# Renormalization of singlet NN-scattering with one pion exchange and boundary conditions

M. Pavón Valderrama, E. Ruiz Arriola

*Departamento de Física Moderna, Universidad de Granada, E-18071 Granada, Spain*

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## Abstract

We present a simple and physically compelling boundary condition regularization scheme in the framework of effective field theory as applied to nucleon–nucleon interaction. It is free of off-shell ambiguities and ultraviolet divergences and provides finite results at any step of the calculation. Low-energy constants and their non-perturbative evolution can directly be obtained from experimental threshold parameters in a completely unique, one-valued and model independent way when the long range explicit pion effects are removed. This allows to compute scattering phase shifts which are, by construction consistent with effective range expansion to a given order in the CM momentum and are free from finite cut-off artifacts. We illustrate how the method works in the $^1S_0$ channel for the one pion exchange potential.
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## 1. Introduction

Effective field theories (EFT) have been successfully investigated in recent years in the context of hadronic and nuclear physics. Their main ingredient has to do with the occurrence of scale separation between long and short distance physics, making the development of a systematic power counting possible. Since the original proposal of Weinberg's [1] to make a power counting in the potential many works have followed implementing such a counting [2–5] with finite cut-offs or proposing a counting in the

*E-mail addresses:* mpavon@ugr.es (M. Pavón Valderrama), earriola@ugr.es (E. Ruiz Arriola).

renormalized *S*-matrix [6,7] which has also been pursued [8]. Both Weinberg and Kaplan–Savage–Wise schemes can be understood as perturbative expansions about infrared fixed points [9] (see also Ref. [10]). In any case, convergence improves under certain conditions [11]. According to Ref. [12] a hybrid counting involving also the chiral limit should be invoked (see also Ref. [13]). For a recent and more complete review on these and related issues see, e.g., Ref. [14] and references therein.

Much theoretical insight has been gained by analysing how short and long distance physics separate for the one pion exchange (OPE) interaction in the singlet $^1S_0$ channel where the scattering length, $\alpha_0 = -23.7$ fm, is much larger than the size of the potential $1/m_\pi = 1.4$ fm. The non-perturbative renormalization

of NN-interaction in this channel has been studied several times in the literature. In Ref. [15] an elegant subtraction method has been developed to construct a finite $T$ matrix for contact, i.e., zero range, interactions added to OPE. Renormalization is indeed achieved by taking the subtraction scale to be much larger than any other mass scale and checking for independence of results in this limit. The resulting description of the $^1S_0$ phase-shift is only valid to very low energies, requiring for inclusion of derivative terms. Unfortunately, the method cannot be easily extended in that case. These derivative interactions can be included within dimensional regularization in the minimal subtraction scheme both in coordinate [16] or momentum [17] spaces. In this latter case a three-parameter fit can be achieved with no explicit two pion exchange contribution. A cut-off regularization has also been introduced in Refs. [18,19]. For the pionless theory, though, the inconsistency between both regularization methods after renormalization has been pointed out for a truncated bare potential [20]. To our knowledge, there is no calculation of OPE where both the effective range expansion is reproduced at a given order in the momentum and finite cut-off artifacts are removed. Momentum space treatments based on the Lippmann–Schwinger equation appear more natural from a diagrammatic point of view within a Lagrangian framework and allow explicit consideration of non-local potentials. In practice, however, in the long range potentials used in NN-scattering are local, and for those the analysis of renormalization in coordinate space may be simpler. In addition, the Schrödinger equation is a second order operator and boundary conditions define a complete solution of the problem in the whole space both inside and outside the boundary. This is equivalent to a sharp separation between the interior and exterior region. This property is naturally formulated in coordinate space for a local potential.

Although the idea of using boundary conditions for NN-scattering is a rather old one (see, e.g., Ref. [21] and references therein), there have been recent works in this regard motivated by the developments within EFT [22–24]. Actually, it has been shown [24] that in the absence of long range forces a low-momentum expansion of the potential within EFT framework for the Lippmann–Schwinger equation is completely equivalent to an effective range expansion and also to an energy expansion of a generic boundary condition

at the origin in coordinate space for the Schrödinger equation. If a long range OPE potential is added we will show below that due to the short distance Coulomb nature of this potential the origin must be reached continuously from above $R \to 0$, $R > 0$ (i.e., excluding the point $R = 0$), in harmony with known theorems on self-adjoint extensions of Schrödinger operators [25].

