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INTRODUCTION

The Eleventh Conference on Stochastic Processes and Their Applications was held at the Université de Clermont-Ferrand II in Clermont-Ferrand, France, over the period 28 June–2 July 1982. The Conference was organized under the auspices of the Committee for Conferences on Stochastic Processes of the ISI's Bernoulli Society for Mathematical Statistics and Probability.

The Conference was attended by 192 scientists coming from France (77), U.S.A. (23), West Germany (20), Sweden (11), The Netherlands (11), Canada (10), Great Britain (9) and several other countries.

The scientific program consisted of 15 invited papers and 65 contributed papers. The following are the abstracts of the papers presented. For papers whose abstracts were not received, or did not conform to the suggested format, only the titles and the authors are listed, and they are marked with an asterisk.

1. INVITED PAPERS

Some Simple Observations about the Malliavin Calculus

K. Bichteler, *University of Texas at Austin, TX*

The solution to a stochastic differential equation depends smoothly on parameters, in the sense of Sp. This together with Girsanov's theorem yields smoothness of the transition function of the solution via simple computation rather easily.

Stochastic Control with Noisy Observations

M.H.A. Davis, *Imperial College, London, United Kingdom*

This paper concerns control of continuous-time stochastic systems where only noise-corrupted measurements of the state variables are available to the controller. This situation involves *filtering* in an essential way since the true 'state' of such a system is the *conditional distribution* of its original state variables given the observations. Some recent approaches to such problems will be outlined. In general the problems are very complicated, but there are special cases, important in operations research, where the 'unobserved part' of the system is unaffected by control action, and this leads to substantial simplifications. Some examples will be given.

Limit Theorems for Measure-Valued Processes

D.A. Dawson, *Carleton University, Ottawa, Canada*

We first describe a measure-valued stochastic diffusion process which arises from a stochastic model of a spatially distributed system of particles. We then consider the transformation of this process under a change of time and spatial scales. Finally, we determine the asymptotic behavior of the system under such rescalings and present the resulting limit theorems.

Markov Fields and 2-Parameter Martingales

H. Föllmer, *Mathematik*, ETH-*Zentrum, Zurich, Switzerland*

The title refers to two very active developments in the theory of 'spatially' indexed stochastic processes. So far there has not been much interaction between these two areas: one reason is that most results on two-parameter martingales rely on assumptions of conditional independence which are typically not satisfied by Markov random fields. In order to illustrate the possibility of going beyond this restriction, we derive an almost sure convergence theorem for two-parameter martingales with respect to the σ-fields generated by a Markov random field. The assumptions are related to Dobrushin's uniqueness condition, and the proof uses a suitable variant of Dobrushin's contraction technique. This variant leads also to new covariance estimates which improve recent results of L. Gross on the exponential decay of correlations.

Martingale Characterization of Probability Measures and Weak Convergence

B. Grigelionis, *Institute of Mathematics and Cybernetics. Vilnius, U.S.S.R.*

The efficiency of weak convergence conditions of the sequences of probability measures on metric spaces depends strongly on used criteria for the relative compactness and for the characterization of the limiting measure. The aim of this lecture is a review of some general results on weak convergence of probability measures based on the martingale characterization of the limiting points.

We shall consider a Polish space $(\mathscr{X}, \mathscr{B}(\mathscr{X}))$ with the definite filtration $\mathbb{B} = \{\mathscr{B}_t, t \geq 0\}$ of σ-subalgebras and a sequence of probability measures $P_{X_n} = P_n \circ X_n^{-1}$, $n \geq 1$, where X_n are random elements defined on the probability spaces $(\Omega_n, \mathscr{F}_n, P_n)$ with the filtrations $\mathbb{F}_n = \{\mathscr{F}_{nt}, t \geq 0\}$ and taking values in $\mathscr{X}$, $n \geq 1$. The limiting measure μ on $\mathscr{B}(\mathscr{X})$ is characterized by the fixed values on $\mathscr{B}_0$ and the assumption that some family of functionals $\{M_t^i(x), t \geq 0, x \in \mathscr{X}, i \in I\}$ are $(\mathbb{B}, \mu)$-local martingales. The conditions of weak convergence are formulated in the terms of this family and the corresponding families of $(P_n, \mathbb{F}_n)$-local martingales $\{M_t^{(n),i}, t \geq 0, i \in I\}$, $n \geq 1$.

As an illustration the weak convergence conditions of semimartingales and point processes will be considered in the terms of their predictable characteristics.

Random Walk Applications from Statistical Physics

C.C. Heyde, *CSIRO-Division of Mathematics and Statistics, Canberra, Australia*

This talk will provide a survey of new developments on various random walk problems in statistical physics particulary those related to transport phenomena emphasis will be given to asymptotic results the use of modern probabilistic techniques and to important unsolved problems.

Statistical Applications of the Theory of Martingales on Point Processes

Niels Keiding, Statistical Research Unit, Danish Medical and Social Science Research Councils

The multiplicative intensity model for multivariate counting processes was formulated by O.O. Aalen in 1978. It was here assumed that the compensator is absolutely continuous and that the derivative factorizes into a product of an observable process and an unknown deterministic intensity to be estimated. The theory of martingales and stochastic integrals was used to study the properties of a natural nonparametric estimator of the integrated intensity. This paper surveys recent developments in this theory such as one- and k-sample tests and regression analysis. In connection with the second topic, attention is drawn to recent attempts to extend the multiplicative intensity model to allow for direct maximum likelihood estimation.

The issues important for biostatistical applications are emphasized. The more general comment is made that this area is a prime example of direct practical application of an area of pure mathematics which was recently developed without specific applications in mind. On the other hand the experience also indicates that close contact with concrete applied-statistical consulting activity is necessary for deriving interesting statistical theory.

Critical Phenomena for Reversible Nearest Particle Systems

Thomas M. Liggett, *University of California, Los Angeles, CA*

Suppose $\beta(l, r)$ is a nonnegative function defined for positive integers l and r. The nearest particle system corresponding to $\beta(l, r)$ is the Markov process on the set of all subsets of the integers which has the following dynamics:

$$A \to A\backslash\{x\} \text{ at rate 1 for each } x \in A$$

and $A \to A \cup \{x\}$ at rate $\beta(l_x(A), r_x(A))$ for each $x \notin A$. Here $l_x(A)$ and $r_x(A)$ are the distances from x to the nearest particle in A to its left and right, respectively. The much studied contact process is the special case obtained by taking $\beta(1, 1) = 2\lambda$, $\beta(l, 1) = \beta(1, r) = \lambda$, and $\beta(l, r) = 0$ if $l, r \geq 2$. Nearest particle systems were introduced in 1977 by Spitzer, who obtained necessary and sufficient conditions on $\beta(l, r)$ for the process to have a reversible invariant measure. These reversible measures turned out always to be renewal measures. After giving a brief summary of the relevant portion of the theory of the contact process, which will serve as part of our motivation, we will consider one parameter families of reversible nearest particle systems with birth rates of the form

$$\beta(l, r) = \frac{\lambda f(l) f(r)}{f(l + r)}$$

where f is a positive logarithmically convex function. There are two particularly interesting initial configurations for the process: all the integers, and the singleton {0}. This gives rise to what we will call the infinite and finite systems, respectively. Among the questions we will discuss in the talk are (a) What are the critical values for the finite and infinite systems? (b) How do the asymptotic properties of the finite and infinite critical systems compare? (c) How does the asymptotic behavior of the finite system depend on λ near the critical value? (d) Are there extremal invariant measures for the infinite system other than the pointmass on $\emptyset$ and the reversible renewal measure (when it exists)? The answers to these questions suggest important problems and conjectures for nonreversible nearest particle systems, most of which are open even in the case of the contact process.

Brownian Local Time on Square Root Boundaries and Slow Points of the Brownian Path

Edwin Perkins, *University of British Columbia, Vancouver, Canada*

By counting the number of times a random walk crosses a square root boundary and then taking weak limits one obtains a 'local time' whose inverse is a stable subordinator of known index. During the level stretches of this local time the 'excursions' of the Brownian path remain inside $\pm c\sqrt{t}$ boundaries. This construction leads to some conditioned invariance principals for random walk and to some very precise information about the slow points of the Brownian path. (Work done jointly with P.E. Greenwood.)

