen clearly in Table 2.

Table2 The error floors, MAP algorithm 18 iterations

MAP/dB	Random	Modified
3.0	1.67×10^{-4}	1.59×10^{-4}
3.5	4.8×10^{-5}	4.2×10^{-5}
4.0	1.6×10^{-5}	1.3×10^{-5}

5 Conclusions

In this paper a modified Code-Matched design is presented. Through analysis and computer simulation some significant conclusions are given:

(1) The modified design method is presented which can effectively reduce the low weight codewords. As a result, the bit error performance of Turbo codes is improved as compared with the original Code-Matched interleaver and *S*-random interleaver.

(2) In Rayleigh fading channel the decoding algorithm combining with the proposed interleaver in this paper shows lower error floor. So it can be said that this kind of interleaver improves the Turbo codes BER performance not only in AWGN channel but also in Rayleigh fading channel. And this kind of interleaver has the same effect for all the four decoding algorithm that the error floor is decreased. So in application whichever decoding algorithm is adopted, it is important to improve the performance by reducing the lower weight codewords.

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