In this Letter we analyze precisely how the energy dependent boundary condition must change as we move the boundary radius for fixed energy to achieve independence of physical observables such as scattering phase shifts. By doing so we are effectively changing the Hilbert space since the wave function in the outer region is defined only from the boundary to infinity. An advantage of this procedure is that we never need to invoke off-shellness explicitly; at any step we are dealing with an on-shell problem. In addition, we work directly with finite quantities and no divergences appear at any step of the calculation when the boundary radius is taken to zero from above. Our approach provides a non-perturbative regularization scheme which, in principle, should be able to accommodate any of the counting schemes proposed in the literature. Rather than making a specific choice, we prefer instead to make a low-energy expansion of the boundary condition at the origin to prove the feasibility of the approach.

## 2. Variable phase equation with boundary conditions

The reduced Schrödinger equation for including OPE in the $^1S_0$ channel for NN-scattering with CM momentum $k$ reads

$$-u_k''(r) + U(r)u_k(r) = k^2 u_k(r), \tag{1}$$

together with the asymptotic condition at infinity

$$u_k(r) \to \sin(kr + \delta(k)). \tag{2}$$

The OPE potential in the $^1S_0$ channel reads

$$U(r) = -\frac{g_A^2 m_\pi^2 M_N}{16\pi f_\pi^2} \frac{e^{-m_\pi r}}{r}. \tag{3}$$

Where $M_N$ is the nucleon mass, $m_\pi$ the pion mass, $f_\pi$ the pion weak decay constant and $g_A$ the nucleon axial

coupling constant. In the numerical calculations below we take $M_N = 938.92$ MeV, $f_\pi = 93$ MeV, $m_\pi = 138$ MeV and $g_A = 1.25$. Our lack of knowledge of the interaction below a certain distance scale $R$ is parameterized in terms of a boundary condition at the matching point $r = R$,

$$u_k'(R) - L(k, R)u_k(R) = 0. \tag{4}$$

In general, this boundary condition depends both on the boundary radius $R$ and the momentum $k$. The value of $R$ separates the whole space into two disjoint regions, an outer region where we assume the interaction to be given by OPE potential, and an inner region where interaction is regarded as unknown.

The boundary condition at $R$, Eq. (4) has a simple physical interpretation. If we switch off the long range piece $U(r)$ above the scale $R$, then the phase shift due to the short distance physics below the scale $R$ is given by

$$\frac{u_k'(R)}{u_k(R)} = L(k, R) = k \cot(kR + \delta(k, R)). \tag{5}$$

It is interesting to see what kind of equation satisfies the short distance phase shift, $\delta(k, R)$, as we steadily move the boundary radius $R$ for a fixed momentum $k$. Using Schrödinger's equation at the boundary $r = R$ we get the variable phase equation,

$$\frac{d\delta(k, R)}{dR} = -\frac{1}{k}U(R)\sin^2(kR + \delta(k, R)). \tag{6}$$

The obvious condition, $\lim_{R\to\infty} \delta(k, R) = \delta(k)$, at infinity must be satisfied. Thus, Eq. (6) describes the evolution of the phase shift as we go down to lower distances, assuming that *both* the long distance potential and the physical phase shift are known. Regardless of whether or not the potential we are considering is realistic at very short distances[1] one can extrapolate the long distance potential to the origin and define the zero range OPE-extrapolated phase shift

$$\delta_S(k) = \lim_{R\to 0^+} \delta(k, R). \tag{7}$$

Being able to take this limit in practice is essential for it means removing any finite cut-off arti-

facts in the long distance force. Actually, the precise manner how this limit is built depends specifically on the OPE potential, Eq. (3), and will be analyzed below. Eq. (6) is well known in potential scattering (for a review see, e.g., Ref. [26]), but it has always been used assuming the trivial initial condition $\delta_S(k) = \lim_{R\to 0} \delta(k, R) = 0$.