Caracterisation des Processus *P*-Stables Fortement Stationnaires à Trajectoires Continues

G. Pisier, *Université Paris VI, Paris, France*

Le théorème de Dudley (67) et Fernique (73) donne une condition nécessaire et suffisante pour la continuité p.s. des trajectoires d'un processus gaussien stationnaire. Dans le cas p-stable ($1<p<2$) on peut généraliser ce théorème pour des processus p-stables fortement stationnaires, par exemple pour des séries de Fourier aléatoires de la forme:

$$\sum_{n\in\mathbb{Z}} a_n\theta_n(\omega)\,e^{int}$$

où $(\theta_n)_{n\in\mathbb{Z}}$ est une suite de v.a. p-stables indépendantes, équidistribuées.

Plusieurs résultats gaussiens (par exemple le lemme de Slepian) admettent des généralisations p-stables, contrairement à ce que l'on pouvait penser jusqu'à présent, au vu de certains exemples.

L'exposé mentionnera aussi certaines application à l'analyse harmonique de ces résultats.

General Nonlinear Models in Time Series Analysis

M.B. Priestley, *University of Manchester Institute of Science and Technology, England*

In this paper we describe a general class of nonlinear time series models called 'state-dependent models' (SDM). The basic structural properties of these models were discussed by Priestley (1980, 1982), and the essential idea underlying their construction is that any general non-linear model which possesses some basic 'smoothness' properties may be described by a *locally linearization* of the form

$$X_t + \phi_1(\mathbf{x}_{t-1})X_{t-1} + \cdots + \phi_k(\mathbf{x}_{t-1})X_{t-k}$$
$$= \mu(\mathbf{x}_{t-1}) + \varepsilon_t + \psi_1(\mathbf{x}_{t-1})\varepsilon_{t-1} + \cdots + \psi_l(\mathbf{x}_{t-1})\varepsilon_{t-l},$$

where $\mathbf{x}_{t-1} = \{X_{t-1}, \ldots, x_{t-k}, \varepsilon_{t-1}, \ldots, \varepsilon_{t-l}\}'$ may be interpreted as the 'state-vector' of process at time $(t-1)$. This leads to a representation similar in form to the conventional linear ARMA time series models, but with the crucial feature that the 'coefficients' in the model are now 'state-dependent'. Thus, the model provides a description of the behaviour of the process appropriate to small deviations of the 'state' from its current position.

It may be shown that the SDM scheme includes, as special cases, the main specific types of non-linear models so far studied, namely, the bilinear, threshold autoregressive, and exponential autoregressive models. Procedures for estimating the functional forms of the model parameters (based on the extended Kalman filter) will be described and illustrated by applications to various data sets.

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M.B. Priestley, State-dependent models: A general approach to non-linear time series analysis, J. Time Series Anal. 1 (1980) 47–71.

M.B. Priestley, On the fitting of general non-linear time series models, in: O.D. Anderson, ed., Time Series Analysis: Theory and Practice I (North-Holland, Amsterdam, 1982) pp. 717–731.

Grandes Deviations et Dynamique des Populations

G. Ruget, *Université de Paris-Sud, Paris, France*

On donne deux applications très différentes des techniques de grandes déviations à l'étude de l'évolution de populations.

Dans la première, pour une population de taille fixée—nommément un modèle d'Ising sur un grand tore, dont les spins intéragissent via un potentiel de Kac—on étudie un évènement rare, la nucléation, qui est le passage d'une phase métastable à une phase stable; on met en évidence une bifurcation suivant la taille du tore, et on décrit les noyaux autour desquels se propage le changement de phase.

La deuxième application concerne le comportement 'presque sûr' d'un processus de branchement spatial inhomogène et dont la loi prend en compte un phénomène de saturation; on s'intéresse surtout à une population comportant plusieurs types d'individus, dont l'évolution est couplée à celle d'un substract régi par une équation parabolique. On montre sur un exemple comment on peut, à la limite des temps grands, espérer une description presque aussi simple que celle donnée par Biggins dans le cas homogène, linéaire, monotype.

2. CONTRIBUTED PAPERS

2.1. Asymptotic properties of Markov processes

Let R_t, $t \geq 0$ be a one-dimensional Markov process with $R_0 > 0$. We are interested in the time of ruin, i.e. a time T for which R_t first becomes negative. The main concern to us is the probability of eventual ruin $\psi(x) = P(T < \infty | R_0 = x)$. In this note we consider the use of martingales in studying $\psi(x)$. Using the stopped process $R_{t \wedge T}$ it can be proved that either $\psi(x) = 1$ or $E(\psi(R_T) | T = \infty) = 0$. Let now v be a nonnegative, nonincreasing real function vanishing at $+\infty$, such that $v(R_{t \wedge T})$ is a martingale with respect to the family of sigma-algebras generated by R_s, $s \leq t$. Then it can be showed that if $\psi(x) > 0$ for all finite x then

$$\psi(x) = v(x)(E(v(R_T) | T < \infty))^{-1}.$$

This formula has been presented in special cases before by Harrison [2] and Gerber [1].

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Ergodic theorems for Subadditive Superstationary Families of Convex Compact Random Sets

Klaus Schürger, *University of Bonn, Federal Republic of Germany*

We consider certain random set analogues of some recent ergodic theorems due to Kingman [5], Krengel [6] and Abid [1]. These results extend, in a way, certain results due to Artstein and Vitale [2] and Hess [3]. Let $\mathscr{C}$ denote the family of all nonvoid compact subsets of $\mathbb{R}^d$ and let $co\,\mathscr{C}$ denote all convex sets in $\mathscr{C}$. Let ρ be the Hausdorff metric on $\mathscr{C}$. A random set is a measurable mapping Y of some probability space $(\Omega, \mathscr{A}, P)$ into the Polish space $(\mathscr{C}, \rho)$. We study families $X = (X_{s,t})$ (the index set being $I = \{(s,t): 0 \leq s < t,\ s,\ t \text{ integers}\}$) of $\mathscr{C}$-valued random sets, which are subadditive (i.e. we have $X_{s,t} \subset X_{s,u} + X_{u,t}$ whenever (s,u), $(u,t) \in I$) and superstationary. In order to explain the concept of superstationarity, conceive X as a random element of the partially ordered Polish space $\mathscr{D} = (\mathscr{C}^N)^N$ (see Kamae, Krengel and O'Brien [4] and Abid [1]). Let T denote the 'shift' in $\mathscr{D}$ (see Abid [1]). Let Q_i denote the distribution of T^iX $(i \geq 0)$. Then X is called superstationary if Q_1 is stochastically smaller than Q_0 (see [4]). It is then shown that if X is a subadditive superstationary family of $co\,\mathscr{C}$-valued random sets satisfying certain integrability conditions, $\lim_{t\to\infty}(1/t)X_{0,t} = Y$ exists a.e. in $(\mathscr{C}, \rho)$ and in the mean. Conditions are given which ensure that Y is constant. An example shows that the above mentioned results will, in general, not hold if the random sets involved are compact but not convex.

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Extreme Values of Markov Chains of Finite Rank

A.H. Hoekstra and F.W. Steutel, *Eindhoven University of Technology, Eindhoven, The Netherlands*

We consider Markov chains $X_0, X_1, \ldots$ with transition probability

$$P(E\,|\,x) = \sum_{j=1}^{r} a_j(x)B_j(E) = \boldsymbol{a}'(x)\boldsymbol{B}(E),$$

as introduced by Runnenburg, and studied more closely in [1]. The n-step transition probabilities take the form (in vector notation)

$$P^{(n)}(E\,|\,x) = \boldsymbol{a}'(x)C^{n-1}\boldsymbol{B}(E),$$

where C is a constant $r\times r$ matrix somewhat like a transition matrix. This Markov chain can be analysed, almost as easily as a finite Markov chain, in terms of eigenvalues and eigenvectors.

If $F_n(y\,|\,x) = P(\max(X_1,\ldots,X_n) \le y\,|\,X_0 = x)$, then F_n has the representation

$$F_n(y\,|\,x) = \boldsymbol{a}'(x)C^{n-1}\boldsymbol{B}(y),$$

where $C(y)$ is a $r\times r$ matrix with $C(y)\to C$ as $y\to\infty$. It turns out that, in the irreducible case and for y sufficiently large,

$$F_n(y\,|\,x) \sim \lambda^n(y), \tag{1}$$

where $\lambda(y)$ is the dominant eigenvalue of $C(y)$. From (1) one proceeds as in the independent case.

Reference

[1] J.Th. Runnenburg and F.W. Steutel, On Markov chains the transition function of which is a finite sum of products of functions of one variable, Math. Centre Report S 304 (Math. Centre Amsterdam, 1962) (abstract in Ann. Math. Statist. 33 (1962) 1483–1484).

