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# A flexible service rule for the dynamic make-to-stock/make-to-order hybrid production system 

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#### Abstract

A dynamic MTS/MTO hybrid system combines make-to-stock (MTS) and make-to-order (MTO) operations with MTS dedicated machines and hybrid machines which can be switched between both operations flexibly. The two classes, priority and ordinary demands, and the demand increases by discounting the price of MTS product are assumed. For the dynamic MTS/MTO hybrid system, a flexible service rule with demand prioritization and pricing rules is proposed. The operating cost and the MTO queue length are evaluated by Markov analysis. The results of analysis showed that the system performance could be improved by the proposed rule.


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## 1. Introduction

In general, production systems can be categorized into two types: make-to-stock (MTS) and make-to-order (MTO) system [1]. The MTS/MTO hybrid systems, which consist of MTS and MTO operations in common facility, are analyzed and used in many fields: e.g. apparel and confection companies, also semiconductor factory. Then, the MTS/MTO hybrid systems are analyzed by some researchers. Chang et al. [2] and Wu et al. [3] investigated integrated devices and semiconductors manufacturing system as hybrid MTS/MTO production systems, where product scheduling and production control are considered. Peña-Perez and Zipkin [4] and Veatch and Véricourt [5]

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Fig. 1. A dynamic MTS/MTO hybrid production system.


Fig. 2. Time variation of the MTS inventory and shortfall.
investigated a multiproduct system with a single machine that can be switched between productions for each product as a two-class queue; i.e. multiclass single-server queue. Chang and Lu [6] analyzed a hybrid MTS/MTO system as a queuing model, in which demands arrive at a single-station production system that determines which system to follow (MTS or MTO). Zhang et al. [7] analyzed a dynamic MTS/MTO hybrid system as a multi-queueing model. In this system, there are two types of machines: MTS dedicated machine and hybrid machine which can be switched between MTS and MTO operations flexibly. This model, however, sometimes has the problems: such as increasing system operating cost and waiting customers at the MTO shop.

To solve such problems, we utilize the flexible service rule proposed by Liu et al. [8]. Moreover, in general, because the demand from customers varies dependent of a price of the products, we also develop discount model that the price is discounted if the inventory level is high. Then, it is expected to reduce excess inventory and suppress the stock-out. Thus, we propose a flexible service rule introduced both demand prioritization and pricing rules, and show the effectiveness of the system performance for the dynamic MTS/MTO hybrid production system.

## 2. Modeling a dynamic MTS/MTO hybrid system

In the dynamic MTS/MTO hybrid system has two virtual shops: MTS and MTO shops. There are $c$ machines in the system: $c-e$ MTS dedicated machines, and $e$ hybrid machines which can be switched between MTS and MTO operations depending on the situation. The framework of this system and time variations of MTS inventory and shortfall are shown in Fig. 1 and Fig. 2, respectively. We describe some assumptions for this system as follows:

- MTS and MTO demands arrive depending on i.i.d. Poisson process with demand rates $\lambda_{s}$ and $\lambda_{\rho}$, respectively. Moreover, MTS demand is classified into two classes [8]: priority demand with demand rate $\lambda_{s}^{p}$, and ordinary demand with demand rate $\lambda_{s}^{o}$. The ordinary demand is only served with priority rate $\eta$. In addition, as MTS demand depends on the price of MTS product, we assume iso-elastic model $\lambda=a p^{-b}$ ( $a>0, b>0$ are constant values and $p$ is a price of product) [9, 10]. Therefore, the MTS demand $\lambda_{s}$ can be expressed by $\lambda_{s}=\left(\lambda_{s}^{p}+\eta \lambda_{s}^{o}\right) \zeta^{-\gamma}$. Then, note that $\zeta$ is a discount rate, not a price.
- Production times of each machine at MTS and MTO shops are exponential distributed with production rate $\mu_{\mathrm{s}}$ and $\mu_{0}$, respectively. In the steady state, $c-e$ dedicated machines are not enough to satisfy the MTS demand, i.e. we assume the condition $(c-e) \mu_{\mathrm{s}}<\lambda_{s}<c \mu_{\mathrm{s}}$.
- We define order-up-to level $S$, safety stock level $s, N=S-s$, and $n=c-e$ (refer to Fig. 2). At time $t$, if MTS inventory level $I(t)=S$, i.e. MTS shortfall level $L(t)=0$, MTS production is stopped. The system operates both hybrid and MTS modes; hybrid mode is switched and switchovered to/from MTS mode if $L(t)=N$ and if $L(t)=n$. In MTS mode, all machines are available for MTS shop. In contrast, in hybrid mode, $c-e$ machines are utilized for MTS shop and others are for MTO shop.
- We assume the service constraint such that backorders for MTS product are accepted although MTS stock-out probability should be less than small enough value $\varepsilon$; that is, $\operatorname{Pr}$ (stock-out) $\leq \varepsilon$.