## 3. Low energy expansion of the boundary condition

The former variable phase equation, Eq. (6) can be cast in a more convenient form by defining the variable $K$-matrix,

$$K(k, R) = k \cot \delta(k, R), \tag{8}$$

yielding

$$\frac{dK(k, R)}{dR} = U(R)\left[K(k, R)\frac{\sin kR}{k} + \cos kR\right]^2. \tag{9}$$

At low energies, however, it can be conveniently parameterized as an effective range expansion, which carries over to the variable phase

$$k \cot \delta(k, R) = -\frac{1}{\alpha_0(R)} + \frac{1}{2}r_0(R)k^2 + v_2(R)k^4 + \cdots \tag{10}$$

one has

$$\frac{d\alpha_0}{dR} = U(R)(\alpha_0 - R)^2, \tag{11}$$

$$\frac{dr_0}{dR} = 2U(R)R^2\left(1 - \frac{R}{\alpha_0}\right)\left(\frac{r_0}{R} + \frac{R}{3\alpha_0} - 1\right), \tag{12}$$

$$\frac{dv_2}{dR} = R^4 U(R)\left\{\frac{1}{4}\left(\frac{r_0}{R} + \frac{R}{3\alpha_0} - 1\right)^2 \right.$$
$$\left. + 2\left(1 - \frac{R}{\alpha_0}\right)\left(-\frac{1}{12}\frac{r_0}{R} + \frac{v_2}{R^3} \right.\right.$$
$$\left.\left. - \frac{1}{120}\frac{R}{\alpha_0} + \frac{1}{24}\right)\right\}. \tag{13}$$

These equations have to be supplemented with some initial conditions $\alpha_0(R_0)$, $r_0(R_0)$ and $v_2(R_0)$ at a given boundary radius, $R_0$. If we take the initial boundary radius, $R_0 = 0$ the set of equations, (11), (12) and (13) express the evolution of the low-energy parameters at short-distances when the long distance potential is

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[1] Two Pion Exchange becomes comparable to OPE at about the distance of $r = 1.5$ fm. So, any extrapolation of Eq. (6) with OPE below 1.5 fm should not be considered realistic.

switched on up to the scale $r < R$. Conversely, if the initial boundary radius is taken to infinity they offer a possibility to determine the short-distance low-energy parameters from the experimental ones by downwards evolution in the variable $R$ when the long distance potential is adiabatically switched off for $r > R$. Notice the very appealing and natural hierarchy in the previous equations; while the distance evolution of the scattering length $\alpha_0$ is autonomous, the remaining low-energy parameters $r_0$, $v_2$, etc. depend on the previous ones. To see the connection with more conventional approaches [6,9], mainly carried out in momentum space, let us consider the region $a \ll R \ll 1/k$, where the potential vanishes, and define the dimensionless logarithmic derivative at zero energy $C_0(R) = 1 - Ru_0'(R)/u_0(R) = \alpha_0(R)/(\alpha_0(R) - R)$ fulfilling the equation

$$RC_0'(R) = -C_0(R)\big(1 - C_0(R)\big) \tag{14}$$

deduced from Eq. (11). Identifying $1/R = \mu$ or $\Lambda$ we reproduce the renormalization group evolution obtained, e.g., in dimensional regularization [6] or sharp cut-off regularization [9] respectively for the four fermion interaction coefficient denoted as $C_0$. A more comprehensive discussion will be carried out in full detail elsewhere [28]. We note also that the evolution in $R$ deduced from Eqs. (11), (12) and (13) is one-valued, in contrast to the multibranched evolution generated by assuming an energy dependent square well potential in the inner region $0 \leqslant r \leqslant R$ as a countert-erm [12].[2] This multivaluedness is irrelevant at low energies but influences the phaseshifts at higher energies. This is an ambiguity typical of inverse scattering problems where knowledge of the amplitude at low-energies, say in the regime of effective range theory, does not uniquely determine the potential but induces a residual dependence of this multivaluedness at higher energies than those used to fix the low-energy parameters (see, e.g., Ref. [25]).