2.2. Applications to physics

Shape and Duration of FM-Clicks

Georg Lindgren, *University of Lund, Sweden*

Noise in FM-receivers can cause the phase of a complex valued wave-form to increase rapidly by an amount 2π. A phase detector represents such a change by an impulse or click which distorts the output signal. The stochastic properties of such clicks in unmodulated carrier plus noise is studied in terms of the conditional behavior of stationary Gaussian processes after level crossings. As the carrier-to-noise power ratio tends to infinity the normalized shape of a positive FM-click is shown to approach the rational function

$$\frac{\zeta_2(\xi + t^2/2)}{(-\xi + t^2/2 + \zeta_1 t)^2 + \zeta_2^2 t^2}$$

where ξ, ζ_1, ζ_2 are random coefficients (having exponential, normal, and Rayleigh distributions) taking different values for each click. The outcomes of this random function illustrate the typical click shapes that have been observed experimentally.

The distribution of the duration of clicks is also derived and shown to be very similar to a Maxwell distribution, although of smaller order for large durations.

Equivalence des Ensembles en Mecanique Statistique sur un Reseau

Marc Pirlot, *Université de l'Etat à Mons, France*

La théorie des fonctions convexes conjuguées est utilisée pour expliquer de façon générale la notion d'équivalence des ensembles en mécanique statistique sur un réseau.

On constate tout d'abord que la pression et l'entropie sont, au signe près, des fonctions convexes conjuguées. De ce fait il résulte que les états donnant une énergie moyenne fixée à une famille quelconque de potentiels et minimisant une fonction 'énergie libre' généralisée sont des états d'équilibre. Mathématiquement, cela revient à montrer que la conjuguée 'partielle' de la pression sur un sous espace de l'espace des interactions est reliée à l'entropie par un principe variationnel.

Il en découle une série de conséquences physiques qui peuvent servir à éclaircir certains usages des physiciens: une formulation précise de l'équivalence des ensembles, l'obtention naturelle de potentiels thermodynamiques (du type énergie libre) présentant un palier (sans devoir faire appel à la règle des aires égales de Maxwell), ainsi qu'une version de la loi des phases de Gibbs.

Boundary Values and the Regularity of the Semimartingale Valued Random Fields

A.S. Ustunel, *2, Bd. Auguste Blanqui, Paris 75013, France*

First we give a general result for checking the existence of the C^{∞}-flows associated to the semimartingale-valued random fields. The next result is the following: Any distribution-valued S-semimartingale can be represented as the boundary value of an analytic semimartingale. Finally using the representation of the distributions as the boundary values of analytic functions and Ito-Tanaka's formula we find the solutions of a Schrödinger type equation with reflection for a large class of distributions as initial condition.

2.3. Gaussian processes

Prediction of Stable Processes: Harmonizability and Moving Averages

S. Cambanis and R. Soltani, *University of North Carolina, NC*

The stationary Gaussian processes are Fourier transforms of Gaussian processes with independent increments; and the purely nondeterministic stationary Gaussian processes are moving averages of Brownian motion. The case of stable processes turns out to be more rich and more complex. We show that the Fourier transforms of stable processes with independent increments form a class of processes distinct from the moving averages of stable motion. For the latter class the prediction problem is relatively simple. The problem of prediction for the class of stationary stable processes that are Fourier transforms of processes with independent stable increments is discussed, and the algorithm for the best linear predictor is given in the discrete-time case.

An Extended Dichotomy Theorem for Sequences of Pairs of Gaussian Measures
G.K. Eagleson, *CSIRO Division of Mathematics and Statistics, Sydney, Australia*

A dichotomy for sequences of pairs of Gaussian measures is proved. This result is then used to give a simple proof of the famous equivalence/singularity dichotomy for Gaussian processes. The proof uses tightness arguments and can be directly applied to the theory of hypothesis testing to show that two sequences of simple hypotheses which specify Gaussian measures are either contiguous or entirely separable.

Extension dans l'Espace de Gauss du Theoreme de H. Brunn (1887)
Antoine Ehrhard, *I.R.M.A., Strasbourg, France*

Soient X un vecteur gaussien à valeurs dans un espace vectoriel mesurable (E, B), f une fonction convexe mesurable de E dans $\mathbb{R}$ et N une variable aléatoire gaussienne réelle; si f^* est la fonction décroissante sur $\mathbb{R}$ telle que $f^*(N)$ ait même loi que $f(X)$ alors f^* est convexe. Cet énoncé généralise à l'espace de Gauss un théorème de H.A. Schwarz, H. Brunn et H. Minkowski. L'exposé aura pour but de justifier cette extension et de l'illustrer par des applications à l'équation de la chaleur.

Vecteurs Aleatoires Gaussiens dans les Espaces de Banach Reguliers
X. Fernique, *Université Louis Pasteur, Strasbourg, France*

On caractérise les vecteurs aléatoires gaussiens à valeurs dans les espaces de Banach E à norme deux fois faiblement dérivable, la dérivée seconde $\{D^2(u), u \in E\}$ étant bornée sur le bord de la boule unité de E; on obtient dans ces conditions:

Théorème. *Soit Γ une fonction de type positif continue sur $E' \times E'$, alors les trois propriétés suivantes sont équivalentes:*

(i) *Γ est la covariance d'un vecteur gaussien X à valeurs dans E,*

(ii) *il existe une décomposition $\sum b_n \otimes b_n$ de Γ telle que*

$$\sum \|b_n\|^2 < \infty,$$

(iii) *il existe une décomposition $\sum b_n \otimes b_n$ de Γ telle que*

$$\sup_{\|u\|=1} \sum \langle D^2(u), b_n \otimes b_n \rangle < \infty.$$

Mesures Majorantes et Theoreme Limites Dans c_0
B. Heinkel, *Université de Strasbourg, France*

On étudie le théorème central-limite et la loi du logarithme itéré pour une variable aléatoire X à valeurs dans c_0. La méthode de démonstration consiste à se ramener à la même propriété limite dans $(C[0,1], \|\cdot\|_\infty)$ pour une variable Y associée à X de façon adéquate. A cette variable aléatoire Y on applique la méthode des mesures majorantes.

Bounded Law of the Iterated Logarithm for the Weighted Empirical Distribution Process in the Non-i.i.d. Case
B. Marcus, *Texas A & M University, TX*

In this joint work with J. Zinn the following theorem is obtained:

Theorem. *Let $\Psi(t)$, $t > 0$ be a nonnegative, nondecreasing, left continuous function with right hand limits. Let $\{X_k\}$ be a sequence of nonnegative, independent random variables and let $\{\varepsilon_k\}$ be a Rademacher sequence, independent of $\{X_k\}$. Let*

$$Z_k(t) = \Psi(t)(I_{[X_k \geq t]} - P(X_k \geq t)), \quad S_n = \sum_{k=1}^{n} \varepsilon_k \Psi(X_k), \quad S_n(t) = \sum_{k=1}^{n} Z_k(t).$$

Let $\{b_n\}$ be an increasing sequence of positive numbers with $\lim_{n\to\infty} b_n = \infty$ and assume that

$$\lim_{n\to\infty} \frac{|S_n|}{b_n} < \lambda \quad a.s. \tag{1}$$

for some $\lambda > 0$. Furthermore, assume that

$$\limsup_{n} \sup_{\Psi(t) > 8\lambda b_n} b_n^{-1} \Psi(t) \sum_{k=1}^{n} P(X_k > t) < 6\lambda. \tag{2}$$

Then

$$\limsup_{n} \frac{\sup_{t>0} |s_n(t)|}{b_n} < 104\lambda \quad a.s. \tag{3}$$

Condition (2) is essentially necessary for (3) and in the standard theorems that give (1) (i.e. Kolmogorov's theorem, etc. . . .), it is easily verified. The theorem is proved using symmetrization, a familiar technique in the study of probability on Banach space.