For the dynamic MTS/MTO hybrid system, we propose a flexible service rule which consists of demand prioritization rule (DPR) and pricing rule (PR). In the DPR, MTS ordinary demand served with priority rate $\eta$ as
assumed above. The following rule for the priority rate based on MTS shortfall $L(t)=k$ is proposed in this paper. The proposed DPR rule:

$$
\eta_{k}=\left\{\begin{array}{cl}
1 & (k \leq N-1)  \tag{1}\\
1-\frac{1-\eta}{s}(k-N) & (N \leq k \leq N+s) \\
\eta & (k \geq N+s)
\end{array}\right.
$$

On the other hand, as the PR, we propose the following rule for a discount rate because MTS demand is dependent of the price of MTS product.

The proposed PR rule:

$$
\zeta_{k}= \begin{cases}\zeta & (0 \leq k \leq m)  \tag{2}\\ 1 & (k \geq m+1)\end{cases}
$$

The flexible service rule combining DPR with PR depends on the value of the decision variables $\Lambda=\{c, e, N, s, \eta$, $\zeta, m\}$. If it is determined, we can evaluate the system performance.

## 3. Analysis of the dynamic MTS/MTO hybrid system

### 3.1. MTS system

As the relationship between MTS inventory and shortfall at time $t: I(t)+L(t)=S$, is completed, the switching rule can be expressed by MTS shortfall instead of MTS inventory, and the MTS shortfall process with the switching process is independent on the MTO process. Then, the MTS shortfall process is identical to the congested staffqueuing model in Zhang [11]. The interarrival time of demand and production time for the item follows exponential distributions. Thus, the Markov process can be formulated by using the definition of a bi-dimensional state $\{L(t)$, $J(t)\}$, where $L(t)$ signifies MTS shortfall level and $J(t)$ indicates the hybrid or MTS modes if $J(t)=0$ or 1 , at time $t$. Then, the state space in the MTS system can be expressed by

$$
\begin{equation*}
\mathbf{S}=\{(k, 0) \mid 0 \leq k \leq c-e\} \cup\{(k, j) \mid c-e+1 \leq k \leq N-1, j=0,1\} \cup\{(k, 1) \mid k \geq N\} \tag{3}
\end{equation*}
$$

Moreover, if $\rho=\lambda_{s}\left(c \mu_{\mathrm{s}}\right)^{-1}<1$, the steady-state probability is defined by $\pi_{k, j}=\lim _{t \rightarrow \infty} \operatorname{Pr}\{L(t)=k, J(t)=j\}$ where $(k$, $j) \in \mathbf{S}$. Then, flow balance equations are developed referring to Zhang [11]. Consequently, by solving these equations, $\pi_{k, j}$ is calculated and the expected MTS shortfall level is obtained: $E(L)=\sum_{(k, j)} \in \mathbf{s} k \pi_{k, j}$. Moreover, representing duration of hybrid and MTS modes as $T_{A}$ and $T_{B}$, respectively, the cycle time $C=T_{A}+T_{B}$ and the expected cycle time $E(C)=E\left(T_{A}\right)+E\left(T_{B}\right)$ are acquired. Thus, we can evaluate the system operating cost $g(\Lambda)$ as follows:

$$
\begin{equation*}
g(\Lambda)=g_{I}+g_{S}+g_{R}+g_{P}=K_{I}(S-E(L))+\frac{K_{S}}{E(C)}+K_{R} \sum_{k=N}^{\infty}\left(1-\eta_{k}\right) \lambda_{s}^{o} \pi_{k, 1}+K_{P} \sum_{k=0}^{m}\left(1-\zeta_{k}\right) \lambda_{s} \pi_{k, 0} \tag{4}
\end{equation*}
$$

where $g_{I}, g_{S}, g_{R}$, and $g_{P}$ are MTS inventory holding cost, switching cost, rejection cost of MTS ordinary demand, and penalty of discounting the price, respectively. Also, $K_{I}, K_{S}, K_{R}$, and $K_{P}$ are cost coefficients of each cost, respectively.

### 3.2. MTO system

This system can be analyzed as an $M / M / e$ vacation queueing model [12]. The feasible/infeasible time of MTO production, $T_{A}$ and $T_{B}$, follow the phase-type distributions with representations $(\boldsymbol{\alpha}, \mathbf{G})$ and $(\boldsymbol{\beta}, \mathbf{H})$, respectively. Here, $\boldsymbol{\alpha}$ is $N$-dimensional zeros row vector with 1 as the $(c-e)$ th element, and $\boldsymbol{\beta}$ is $N^{\prime}$-dimensional zeros row vector with 1 as the $(M+1)$ th element (Note that $N^{\prime}=M+N-c+e$, and $M$ is a large number representing the maximum amount of MTS shortfall above $N$ ). Further, $\mathbf{G}$ and $\mathbf{H}$ are $N \times N$ and $N^{\prime} \times N^{\prime}$ dimensional matrices, respectively, as follows:


Fig. 3. Transition diagram of the MTO system.