Before presenting the numerical results (11), (12) and (13) we analyze first the short and long distance behaviour. At short distances $R \ll 1/m_\pi$ the OPE potential behaves like the Coulomb potential. Eq. (11)

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[2] If one assumes the square well potential $U = U_0 + k^2U_2 + \cdots$ in the region $0 \leqslant r \leqslant R$ and matches the logarithmic derivative of the regular solution in powers of $k$ with Eq. (5) one gets $1/(R - \alpha(R)) = \sqrt{U_0(R)} \cot(\sqrt{U_0(R)}R)$ which is multivalued in $U_0(R)$.

can be easily solved in two cases, $\alpha_0 \ll R$ and $\alpha_0 \gg R$. In the first case we get

$$\alpha_0(R) = \alpha_0(R_0) - \frac{g_A^2 m_\pi^2 M_N}{32\pi f_\pi^2}\big(R^2 - R_0^2\big), \quad \alpha_0 \ll R, \tag{15}$$

where the limit $R_0 \to 0$ can be taken. In the second case one solution behaves as

$$\alpha_0(R) = \frac{\alpha_0(R_0)}{1 + \alpha_0(R_0)g_A^2 m_\pi^2 M_N/(16\pi f_\pi^2)\log(R/R_0)}$$

$$\to \frac{16\pi f_\pi^2}{g_A^2 m_\pi^2 M_N}\frac{1}{\log(R/R_0)}, \quad \alpha_0 \gg R, \tag{16}$$

where $R < R_0 \ll 1/m_\pi$. As we see, $\alpha_0(R)$ goes to zero very slowly and with $\alpha_0'(R) \to -\infty$ at short distances, which in momentum space corresponds to the ultraviolet limit. Eq. (16) agrees with the perturbative analysis in momentum space of Ref. [6]. It is easy to see that the first case, Eq. (15), corresponds to selecting the regular solution at the origin, whereas Eq. (16) is the generic case, which always contains an admixture of the irregular solution. Obviously, the regular case is exceptional and for that particular situation one can integrate from the origin starting with the trivial initial condition $\delta(k, 0) = 0$ up to infinity. The result corresponds to a pure OPE interaction, with no short-distance interactions. The important thing to note here is that no matter what the initial value of $\alpha_0$ was at infinity (except for the exceptional case discussed before), removing one-pion exchange goes into the same value at the origin, as implied by Eq. (16). This also means that any small deviation of the $\alpha_0(R_0)$ at small distances results in huge variations at infinity. Thus, removing OPE results in a extreme fine tuning of the low-energy parameters at short distances.

We analyze now the long distance behavior. Clearly, when $R \gg 1/m_\pi$ we have $\alpha_0'(R) = 0$, Eq. (11), and we approach quickly the asymptotic value $\alpha_0(\infty)$. For such long distances we can always use perturbation theory to solve the equations backwards. For scattering lengths which are small, i.e., $\alpha_0 \ll 1/m_\pi$ we may neglect $\alpha_0(R)$ with respect to $R$ and get

$$\alpha_0(R) - \alpha_0 = -\int_R^\infty U(R)R^2\,dR + \cdots. \tag{17}$$
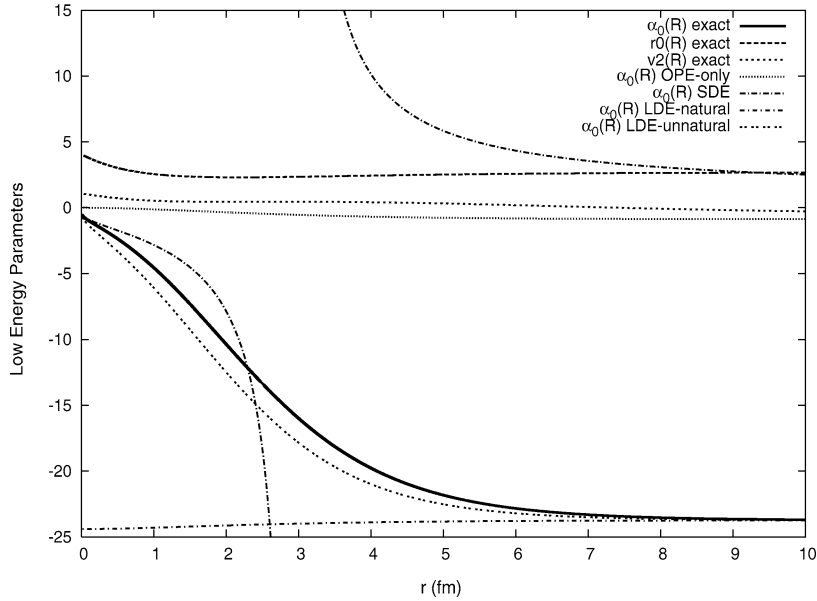
Fig. 1. Evolution of the scattering length $^1S_0$ NN-threshold parameters $\alpha_0(R)$ (in fm), $r_0(R)$ (in fm) and $v_2(R)$ (in fm$^3$) from the asymptotic values at infinity (which we take in practice $R_\infty = 20$ fm) when OPE effects are removed down to the origin. $\alpha_0 = -23.73$ fm and $r_0 = 2.68$ fm and $v_2 = -0.48$ fm$^3$. Solutions of Eqs. (11)–(13) are labelled as "exact". The extrapolated values at the origin when OPE effects are removed are $\alpha_{S,0} = 0$, $r_{S,0} = 4.04$ fm, and $v_{S,2} = 1.07$ fm$^3$. We also show some approximations for $\alpha_0(R)$. OPE means one-pion-exchange only and corresponds to integrate Eq. (11) from the origin to infinity with the boundary condition $\alpha_0(0) = 0$. SDE means short distance expansion as given by Eq. (16). LDE correspond to a long distance expansion, Eq. (17) (natural case) and Eq. (18) (unnatural case), respectively.

