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Research on the Critical Value of Traffic Congestion Propagation Based on Coordination Game

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Abstract

In order to analysis the process of congestion diffusion in traffic network and obtain the critical value of traffic congestion, this paper presented a coordination game model for traffic congestion diffusion. Through the coordination game between network individuals and the decision-making behavior of travelers in traffic congestion, this paper described the process of congestion diffusion in traffic network. And according to the process of pass through the behavior of adjacent nodes in network, built a traffic network congestion diffusion model, derived the critical condition of traffic congestion diffusion with the method of probability generation function. Finally, this paper built a simulation model to analysis the influence on the traffic congestion imposed by the road network structure and the distribution of node degree. When the results of simulation is same with the results of analytical analysis, and it can reflect the dynamic information of traffic congestion, then it is found that the influence of the congested node on other adjacent nodes is the key point to find the critical condition of traffic congestion diffusion, when the influence reaches to a certain extent, a massive traffic congestion will be generated.

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Keywords: congestion diffusion; coordination game; cascade; probability generation function; critical value of propagation

1. Introduction

With the improvement of people's living standard, the demand of traffic has been increased. Backward transportation system cannot meet people's needs; the problem of traffic congestion has become more and more serious. Traffic congestion is caused when traffic demand exceeds the available capacity of the traffic system. A part

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of road congestion can lead to the congestion spread the whole traffic network rapidly and affect other roadways. Such kind of continuous fault event caused the paralysis of the whole urban traffic network. It is the process of phase transition for the network from the normal state to the congestion state [1,2], the key point to control traffic congestion is to find the critical value between the normal state and the congestion state. [3,4] has studied the critical value of load in scale-free network. Lobos [5]has studied the critical value of congestion delay time and node self-healing time in the network. Few scholars study the critical value of traffic congestion from the prospect of strategic interaction between individuals in traffic network. This paper built a coordination game model of traffic congestion, and studies the critical value of traffic congestion with the method of probability generation function, then some control strategies is proposed at the end of paper.

2. Research on the Critical Value of Traffic Congestion Propagation Based on Coordination Game

2.1 Propagation Model of Traffic Congestion

In the state of relative saturated traffic flow, a congested node will have some potential effect on other nodes in traffic network; these nodes which obtained the congestion information will take different decisions. Assuming after node v_v come into the state of congestion, the adjacent nodes get this information and other nodes won't influence by this node, analysis the behavior of the adjacent nodes.

As shown in figure 1(a), node v_v next to node v_a、 node v_b and node v_c, there are 6 kinds of traffic flow traffic flow: $l(av)$ from node v_a to node v_v; traffic flow $l(va)$ from node v_v to node v_a; traffic flow $l(bv)$ from node v_b to node v_v; traffic flow $l(vb)$ from node v_v to v_b; traffic flow $l(cv)$ from node v_c to v_v; traffic flow $l(vc)$ from node v_v to v_c. After one node under the state of congestion, the adjacent nodes have known the information, traffic flow $l(va)$, $l(vb)$ and $l(vc)$ from node v_v won't be influenced, and traffic flow $l(av)$, $l(bv)$ and $l(cv)$ from the adjacent nodes will flow to their own adjacent nodes to find a new way instead of flow to the node v_v. As shown in figure 1(b), traffic flow $l(av)$ has become traffic flow $l(av)_1$ and traffic flow $l(av)_2$, and

$$l(av) = l(av)_1 + l(av)_2 \tag{1}$$

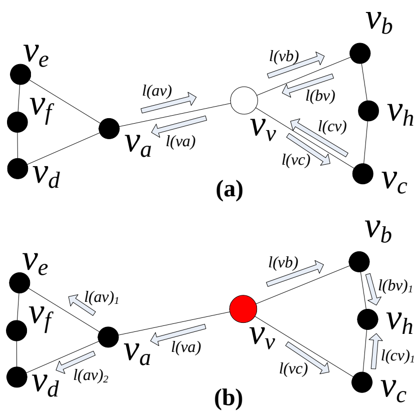

Fig. 1. Changes of node flow.

To the adjacent node v_i of any congested node, after influenced by this node, the traffic flow $l(iv)$ which flow to the congested node v_v will flow out to other $m-1$ of normal nodes, it can be described as:

$$l(iv) = l(iv)_1 + l(iv)_2 \cdots + l(iv)_{m-1} \tag{2}$$

Build a model to describe the changes of flow [7]. In order to solve the model effectively, network coordination game model is introduced into the model: In one traffic network, each node has two different states, C (C represents the node is in state of congestion) and N (N represents the node is in state of normal). $Inf1$ is the influence of node v_i that connects to the congested node, $Inf2$ is the influence of this node that connects to the normal node, it can be described as:

$$v(Influence)_i = \begin{cases} Inf1 \text{ connect with congested node} \\ Inf2 \quad \text{connect with normal node} \end{cases} \tag{3}$$

Analysis all of adjacent nodes of any node v, some of them may in state of congestion, the rest of them are in normal state, then the node v will influenced by all of congested nodes and normal nodes. Describe this situation with a coordination game [8], node v and node w interact each other, A and B is the situation that may occur. And the corresponding returns are as follows:

- If node w is in state of A, then the return $a > 0$ when node v choose A, the return $a = 0$ when choose B.
- If node w is in state of B, then the return $b > 0$ when node v choose B, the return $b = 0$ when choose A.