$$
\begin{align*}
& \mathbf{G}=\left(\begin{array}{cccccccc}
-\lambda_{s} & \lambda_{s} & & & & & & \\
\mu_{s} & -\left(\mu_{s}+\lambda_{s}\right) & \lambda_{s} & & & & & \\
& 2 \mu_{s} & -\left(2 \mu_{s}+\lambda_{s}\right) & \lambda_{s} & & \ddots & & \\
& & \ddots & \ddots & & \ddots & \\
& & & (c-e) \mu_{s} & -\left((c-e) \mu_{s}+\lambda_{s}\right) & \lambda_{s} & \\
& & & \ddots & \ddots & \ddots & \\
& & & & & & (c-e) \mu_{s} & -\left((c-e) \mu_{s}+\lambda_{s}\right)
\end{array}\right)  \tag{5}\\
& \mathbf{H}=\left(\begin{array}{cccccccc}
-c \mu_{s} & c \mu_{s} & & & & & & \\
\lambda_{s} & -\left(c \mu_{s}+\lambda_{s}\right) & c \mu_{s} & & & & & \\
& \lambda_{s} & -\left(c \mu_{s}+\lambda_{s}\right) & c \mu_{s} & & & \\
& & & \ddots & \ddots & \ddots & & \\
& & & & \lambda_{s} & -\left((c-1) \mu_{s}+\lambda_{s}\right) & (c-1) \mu_{s} & \\
& & & & \ddots & \ddots & \ddots & \\
& & & & & & \lambda_{s} & -\left((c-e+1) \mu_{s}+\lambda_{s}\right)
\end{array}\right) \tag{6}
\end{align*}
$$

Then, the expected durations of hybrid and MTS modes are calculated by $E\left(T_{A}\right)=-\boldsymbol{\alpha} \mathbf{G}^{-1} \boldsymbol{e}$ and $E\left(T_{A}\right)=-\boldsymbol{\beta} \mathbf{H}^{-1} \boldsymbol{e}$, respectively, where $\boldsymbol{e}$ is ones column vector. Let $\mathbf{I}$ be an identity matrix and assuming steady state probability vector with $k$ waiting customers at MTO shop is $\boldsymbol{p}_{k}=\boldsymbol{p}_{0} \mathbf{R}^{k}(k \geq 0)$ with rate matrix $\mathbf{R}$, the average MTO queue length $E\left(Q_{o}\right)$ can be obtained by solving flow balance equations derived from the transition diagram shown in Fig. 3 and normalization condition $\sum_{k=0}^{\infty} \boldsymbol{p}_{k} \boldsymbol{e}=1$, as follows:

$$
\begin{equation*}
E\left(Q_{o}\right)=\boldsymbol{p}_{0}(\mathbf{I}-\mathbf{R})^{-2} \boldsymbol{e} \tag{7}
\end{equation*}
$$

## 4. Numerical experiments

The reduction effects of the proposed flexible service rule are investigated by numerical experiments. Fig. 4


Fig. 4. Relationships between system operating cost and average MTO queue length for (a) $\beta=0.1$, (b) $\beta=1.0$, (c) $\beta=2.5$.
shows the results of numerical experiments for $\beta=0.1,1.0$, and 2.5 . Other parameters and decision variables were then set as $\lambda_{s}^{p}=8, \lambda_{s}^{p}=25, \lambda_{o}=3, \mu_{s}=4, \mu_{o}=2, K_{I}=1.0, K_{S}=75, K_{R}=30, K_{P}=10, c=11, e=4, N=\{25,30, \ldots$, $70\}, \eta=\{0.0,0.1, \ldots, 1.0\}, \zeta=\{0.5,0.6, \ldots, 1.0\}, m=8$ and $s$ is the minimal safety stock level which satisfies the system constraint $\operatorname{Pr}($ stock-out $) \leq \varepsilon[7]$.

In Fig. 4, it can be claimed that all the proposed rules improve the system performance. That is $g$ is reduced for large $E\left(Q_{o}\right)$ under only PR and any $E\left(Q_{o}\right)$ under only DPR. Further, the combination of them (DPR-PR) can make the system performance much better than either DPR or PR and previous policy. Regarding $E\left(Q_{o}\right)$, DPR-PR show better performance than the previous policy because hybrid machines have been utilized in MTO shop for much longer term per the cycle time itself although cycle time becomes shorter by applying a flexible service rule.

As $\beta$ increases, DPR-PR shows much better system performance than other rules. Under $\beta=2.5$, effectiveness of DPR-PR is slightly greater than that of PR. As a result, DPR-PR is effective in entire domain with respect to $\Lambda$ and large $\beta$. Therefore, it can be claimed that the proposed policy, especially the combination of the demand prioritization and pricing rules, improves the performance in a dynamic MTS/MTO hybrid system.

## 5. Conclusions and future research

In this paper, we proposed a flexible service rule that consists of demand prioritization rule (DPR) and pricing rule (PR) for the dynamic MTS/MTO hybrid production system. Numerical experiments show that either DPR or PR could improve only a little, in contrast, the combination of them could reduce system operating cost and average MTO queue length extremely and comprehensively. Thus, we can conclude that the proposed flexible service rule has good effectiveness for the dynamic MTS/MTO hybrid production system.

We are considering some issues of the dynamic MTS/MTO hybrid system for future research. We intend to enlarge a flexible service rule. Furthermore, we can improve the system performance by developing a switching policy which depends not only the state of MTS shop but also that of MTO shop.

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