For unnatural scattering lengths, $\alpha_0 \gg 1/m_\pi$ we make the opposite approximation, and get

$$\frac{1}{\alpha_0(R)} - \frac{1}{\alpha_0} = -\int_R^\infty U(R)\,dR + \cdots. \tag{18}$$

The previous Eqs. (17) and (18) hold irrespectively of the strength of the potential, provided $R$ is sufficiently large. Similar approximations for the remaining low-energy parameters will be discussed elsewhere [28].

The numerical evolution of $\alpha_{S,0}(R)$ and $r_S(R)$ starting with the experimental values, $\alpha_0 = -23.739$ fm, $r_0 = 2.68$ fm and $v_2 = -0.48$ fm$^3$ (see Rentmeester as quoted in Ref. [4]) down to the origin according to Eqs. (11)–(13) is shown in Fig. (1).[3] We also show the perturbative estimate in the case of large

and small scattering lengths based on a long distance expansion Eq. (17) (natural case) and Eq. (18) (unnatural case), respectively, as well as our short distance estimate, Eq. (16). In the case of $\alpha_0(R)$ we observe a huge change from infinity down to the origin, although remains unnatural, $\alpha_0(R) \gg R$. Numerically we confirm our theoretical expectation that $\alpha_{S,0}(0) = 0$ (see Eq. (16)). This simply means that the bare contact interaction becomes arbitrarily small as the OPE potential is switched off. This is, however, not the case for the bare derivative interaction, as expected from our estimate, Eq. (16). Our numerical values extrapolated to the origin are

$$\alpha_{S,0} = \alpha_0(0^+) = 0, \qquad r_{S,0} = r_0(0^+) = 4.04 \text{ fm},$$
$$v_{S,2} = v_2(0^+) = 1.07 \text{ fm}^3. \tag{19}$$

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[3] In practice results are insensitive for long distance cut-off of $R_\infty = 20$ fm. In the case of the short distance cut-off we can go down to $R_S = 0.0001$ fm without much effort but results are fairly insensitive to the short distance radius already at $R_S = 0.1$ fm, where we have $\alpha_{S,0} = -0.9865$ fm, $r_{0,S} = 3.780$ fm and $v_{2,S} =$

0.994 fm$^3$. For shorter distances Eq. (16) provides an accurate estimate for $\alpha_0(R)$. Taking larger values of $R_S$ builds in finite cut-off effects. Actually $R_S \gg 1/m_\pi$ corresponds exactly to effective range expansion.

This is the initial condition which, in principle, has to be supplemented in Eqs. (11)–(13) in order to get the experimental results (see also discussion below). The work of Ref. [27] uses a two Yukawa model to extract the short-distance low-energy parameters. This is done by fitting the data and then switching off the OPE contribution, yielding $\alpha_{S,0} = -1.72$ fm, $r_{S,0} = 1.60$ fm and $v_{S,2} = -0.024$ fm$^3$. In Ref. [12] an attempt to determine the short-distance parameters based on the three Yukawa model yields $\alpha_{S,0} = -3.38$ fm, $r_{S,0} = 2.60$ fm and $v_{S,2} = 0.313$ fm$^3$. The short-distance scales in that calculation are $R_\sigma = 2/m_\sigma = 0.80$ fm and $R_\rho = 2/m_\rho = 0.46$ fm. For that range we get $\alpha_{S,0} = -3.6, -2.21$ fm, $r_{S,0} = 2.7, 3.1$ fm and $v_{S,2} = 0.59, 0.74$ fm$^3$, respectively, in qualitative agreement with Refs. [12,27]. Note, however, that our way of determining the short-distance low-energy parameters does not require any specific model at short distances.