The Rate of Convergence of Extremes of Stationary Normal Sequences

Holger Rootzén, *University of Copenhagen, Denmark*

Let $\{\xi_t; t = 1, 2, \ldots\}$ be a stationary normal sequence with zero means, unit variances, and covariances $r_t = E\xi_s\xi_{s+t}$, let $\{\hat{\xi}_t\}$ be independent and standard normal, and write $M_n = \max_{1 \leq t \leq n} \xi_t$, $\hat{M}_n = \max_{1 \leq t \leq n} \hat{\xi}_t$. In this talk we give bounds on $|P(M_n \leq u) - P(\hat{M}_n \leq u)|$ which are roughly of the order $n^{-1}(1-\rho)(1+\rho)^{-1}$, where ρ is the maximal correlation, $\rho = \sup\{0, r_1, r_2, \ldots\}$. It is further shown that, at least for m-dependent sequences, the bounds are of the right order and, in a simple example, the errors are evaluated numerically. Bounds of the same order on the rate of convergence of the point processes of exceedances of one or several levels are obtained using a 'representation' approach (which seems to be of rather wide applicability). As corollaries we obtain rates of convergence of several functionals of the point processes, including the joint distribution function of the k largest values amongst $\xi_1, \ldots, \xi_n$.

Comparaison des Mesures 'Gaussiennes' Sur l'Espace Projectif

G. Royer, *Université de Clermont II, France*

Soit $\mathbb{P}(E)$ l'espace projectif d'un espace vectoriel E (de dimension infinies). μ et ν deux mesures gaussiennes sur E, $\tilde{\mu}$ et $\tilde{\nu}$ leurs images canoniques sur $\mathbb{P}(E)$. On montre que si $\tilde{\mu}$ et $\tilde{\nu}$ ne sont pas étrangères, il existe une homothétie h de E telle que $h(\mu)$ soit équivalente à ν. Quelques propriétés d'une moyenne $M(a_1, \ldots, a_n)$ apparaissant dans les calculs sont rassemblées.

Ensembles Polaires de Processus Gaussiens

M. Weber, *Université de Strasbourg, France*

Soit X un processus gaussien normalisé à valeurs dans $\mathbb{R}^d$ à composantes indépendantes de même loi et à trajectoires continues; soit K un compact de $\mathbb{R}^d$. On étudie les conditions sous lesquelles K est polaire relativement à X.

2.4. Statistics for processes

Infinite-Dimensional Diffusions from the Itô Point of View

Thomas Barth, *Universität Essen, BRD*

An attempt is made to understand the motion of an infinite interacting particle system as a diffusion in the sense of Itô, i.e. as the solution of a stochastic differential equation with state space $\mathbb{R}^I$ where I is a countable index set. The space $\mathbb{R}^I$ is considered as the union of Banach spaces $E_\alpha = \{x : \sum_{i \in I} \alpha_i x_i^2 < \infty\}$ with weights $\alpha_i > 0$. For any E_α the stochastic integral with respect to a system $(W_i)_{i \in I}$ of independent one-dimensional Brownian motions is defined independently of the weights: it is the same in any $E_\beta \supset E_\alpha$. Thus the notion of a solution of a stochastic differential equation is independent of the spaces E_α. An Itô formula is derived, and an existence and uniqueness theorem for solutions is proved under one-sided conditions for the drift, a monotonicity and a linear growth condition.

This framework is sufficiently large to include interesting examples. The continuous spin model on the lattice $I = \mathbb{Z}^d$ of Doss–Royer satisfies the hypotheses of our theorem, and the motion can be constructed directly by stochastic differential equation methods.

Identification de Modèles de Séries Chronologiques Fondée sur la Loi du Logarithme Itéré pour les Rapports de Vraisemblance. Etude Non Asymptotique

Malek Bouaziz et Bernard Prum

L'ajustement d'un modèle paramétrique (ARMA, par exemple) à une série chronologique pose le problème de la décision de la taille du modèle (type de l'ARMA, par exemple). Pour diverses raisons, parmi lesquelles prédomine la difficulté du choix des niveaux dans un test multiple [2, 3], Akaike [1] a proposé une démarche dans laquelle s'effectuent simultanément l'identification du modèle et l'estimation des paramètres. Cette démarche se fonde sur la pénalisation de la log-vraisemblance d'un modèle par une fonction croissante du nombre de paramètres de ce modèle. Elle s'étend d'ailleurs à bien d'autres domaines que les séries chronologiques.

Suivant cette idée, nous nous plaçons d'abord d'un point de vue asymptotique: nous calculons la vitesse de convergence des rapports de vraisemblance de processus gaussiens, appliquant pour cela à certaines formes quadratiques un principe fonctionnel de la loi du logarithme itéré.

Dégageant un cadre formel d'estimation ensembliste adapté au problème, nous cherchons quelles conditions doit satisfaire la pénalisation de la Log-vraisemblance pour que l'on écarte les modèles 'sous-vraisemblables' et 'sur-vraisemblables'. On en déduit des estimateurs fortement consistants du modèle minimal, améliorant les estimateurs non consistants de [1], et généralisant les procédures de [4, 5].

Nous étudions ensuite la validité de ces résultats pour des échantillons de taille finie, ce qui nous ramène à des problèmes de tests entre modèle. L'étude est développée dans certains exemples d'ARMA.

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Estimation in Multitype Branching Processes

J.P. Dion and K. Nanthi, *Université du Québec à Montréal, Canada*

In this paper we consider a positively regular multitype Galton–Watson process, originating from a single ancestor. We obtain asymptotically normal and/or consistent estimators for the following parameters of the process: the growth rate, its right eigenvector, the probability of extinction, the asymptotic variances of Becker's (1977) and Asmussen and Keiding's (1978) growth rate estimators, the age of the process. With the exception of the estimator for the asymptotic variance of Becker's estimator, which is based on the successive generation sizes only, these estimators are based on a sample intermediate between Becker's scheme and Asmussen and Keiding's full information sampling.

Processus Autoregressifs Generaux

Paul Doukhan, *Université de Rouen, France*

Un tel processus (X_n) à valeurs dans $\mathbb{R}^h$ est donné par un entier k, une fonction $f:(\mathbb{R}^h)^k \to \mathbb{R}^h$ (continue et bornée), par un bruit (ε_n) (suite de variables i.i.d.) et par la relation

$$X_{n+k+1} = f(X_{n+1}, \ldots, X_{n+k}) + \varepsilon_n.$$

Le processus $Y_n = (X_{n+1}, \ldots, X_{n+k+1})$ est alors Markovien; le but de ce travailest d'estimer la loi limite de Y_n ainsi que la fonction f au vu des observations. Des estimateurs à noyaux, non paramétriques, donnent alors des vitesses de convergence des risques quadratiques et intégrés sur une partie de l'espace de l'ordre de $n^{-2/(2+kh)}$. De tels risques paraissent optimaux. On étudie ensuite la validité pratique des résultats de convergence à l'aide de simulations. Enfin on étudie la relation qui existe, si $k=1$, entre les mesures invariantes par f et la mesure limite du processus.

Cadre d'étude pour des Modèles Non linéaires

D. Guegan

A partir de la présentation d'un certain nombre de propriétés probabilistes du processus non linéaire (X_t) (appelé de fait processus bilinéaire) régi par l'équation:

$X_t = aX_{t-2}\varepsilon_{t-1} + \varepsilon_t$ où (ε_t) est une suite de variables aléatoires indépendantes gaussiennes centrées, nous montrons comment les techniques hilbertiennes sont insuffisantes pour l'étude de ce type de modèles.

On peut alors envisager d'étudier ces modèles dans un cadre plus large, celui des chaos de Wiener. Dans ce cadre là nous proposons un essai de classification des processus englobant les processus linéaires et bilinéaires. Nous nous interessons principalement aux notions de processus réguliers, singuliers, processus d'innovations.

Estimation and Reconstruction for Binary Markov Processes

Alan F. Karr, *Johns Hopkins University, Baltimore, MD*

Given a Markov process X with state space $\{0, 1\}$, we treat parameter estimation for the transition intensities and state estimation, i.e. reconstruction by means of conditional expectations, of unobserved portions of the sample path, based on various forms of partial observation of the process. Suitable parameter estimators are shown to be strongly consistent and asymptotically normal. State estimators are computed explicitly and represented in recursive form. Observation mechanisms include regular samples with time jitter, Poisson samples with state 0 unobservable, temporally averaged observations, and observation of a random time change of the underlying process. In all cases the law of the observability process may be partly unknown. The problem of state estimation using estimated parameters is also examined.

About Linear and Bilinear Stochastic Differential Equations

A. le Breton, *Laboratoire I.M.A.G. B.P. 53X-38041 Grenoble Cédex, France*

We are concerned with a multidimensional differential model for stochastic processes in continuous time of the following form

$$\mathrm{d}X_t = [A_0(t)X_t + a_0(t)]\,\mathrm{d}t + \sum_{j=1}^{d} [A_j(t)X_t + a_j(t)]\,\mathrm{d}W_t^j;$$

$$t \geq 0;\ X_0 = X(0),$$

where $W = (W^1, \ldots, W^d)'$ is a standard brownian motion in $\mathbb{R}^d$ and A_j (resp. a_j) are matrix (resp. vector) valued deterministic functions satisfying suitable conditions. Such so-called bilinear or general linear models including the usual linear case have been considered in continuous time stochastic systems theory.

Here we are interested both in structural properties and statistical analysis of the model:

Using a method similar to that of [1] for linear systems, the mean and covariance functions of the process are computed. Necessary and sufficient conditions for second-order stationarity are provided. A linear representation of the state process with respect to a wide-sense Wiener process is obtained.

The problem of filtering is investigated. The equations for the optimal linear filter are derived. The problem of parameter estimation in view of the observation of one trajectory of the process is considered in the case when the system is autonomous. The maximum likelihood method provides consistent and asymptotically normally distributed estimates for the drift parameters when the process is ergodic. The case when the process is not ergodic is discussed in the linear model.

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Asymptotique des Modeles avec Rupture

Dominique Picard, *Université d'Orsay, France*

On étudiera ici le comportement des méthodes de vraisemblance dans les modèles statistiques quand intervient un brutal changement au cours de l'expérience (maladie, crise économique . . .). Plus précisément on se posera deux types de problèmes:

(1) Au vue d'une série d'observations, décider s'il y a eu ou non changement.

(2) Si on sait qu'il y a eu changement en déterminer les paramètres (instant, amplitude).

A travers l'exemple des méthodes de vraisemblance, dans le cas d'observations indépendantes, on montrera que le paramètre 'instant de rupture' possède un comportement statistique très particulier, d'une part parce qu'il viole les conditions de régularité minimales généralement requises, d'autre part, du fait de sa liaison très particulière avec le paramètre amplitude de la rupture.

Les résultats s'obtiennent à partir d'un Principe d'Invariance sur la suite des processus des rapports de vraisemblance qui généralise le résultat d'Ibragimov–Hashminsky: l'analyse fine du comportement de cette suite processus, permet d'opérer une renormalisation des tests de vraisemblance traditionnels dont le niveau n'était pas approximable; elle permet de plus de 'reparamètriser' le modèle en présence d'une rupture pour évaluer la loi asymptotique globale des paramètres du changement.

2.5. Point processes

Life Distribution Properties of Devices Subject to a Lévy Wear Process

M. Abdel-Hameed, *University of North Carolina at Charlotte, NC*

Assume that a device is subject to wear. Over time the wear is assumed to be a one-sided Lévy process (X_t). Suppose the device has a threshold X' with right-tail probability $\bar{G}$. Let ζ be the failure time of the device and $\bar{F}^x$ be its survival probability given that $X_0 = x$. It is shown that life distribution properties of $\bar{G}$ are inherited as corresponding properties of $\bar{F}^x$ for each $x \in R_+$. Optimal replacement policies for such devices are discussed for suitably chosen cost functions when the failure rate of $\bar{G}$ is bounded and continuous a.e.

Poisson Statistics in Random Graphs

A.D. Barbour, *Gonville and Caius College, Cambridge, U.K.*

Many statistics of a large sparse classical random graph are known approximately to follow a Poisson distribution. Here, motivated by a problem in statistical epidemiology, a method is discussed whereby such results can be extended to graphs with less restrictive structure than the classical random graphs.

A Rank Test For Nonhomogeneous Poisson Processes

David McDonald, *University of Ottawa, Canada*

Consider q independent replications $\{N_i(t), 0 \leq t \leq 1\}_{i=1}^{q}$ a simple point process. Given that for $i = 1, \ldots, q$, $N_i(1) = n_i$, we can define $R_{i(j)}$ to be rank of the jth arrival time of process N_i where the ranking is done among the $n_1 + n_2 + \cdots + n_q = n$ arrival times. If the N_i are nonhomogeneous Poisson processes these ranks are those obtained by randomly allocating the integers $\{1, \ldots, n\}$ into groups of $\{n_i\}_{i=1}^{q}$ and then reordering each group into increasing order. The jth largest in group i thus has rank $R_{i(j)}$.

Since we may take $q \to \infty$ but the n_i are small a new statistic is defined to test for this randomness:

$$\mathscr{R} = \sum_{i=1}^{q} \sum_{j=1}^{n_i} \left(\frac{R_{i(j)}}{n+1} - \frac{j}{n_i + 1} \right)^2 .$$

The mean and variance of this rank statistic are calculated. Tables are given for small values of q. Finally a test for the class of nonhomogeneous Poisson processes is given.

The asymptotic normality of $\mathcal{R}$ is established using a martingale difference argument.

Negative Binomial Point Processes: Structure and Related Topics

G. Gregoire, *University of Grenoble, France*

The purpose of this talk is to construct a particular class of point processes and to investigate its properties. The idea is to construct point processes, on a rather general space X, for which all finite-dimensional distributions are usual negative binomial finite-dimensional distributions. Firstly, using an urn model, we construct mixed sample processes with the required property and we point out that there is in fact a more general class of point processes which enjoys this property. The distributions, denoted by $\mathrm{BN}(r, \nu)$ where ν is the intensity measure and r a positive parameter, appear to be mixed Poisson processes distributions.

By use of direct methods, we derive all the f.d. distributions, we observe that $\mathrm{BN}(r, \nu)$ is an infinitely divisible distribution and give its canonical representation. Studying conditioning properties, we show that, for a point process ξ, $\mathrm{BN}(r, \nu)$-distributed, similar results hold for conditional distributions given $\xi(B) = k$ and for Palm Probabilities. Generalizing a well-known convergence result we prove that, when $r_k\nu_k$ converges to ν, $\mathrm{BN}(r_k, \nu_k)$ converges to the Poisson processes distribution with intensity measure ν.

Examples in the case $X = \mathbb{R}, \mathbb{R}_+, \mathbb{R}^2$ are presented, the properties of the stationary $\mathrm{BN}(r, \nu)$ are scrutenized, and cluster representations are given.

Finally, we study some statistical questions as filtering in the case $X = \mathbb{R}^2$.

Processus Ponctuels et (Pre)Visibilité

P.C.T. v.d. Hoeven, *Subfaculteit Wiskunde, RUL, Nederland*

On connaît la théorie, qui mène au compensateur prévisible [1] (Filtration, tribu prévisible, théorème de section prévisible, mesures aléatoires prévisibles, projection prévisible et prévisible duale, expression limite pour le compensateur).

D'autre part on sait, qu'on peut définir l'intensité conditionnelle d'un processus ponctuel sur un espace LCD quelconque comme une limite [4] et que sous certaines conditions celle-ci est l'unique mesure aléatoire qui satisfait une équation intégrale [3].

Il se trouve que l'intensité conditionnelle et le compensateur prévisible se ressemblent beaucoup. Cette analogie sera mise en clarté et utilisée pour introduire la notion de visibilité [2] pour des processus ponctuels sur un espace LCD: Si l'on définit la tribu visible et des mesures aléatoires visibles un théorème de section visible et la projection visible de processus entrainent l'existence d'une projection visible duale, qui n'est autre que l'intensité conditionnelle.

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'Duality' Relations for Stationary Semi-Markov Risk and Queuing Models

Jacques Janssen, *Université Libre de Bruxelles, Belgium*

For the semi-Markov model in risk and queuing theories, we introduce the natural extension of the concept of stationarity in classical models (i.e. the passage of a renewal process to a general stationary renewal process for the arrivals).

It is shown that some interesting relations can be obtained between positive capital risk models using a new concept of duality called $*$-duality.

Numerical Results for Many-Server Queues

J.H.A. de Smit, *Twente University of Technology, The Netherlands*

We discuss a numerical factorization technique which may be applied to a large class of stochastic models. The method is described in more detail for the many-server queue with hyper-exponential service times.

A computer program has been written for $GI|H_2|s$. We report about the performance of this program and the results it has yielded.

Asymptotic Theorems for the Effect of Superpositions of Markov-Modulated Poisson Processes on Single-Server Queues

David Y. Burman and Donald R. Smith, *Bell Laboratories, Holmdel, NJ*

Suppose that the 'burstiness' of an arrival stream is measured in the following manner: compute the expected delay of a customer in a single server queue offered the given traffic and divide by the expected delay of the single server queue offered Poisson traffic of the same average rate. We show how (in either light or heavy traffic) to relate the burstiness of superposed traffic to the burstiness of the sources.