## 4. $^1S_0$-phase shift

Once the short distance parameters are known one may compute the phase shifts to any order of the approximation in a $k^2$ expansion of the initial condition *without any additional parameter fitting* by integrating Eq. (9) upwards with a suitable initial condition at a short distance initial value radius, $R = R_S$,

$$K_S(k) = k \cot \delta_S(k)$$
$$= -\frac{1}{\alpha_{S,0}} + \frac{1}{2}r_{0,S}k^2 + v_{2,S}k^4 + \cdots. \quad (20)$$

The standard way of proceeding is to determine the low-energy constants or equivalently the short distance parameters directly from a fit to the data in a given energy window and then recompute the threshold parameters. This builds in some systematic error, unless the energy window is small enough as to make this uncertainty comparable to the experimental error. An advantage of avoiding a fit is that one can prevent spurious and/or multiple minima; our solution is essentially unique. Moreover, since by construction at a given order in the $k^2$ expansion the low-energy behavior of the phase shift is reproduced up to the same order in $k^2$, the possibility of getting even slightly different threshold parameters due to a fit in

the intermediate energy region is precluded. Actually, our procedure would coincide with the standard one, if the fit was carried out in the region where an effective range expansion holds ($k < 60$ MeV if $v_2$ is included).

Due to the fact that the origin is a fixed point for the running scattering length, i.e., $\alpha_0(R) \to 0$ for $R \to 0$ regardless of the value of $\alpha_0 = \alpha_0(R = \infty)$, Eq. (16), one must integrate the equations from very small distances upwards, using the value of $\alpha_0(R)$ at that distance. It is important to realize that a tiny mismatch in the value of $\alpha_0$ close to the origin results in a complete different value of $\alpha_0$ and also of the phase shift at infinity.

In Fig. (2) we show the results for the phase shift depending on the number of terms kept in the low-energy expansion at short distances (LO first term, NLO first two terms and so on in Eq. (20)). Our results exhibit a good convergence rate. For comparison we also depict the effective range expansion results without explicit pions, which is expected to work at low energies only, and corresponds to make $R_S \to \infty$ in our approach. As we see, the effect of introducing pions always improves the results. This can be fully appreciated at NNLO, where ER does a poor job above CM momenta $\sim 100$ MeV, but explicit OPE effects enlarge the energy range up to about $\sim 140$ MeV $\sim m_\pi$ where we expect explicit two pion exchange contributions to start playing a role.