Distributional Coupling and Time Dependent Regenerative Processes

Hermann Thorisson, *Chalmers University of Technology, Goteborg, Sweden*

A distributional coupling concept is defined for continuous time stochastic processes with a general state space, and applied to processes having a certain non-time-homogeneous regeneration property: regeneration occurs at random times S_0, $S_1, \ldots$ forming an increasing Markov chain, the post-S_n process is conditionally independent of $S_1, \ldots, S_{n-1}$ given S_n, and the conditional distribution is independent of n. The coupling problem is reduced to an investigation of the regeneration times $S_0, S_1, \ldots$, and a successful coupling is constructed under the condition that the recurrence times $S_{n+1}-S_n$ given that $S_n = s$, $s \in [0, \infty)$, are stochastically dominated by a random variable with finite first moment, and that the distributions $R_s(A) = P(S_{n+1}-S_n \in A \mid S_n = s)$, $s \in [0, \infty)$, have a common component which is absolutely continuous with respect to Lebesgue measure (or aperiodic when the S_n:s are lattice-valued). This yields results on the tendency to forget initial conditions, as time tends to infinity. In particular, results on the tendency towards equilibrium are obtained, provided the post-S_n process is independent of S_n. The theorems proved cover uniform convergence of distributions, means and intensity measures. Uniform convergence rates are also obtained, under moment conditions on the R_s:s and the times of the first regeneration.

2.6. *Stochastic differential equations*

Formule de Itô pour Certains Proccssus Indexés par une Partie de $\mathbb{R}^d$

Marie-France Allain, *Université de Rennes* I, *France*

Pour certains processus X indexés par une partie de $\mathbb{R}^d$, il est possible de définir des processus $X^{(k)}$, $k = 1 \ldots m$ auxquels sont associées des mesures stochastiques, de telle sorte que, pour $T =]0, a]$, X continu et nul sur les axes, $f \in \mathcal{C}^b_m$, on ait une formule de Itô:

$$\forall t \in T \quad f(X_t) = \sum_{k=1}^{m} \frac{1}{k!} \int_{]0,t]} f^{(k)}(X_s)\, dX_s^{(k)}$$

A Submartingale Type Inequality with Applications to Stochastic Evolution Equations

Peter Kotelenez, *Universität Bremen, West Germany*

On a fixed time interval $[0, T]$ we consider $\int_{[0,t]} U'_s \Phi_s(\omega)\, dM_s(\omega)$, where M is a square integrable martingale with values in a Hilbert space K, $\Phi_s(\omega)$ suitable

operators from K to H, another Hilbert space, and U_s^t is a mild evolution operator on H. Under the assumption that there is a $\beta \geq 0$ such that $\|U_s^t\|_{\mathcal{L}(H)} \leq e^{\beta(t-s)}$ we prove a submartingale type inequality for that integral. As applications we show that it has a continuous version, if M has, or cadlag if M is cadlag, and we prove that it is stable under some additional assumption on U_s^t. Finally we obtain an existence and uniqueness theorem for an SPDE.

Key words: stochastic evolution equation, mild solution, sample path continuity, stability of the integral.

On Coupling of Diffusion Processes

Torgny Lindvall, *Chalmers University, Sweden*

Since the paths of diffusions are continuous, the coupling method is well fitted to be used in the study of one-dimensional such processes. Let $Q_\lambda(t)$, $Q_\mu(t)$ be the distributions of a diffusion $X(t)$, $t \geq 0$, when λ, μ respectively are initial distributions. Without restriction, it may be assumed that we are on natural scale. The coupling method is used to prove that

(i) $\|Q_\lambda(t) - Q_\mu(t)\| \to 0$ as $t \to \infty$, except in a few particular cases, and

(ii) $t^\alpha \|Q_\lambda(t) - Q_\mu(t)\| \to 0$ as $t \to \infty$ under appropriate moment conditions on λ, μ and the speed measure of the diffusion; $\|\cdot\|$ denotes total variation norm.

The Stability Diagram of Linear Stochastic Systems

L. Arnold, H. Crauel and V. Withstutz, *Universität Bremen, Germany*

Let $\dot{x} = A(t)x$ be a linear stochastic system with $d \times d$-matrix valued stationary and ergodic diffusion process $A(t)$. Let $\lambda = \overline{\lim}_{t\to\infty}(1/t)\log|x(t)|$ be the (uniquely determined) Lyapunov number of the system measuring its degree of stability. We aim at clarifying the dependence of λ on the systematic and statistical parameters of the system (stability diagram), using methods of asymptotic analysis of stochastic differential equations.

In particular, we investigate how the Lyapunov number of the random oscillator $\ddot{y} + 2\beta\dot{y} + (c + \xi(t))y = 0$ depends on the damping parameter β and the noise intensity $\sigma^2 = E\xi^2$. It is known that due to the noise the undamped random oscillator is not even marginally stable, but always unstable ($\lambda > 0$). However, by turning on the noise to a certain degree one can improve the stability property of the system (i.e. lower λ), if—in the damped case—β is not too small, or—in the undamped case—c is not too large.

2.7. General theory of processes

Markov Properties of Processes with Two-Dimensional Parameter

Markus Dozzi, *University of Berne, Switzerland*

Markov properties of stochastic processes $(X_z; Z \in I)$ with $I = \mathbb{R}^2$ or $\mathbb{Z}^2$ are considered. Emphasis is on two-parameter analogies of the local Markov property $((Y_t; t_0 < t < t_1)$ conditionally independent of $(Y_t; t < t_0$ or $t > t_1)$ given $(Y_{t_0}, Y_{t_1}))$ for processes $(Y_t; t \in \mathbb{R})$. Relations with the Markov properties of two-parameter processes already investigated are given. The stationary Gaussian processes, satisfying a local Markov property, are characterized.

Quelques Resultats sur les Variations de Champs Gaussiens Stationnaires a Indice dans $\mathbb{R}^2$. Application a l'Identification

Xavier Guyon, *Université de Paris-Sud, France*

Soit X un champ gaussien stationnaire sur $\mathbb{R}^2$. Alors que pour un processus à temps réel on ne dispose en gros que d'une seule variation, on disposera pour X d'un plus grand nombre de variations dépendant de paramètres auxiliaires controlables: variation produit sur deux droites parallèles distantes de d, variation quadratique sur un axe faisant un angle α un axe de référence, variation superficielle sur une grille de rapport λ (λ est le rapport des côtés des rectangles élémentaires de la grille). Les limites dans L^2 de telles variations sont étudiées, leur comportement étant lié à celui de la covariance de X à l'origine. Ces variations se révèlent être un bon outil pour l'identification de modèles paramètriques de champs sur $\mathbb{R}^2$.

Some Results in Stochastic Riemannian Geometry

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In [1] there is a description, and in [2] a rigorous proof, of a probabilistic version of a theorem of Goldberg, Ishihara, and Petridis. The theorem generalizes Picard's little theorem to the context of Riemannian manifolds and harmonic maps. The probabilistic version uses Brownian motion.

Other applications of these methods provide nice proofs of a generalised Liouville's theorem, and of existence of limiting direction for Brownian motion on

a 2-dimensional manifold of negative curvature with extremely weak conditions on the curvature. These further applications are discussed.

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Transformees de Burkholder et Sommabilite de Martingales a Deux Parametres

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Nous présenterons dans cet exposé quelques résultats sur les transformées de Burkholder de martingales bidimensionnelles, dont le suivant: toute martingale à deux paramètres dont la variation quadratique est intégrable converge presque sûrement ainsi que toutes ses transformées de Burkholder. Ce résultat est une conséquence de l'extension bidimensionnelle de certaines inégalités de Burkholder reliant une martingale à sa variation quadratique. Une application aux semi-martingales à deux indices sera donnée.

Remarks on the Regularity of Two-Parameter Processes

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Conditions for a two-parameter stochastic process to possess regular paths are much more difficult to realize than in the one parameter case. Tools for the resolution of this problem are: the notion of random increasing path and a generalization of the optional stopping theorem which allow one to study the processes on these random paths and to make use of results in one-parameter process theory. The most interesting class of processes which has a property of regularity or continuity is the class which possesses a martingale property. However, if we introduce the partial order induced by the cartesian coordinates, a submartingale, even if bounded, does not have right-continuous modification, except under very stringent assumptions on the filtration. In order to achieve regularity for almost every trajectory, we require a supplementary condition—either a condition on the conditional variation of the process, or by decomposition and approximate laplacian. Different kinds of quasimartingales and potentials are surveyed and studied and the relation between the regularity of a quasi-martingale and the continuity of the associated integrable variation is proved.

On the Distribution of a Double Stochastic Integral

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Let $\{W(z), z \in T\}$, $T=[0,1]^2$, be a Wiener process with a two-dimensional parameter. Denote by D the set

$$\{(z,z')\in T\times T: z=(x,y),\ z'=(x',y'),\ x\leqslant x' \text{ and } y\geqslant y'\}.$$

Itô's differentiation formula states that

$$\tfrac{1}{2}(W_{11}^2-1)=\int_T W\,\mathrm{d}W+\int_T\int_T 1_D\,\mathrm{d}W\,\mathrm{d}W.$$

Contrarily to what happens in the one-parameter case, we cannot attain from this expression the distribution of the random variables $K=\int_T W\,\mathrm{d}W$ and $J=\int_T\int_T 1_D\,\mathrm{d}W\,\mathrm{d}W$. In this paper we discuss the common law of these variables.

First, we compute the eigenvalues $\{\mu_k\}_{k=1}^{\infty}$ of the symmetric kernel $\tfrac{1}{2}(1_D(z,z')+1_D(z',z))$, and J has the law of the sum $\sum_{k=1}^{\infty}\mu_k(Z_k^2-1)$, where Z_k is a sequence of i.i.d. standard normal variables. In addition, the characteristic function of J is

$$\mathrm{e}^{-\mathrm{i}t/4}\left(\prod_{k=1}^{\infty}\cos\frac{\mathrm{i}t}{(2k-1)\pi}\right)^{-1}\left(1-4\sum_{k=1}^{\infty}\frac{1}{(2k-1)\pi}\tan\frac{\mathrm{i}t}{(2k-1)\pi}\right)^{-1/2}.$$

If $\{X^n(t), t\in[0,1], n\geqslant 1\}$ and $\{Y^n(t), t\in[0,1], n\geqslant 1\}$ are two independent infinite dimensional Brownian motions, the sequence

$$n^{-1}\sum_{i,j=1}^{n}\left(\int_0^s X^i\,\mathrm{d}X^j\right)\left(\int_0^t Y^i\,\mathrm{d}Y^j\right)$$

converges weakly to the process $\int_{[0,s]\times[0,t]} W\,\mathrm{d}W$. This result provides a method to evaluate the moments of K, and to exhibit a martingale array having this non-symmetric limit distribution.

Bilateral Approach to Killing of Markov Process

Michael Taksar, *Stanford University, CA*

Given a Markov process X in the state space E, we can construct a new process $K(X)$ on any good subset D of E, killing the original process at the first time it exits D. But stationarity of the process is lost under such a transformation.

We study another operation Q, which transforms a stationary Markov process X (a Markov process under a stationary distribution and with time parameter set $]-\infty,+\infty[$) in the space E into a process of the same type with a state space $D\subset E$. But unlike the initial process X, the process $Q(X)$ has random birth and death times, and the corresponding measure in the space of paths can be infinite. The transition probabilities of the process $Q(X)$ are equal to those of $K(X)$; and the one-dimensional distributions of $Q(X)$ and X are equal on D. But in contrast to K, the operation Q is invariant under time reversal.

The most interesting is the inverse problem: for a given stationary Markov process Y in the space D to construct a conservative stationary Markov process in space $E \supset D$ such that $Q(X) = Y$. This is solved and as a corollary we have the following result.

If T_t is a contraction semigroup and ν is finite excessive measure then there exists a conservative semigroup $\tilde{T}_t \geq T_t$ and ν is invariant with respect to T_t.

On Double Stochastic Integration

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We study random integrals of the form $\iint f(x, y)M(\mathrm{d}x)M(\mathrm{d}y)$ where M is either a Brownian motion or a stable process. The structure of associated product random measure is investigated. The results extend earlier work of Kac and Siegert, Varberg and others and are related to the study of infinite quadratic forms in random variables. It is a joint work of J. Szulga, J. Rosinski and the author (or some combination thereof).

2.8. Filtering and stochastic control

Optimal control of Markov Processes

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Firstly we consider Markov decision chains with a denumerable state space and compact metric action spaces.

We assume the usual continuity properties of the immediate costs and of the transition probabilities. In [2] the existence of a finite solution to the average cost optimality equations and the existence of an average cost optimal stationary policy was shown for the case where the underlying Markov chains have a single ergodic set. In this paper we generalize both results to the multichain case. A counter-example shows that for the multichain case the simultaneous Doeblin condition [3] is not sufficient. However, the continuity of the ergodic potential suffices.

Under this assumption the Markov decision problem can also be solved for more sensitive optimality criteria as bias—and Backwell—optimality. Secondly we will sketch how the previous results can be extended to more general optimal control models. Special attention will be paid to Markov decision drift processes ([4]).

To conclude we remark that in [1] and [4] only existence results for optimal policies are derived. However, for the computation of optimal policies the stronger result of the existence of finite solutions of the optimality equations is necessary.

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Some Filtering Problems for Counting Observations in Software Reliability

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In order to obtain a comprehensive framework to software reliability problems [1] martingale theory is applied. Then the error and failure processes are modelled as jump processes. Several filtering problems are now formulated which in most cases lead to a finite dimensional solution. Some examples are given.

Conditional densities thereby obtained, may be used for maximum likelihood estimation and computing various reliability measures [2].

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Equation de Filtrage pour des Processus a Valeurs Distributions

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Le filtrage des processus à valeurs dans des espaces de dimension infinie a été considéré dans [2], [3], [4] etc., dans le cadre des processus à valeurs hilbertiennes.

On se propose de l'étudier ici pour des processus à valeurs distributions. Dans ce but, on étend l'intégration stochastique dévéloppée dans [5] au cas des processus à valeurs opérateurs par rapport à un mouvement brownien à valeurs distributions. Les théorèmes de Girsanov et de représentation sont aussi considérés pour l'obtention de l'équation de filtrage non-linéaire. Le modèle étudié est similaire à celui de [1] dans le cas des processus de dimension finie. Les résultats peuvent être utilisés pour le filtrage des processus à deux indices.

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Stability of Optimal Stopping and Optimal Replacement Problems

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A typical optimal stopping problem can be viewed as a triple (X, g, M) where X is a Markov process with state space $(E, \mathscr{E})$, $g \in \mathscr{E}$ and M is a class of Markov times. For $\varepsilon \geqslant 0$, the ε-optimal problem is to find a stopping time τ_ε such that $E^x g(X_{\tau_\varepsilon}) \geqslant \pi_X g(x) - \varepsilon$ where $\pi_X g(x) = \sup_{\tau \in M} E^x g(X_\tau)$. Problems of this type often arise in connection with Markov decision processes. In particular, recent work has been done on various optimal replacement models whose cost minimization problems can be translated into associated optimal stopping problems.

It is trivial to construct examples of families of stopping problems (X^α, g, M), $\alpha \in [0, 1)$, such that, in one reasonable sense of convergence, $X^\alpha \to X^0$ as $\alpha \to 0$, but such that nearly optimal times for 'close' X^α are not nearly optimal for X^0 and vice versa. In these examples the state spaces are even finite.

In this work, metrics on *appropriate* classes of processes are defined with respect to which it is reasonable to discuss the stability of optimal stopping and optimal replacement problems. Among those processes shown to be stable are those with a finite state space, the one and two dimensional Brownian motion and the classical symmetric and nonsymmetric random walks on the integers. An example is given of an unstable process.

On Sequential Bayesian and Minimax Decision Problems based on the Observation of Stochastic Processes

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For a large class of sequential decision problems in continuous time the problem of finding an optimal sequential decision procedure is reduced to an optimal stopping problem for a certain stochastic process (X_t). The form of the process (X_t) involves integrals over the parameter space Θ. Appropriate conditions yield the existence of an optimal stopping time for the process X. Some theoretic problems in the construction of the general Bayesian model and of the process X are pointed out (e.g. existence of the posterior distribution in the sequential case; regularity of semimartingales). The following observation processes are studied: (1) continuous semimartingales, (ii) some semimartingales with Gaussian martingale part (e.g. Gaussian Markovian processes), (iii) multivariate point processes (e.g. compound Poisson processes, queuing processes), (iv) generalized Gamma processes, (v) multinomial processes in discrete time. A closed form of the process (X_t) can be derived if conjugate priors and appropriate but rather general loss functions are used. Then an optimal stopping time always exists. In special cases the optimal stopping time can be given explicitly. In those cases also an optimal sequential Minimax procedure can be obtained.