An interesting point to note at this stage is that if $\alpha_{S,0} = 0$ with other short-distance low-energy parameters fixed, we would inevitably get $\delta_S(k) = n\pi$, as deduced, for instance, from Eq. (20). If we solve the variable phase equation with that condition at $R = 0$ up to $R = R_\infty \gg 1/m_\pi$ we get the result (also shown in Fig. (2) for comparison) corresponding to a regular OPE with the regular boundary condition $u_k(0) = 0$ instead of the mixed boundary condition of Eq. (4) at $R = 0$. The puzzle is resolved by realizing that the limiting procedure in the boundary condition and the solution do not commute; the limit $R \to 0^+$ implies $\delta'(k, R) \to \infty$ whereas starting at $R = 0$ requires $\delta(k, R) \sim R^2$ producing instead a bound derivative $\delta'(k, R) \sim R$ (see Eq. (6)). This discontinuous dependence of the boundary condition on the boundary radius at $R = 0$ agrees with rigorous theorems on self-adjoint extensions of Schrödinger operators (see, e.g., Appendix D of Ref. [25]).
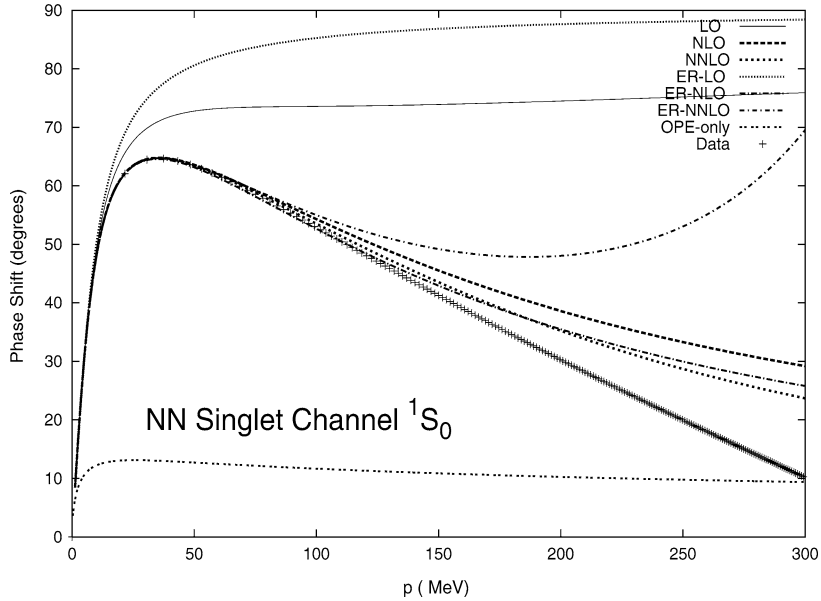
Fig. 2. Predicted phase shifts according to Eq. (9) when OPE potential is switched on and the initial condition is a low-energy expansion of the $K$-matrix at short distances (see Eq. (20) in the main text). LO means keeping $\alpha_{S,0}$ only, NLO keeping $\alpha_{S,0}$ and $r_{0,S}$ and NNLO keeping $\alpha_{S,0}$, $r_{0,S}$ and $v_{2,S}$. The short range parameters are directly determined by evolving the low-energy parameters from their experimental values $\alpha_0 = -23.73$ fm and $r_0 = 2.68$ fm and $v_2 = -0.48$ fm$^3$. ER-LO, ER-NLO and ER-NNLO corresponds to a pure effective range expansion keeping $\alpha_0$ only, $\alpha_{S,0}$ and $r_0$, $\alpha_0$, $r_0$ and $v_2$, respectively. OPE-only corresponds to OPE without short-distance contributions. No further fit is involved. Data are the PWA from Ref. [29].

## 5. Conclusions

In the present Letter we have analyzed the renormalization of the OPE interaction in the presence of contact and derivative interactions of any order for NN scattering. In order to do that we have derived an equation for the evolution of an energy dependent boundary condition in coordinate space as a function of the boundary radius. The resulting equation shares many properties with renormalization group equations and can be interpreted in terms of the phase shift produced by eliminating OPE from infinity to the boundary radius, which eventually is taken to zero. Two advantages can be deduced from this framework: no divergences appear and there is no need to consider off-shell extrapolations. This allows to set up equations for the running low-energy parameters as a function of the boundary radius. Using the experimental values for the low-energy parameters, which correspond to an infinity boundary radius, we extract in a unique and model independent way the corresponding short-distance parameters. Our numerical values agree with

other determinations based on specific models for the short-distance interaction. As we get closer to the origin we find a fixed point structure, triggered by the non-vanishing contribution of the irregular solution. This requires a fine tuning of the short-distance low-energy parameters. After that we integrate the running phase shift upwards and determine without any additional fit the $^1S_0$ phase shift. The OPE plus contact and derivative interactions to NNLO is able to describe the $^1S_0$ phase shift up to C.M. momentum of about 140 MeV, which coincides with the opening of the two pion exchange left cut channel. Above that momentum explicit two pion exchange effects should set in.

As suggested by Weinberg [1], one of the most interesting aspects of the EFT chiral approach to nuclear phenomena concerns the study of reactions such as $\pi d$ scattering, and the possibility of making model independent predictions. An indispensable prerequisite for this, in any EFT scheme, is a good knowledge of NN-interaction. Although nothing prevents from extending our framework for other processes beyond NN-scattering, it remains to be seen whether the approach

presented here can successfully tackle these reactions. The results presented in this Letter are very encouraging and suggest several improvements and extensions still within the NN-sector. Explicit two pion exchange effects are expected to contribute significantly at about 1.5–2 fm, so our results should not be considered realistic below that scale, or equivalently above CM momenta of about 100–150 MeV, as it seems to be the case. In addition, our description should be enlarged to include all partial waves. Work along these lines will be presented elsewhere [28].

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