Instantaneous Control of Brownian Motion

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A controller continuously monitors a storage system, such as an inventory or bank account, whose content $Z = \{Z_t, t \geq 0\}$ fluctuates as a (μ, σ^2) Brownian motion in the absence of control. Holding costs are incurred continuously at rate $h(Z_t)$. At any time, the controller may instantaneously increase the content of the system, incurring a proportional cost of r times the size of the increase, or decrease the content at a cost of l times the size of the decrease. We consider the case where h is convex on a finite interval $[\alpha, \beta]$ and $h = \infty$ outside this interval. The objective is to minimize the expected discounted sum of holding costs and control costs over an infinite planning horizon.

It is shown that there exists an optimal control limit policy, characterized by two parameters a and b $(\alpha \leq a < b \leq \beta)$. Roughly speaking, this policy exerts the minimum amounts of control sufficient to keep $Z_t \in [a, b]$ for all $t \geq 0$. Put another way, the optimal control limit policy imposes on Z a lower reflecting barrier at a and an upper reflecting barrier at b. The cumulative control is the local time at the reflecting barriers.

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Markov Decision Processes, Time-Discretization

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Consider a continuous-time Markov decision process $(s, X_R(s))_{s \geq 0}$ with infinitesimal operator A_u under decision u and with R a Markov po icy. A costrate function $f(s, x, u)$ is given.

To derive approximations and structure of optimal expected costs and policies the method of time-discretization will be applied.

For that, by means of discrete-time generators A_u^k under decision u and policy $R_k = R$, a kth discrete-time markov decision process $(jk^{-1}, X_{R_k}(jk^{-1}))_{j=0,1,2,\ldots}$ is constructed.

Approximation lemmas are given, which require for $k^{-1} \to 0$ convergence conditions for the discrete-time generators, to show convergence of semigroups, corresponding to transition probabilities, and of expected costs. Moreover, they provide convergence bounds and allow for compact convergence conditions, hence extending results known previously.

2.9. Random walk

Random Walks on Finite Groups: Probabilistic Techniques and Card-Shuffling Examples

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The asymptotic theory of random walks on finite groups G is well understood. We discuss some features of the nonasymptotic behavior:

How large must n be to make the transition probabilities $p_{i,j}^{(n)}$ approximately uniform?

What are the first hitting time distributions?

How long does the random walk take to visit every state?

The informal idea of 'the time for $p_{i,j}^{(n)}$ to approach uniformity' can be formalised as a parameter τ, which can be estimated by coupling techniques. We illustrate with examples of random card-shuffling. These examples have the property

(*) τ is small compared to $\# G$.

For any random walk define R to be the mean number of visits to the initial state before time τ. If $\# G$ is large and (*) holds, several features of the random walk involve essentially only R:

From the uniform start, the first hitting time on a state has approximately the exponential distribution, mean $R \# G$.

The time to visit all states is approximately $R \# G \log(\# G)$.

In the card-shuffling examples, R can be estimated using exponential martingales.

Resource Depending Branching Processes

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We shall investigate a population model where the individuals' survival until the age of reproduction depends on the amount of resources they inherit from their ancestors.

Consider a population starting with Z_0 individuals which create a certain environment of resources, a resource space R_0, say. Each offspring is supposed to need a random stock of resources to be able to develop until the age of fertility. If it cannot receive it, then it will emigrate without leaving offspring. Those who stay in the population will form the first generation ($\# = Z_1$), they create a new resource space R_1 and so on. The reproduction law of those who stay will be the same for all generations. The individual needs will be i.i.d. random variables within each fixed generation but may depend on the generation as well as on the available resource space.

We are primarily interested to know how many resources each generation has to provide for its children such that the process may survive forever. We shall distinguish between the case where children can profit from the whole resource space left by the preceding generation and the case where they are confined to their father's heritage.

Une Loi du Logarithme Itéré pour les Chaines de Markov sur $\mathbb{N}$ Associées a des Polynomes Orthogonaux

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Let X_n be a random walk on the set $\mathbb{N}$ of the positive integers, namely a Markov chain with stationary transition probabilities given by $p_{i,i+1} = p_i$, $p_{i,i-1} = 1 - p_i$ ($p_i \in]0, 1[$ for $i \geq 1$) and $p_{0,1} = 1$. When $p_i = \frac{1}{2}$ (for every $i \geq 1$), one has the following (well-known) result:

$$\text{I.L.L.} \quad \limsup_{n \to +\infty} \frac{X_n}{\sqrt{2n \log \log n}} = 1 \quad \text{a.s.}$$

If we perturb the preceding transition probabilities with asymptotic damping i.e. $p_i = \frac{1}{2}(1+\varepsilon_i)$ and $\varepsilon_i = \lambda/i + o(1/i)$ $(i \to +\infty)$, then we show that the I.L.L. is unchanged if $\lambda > 0$. When $\lambda \le 0$, we can only prove that the lim sup is smaller than 1. Our methods of investigation are based on Fourier analysis deriving from Gegenbauer's polynomials and they can be applied to a large class of Markov chains on $\mathbb{N}$ for which we give the C.L.T. and the I.L.L.

Transport de Particules dans un Fluide Turbulent

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On étudie le transport de particules solides dans un fluide turbulent qui s'écoule dans un bassin, en utilisant le modèle d'une randonnée aléatoire. On peut obtenir quelques formules générales pour les probabilités de sédimentation et de sortie de ces particules, et aussi quelques résultats dans le cas où leur mouvement est toujours non-négatif. On démontre que l'identité de Wald donne quelques résultats approximatifs pour ces probabilités. On obtient finalement des formules pour les probabilités de sédimentation dans diverses positions au fond du bassin.

Periodically Varying Branching Processes

Peter Jagers and Olle Nerman, *Chalmers University of Technology, Sweden*

Consider a supercritical general branching process where the distribution of the birth process of an individual depends upon the chronological time in a periodic manner, say with period 1 (or 24 h). This situation occurs in several biological populations.

Under decent conditions it is shown that as time passes the population composition stabilizes around a periodically varying, but otherwise stable limit structure.

The argument has three steps. First the existence is established of a generalized Perron–Frobenius root and the corresponding eigen-functions. Then by use of a periodic renewal argument the asymptotics of the expected population is deduced. Finally matringale and law-of-large-numbers arguments yield the asymptotics of the branching process itself.

The Stable Pedigrees of Critical Branching Population

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In "The stable doubly infinite pedigree process of supercritical branching population", by this author and Peter Jagers, the concept of an ego-centered doubly infinite pedigree space corresponding to a branching life space was introduced together with the stable pedigree process, arising as the limit when sampling of an ego is performed among all those born up to time t in a supercritical branching process and $t \to \infty$. Further, the stable pedigree processes corresponding to sampling among certain subpopulations, e.g. the population of those alive, were given. Consider now a critical branching process. Suppose that the life spans have finite expectations. Then we shall define the stable pedigree measure $\tilde{P}$ corresponding to sampling among those alive, and show that this measure arises as the conditional limit (in several senses) when sampling of an ego is performed among those alive at time t, given non-extinction before t, and $t \to \infty$.

A Class of Transformations

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Pour une classe de transformations dilantantes de l'intervalle unité, nous montrons un théorème de la limite locale pour les processus $(f \circ T^n)_{n \in \mathbb{N}}$, où f est une fonction à variation bornée.

Nous monotons d'autre part que la vitesse dans le théorème de la limite centrale est $1/\sqrt{n}$.

Random Limit Theorems for Random Walks Conditioned to Stay Positive

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Let $\{X_k, k \geq 1\}$ be a sequence of independent identically distributed random variables with $EX_1 = 0$, $EX_1^2 = \sigma^2 < \infty$ and let $\{N_n, n \geq 1\}$, $N_0 = 0$ a.s. be a sequence of integer-valued random variables. Form the random walk $\{S_{N_n}, n \geq 1\}$ by setting $S_0 = 0$ and $S_{N_n} = X_1 + X_2 + \cdots + X_{N_n}$, $n \geq 1$. We investigate the limit behaviour of

$$P[S_{N_n} < x\sigma\sqrt{N_n} \mid S_1 > 0, S_2 > 0, \ldots, S_{N_n} > 0].$$