The problem that node v has to face is: if some of his adjacent nodes choose A, and some adjacent nodes choose B, then the decision of node v is depend on the relative number of each decisions and the corresponding returns of a and b. Assuming in all of adjacent nodes of node v, a ratio of p adjacent nodes chooses A, a ratio of $Inf2 = 2$ adjacent nodes chooses B, that is to say if node v have d of adjacent nodes, then pd of nodes choose A, $(1-p)d$ of nodes choose B as shown in figure 2. Therefore the return is pda when node v chooses A; the return is $(1-p)db$ when choosing B. Then, the decision of node v can meet the following equation:

$$\begin{cases} pda \geq (1-p)db \quad \text{choose A} \\ pda < (1-p)db \quad \text{choose B} \end{cases} \tag{4}$$

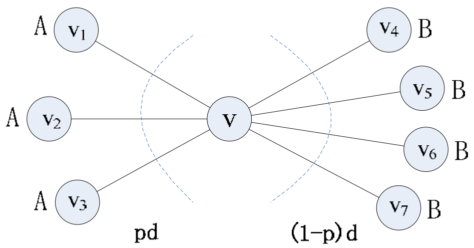

Fig. 2. Node v makes decision depend on the decision of adjacent nodes.

From equation (4), it is easy to know that if at least a ratio of $q = b/(a+b)$ adjacent nodes choose A, then node v will choose A too. For the network, choose A is more attractive when q value is low. So, only if a small part of adjacent nodes are intend to choose A, almost all of nodes in network will choose A too. And then the transformation model between local game and overall network is established. In the process of congestion diffusion, if the congested node have enough influence ($Inf1$) on its adjacent nodes, it will cause cascade congestion in most nodes of the network.

2.2 Propagation Process of Traffic Congestion

There are two distinct equilibrium of the coordination game in propagation of traffic congestion: the first one is all of nodes are come into the state of congestion, another one is all of nodes are in the state of normal. Assuming there is an initial set of nodes is in state of congestion and the rest of nodes are normal nodes. As time goes forward, each node play game according to the critical value equation, and decide whether to convert between congestion state and normal state until each node are in state of congestion or reach to a state that all of nodes are no longer changes.

Describe the process though traffic network in figure 3.

- Assuming the structure of network is as shown in figure 3(a), set $Inf_1 = 4$, $Inf_2 = 3$ in coordination game, which means the influence of congestion state is 4/3 times of the influence of normal state. According to the critical value equation, when a node at least has a ratio of $q = 3/(3+4) = 3/7$ adjacent nodes is in state of congestion, the node will come into congestion state from normal state.

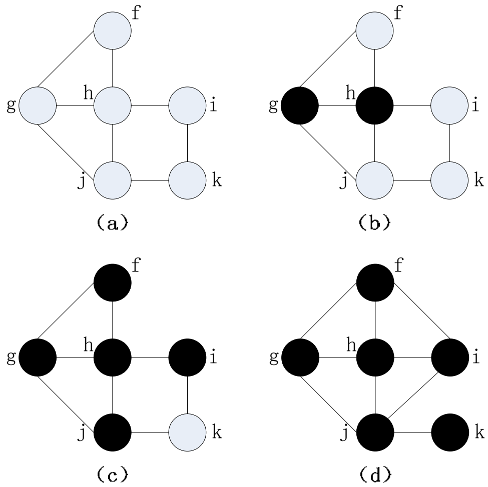

Fig. 3. Propagation of cascade congestion.

- Assuming node g and node h are the initial congested nodes in the network, the rest of nodes are normal node as shown in figure 3(bold line round represents the congested nodes, thin line round represents the normal nodes). Next, each node will play game according to the critical value equation, node r , i and j will convert into the state of congestion, three of them have a ratio of $1 > 3/7$ 、 $1/2 > 3/7$ and $2/3 > 3/7$ adjacent nodes are in the state of congestion respectively. Node k won't have any change, because for him, only a ratio of 1/3<2/5 adjacent nodes are in state of congestion.
- Next, owing to node k have a ratio of $1 > 3/7$ adjacent nodes are in state of congestion; neither he will come into the congestion state. The whole process will end with all of nodes are come into congestion state.

This is a typical cascade, node $h_1(s)$ and node $h_1(s)$ cannot make node k convert into the state of congestion, but once they converted node i and node $h_1(s)$, and then they will have enough influence on node k , the whole process will come to end when all of nodes are no longer changes. What kind of situation will make all of nodes come into the state of congestion at the end in general traffic network? Obviously, the critical value in traffic congestion is the key point to control the whole traffic, and this critical value will be analyzed at the next part.

3. Analyses on the Critical Value of Traffic Congestion Based on the Coordination Game

Initial congestion in traffic network may happen in any node. As shown in figure 4, node v_v is a congested node; the degree is the key feature of the node v_v . $p(k)$ represents the distribution of degree in network node. In traffic network, $g_0(s)$, the probability generating function [9] of a random congested node generate child congestion edges, can be represents as:

$$g_0(s) = \sum_{k=0}^{\infty} p(k)s^k \quad (5)$$

p_2 is a another random variable that represents the probability of an edge connect to the node which degree is $d = k$, it can be represented as:

$$p_2 = \frac{kp(k)}{\langle k \rangle} \quad (6)$$

The probability generating function of this random variable is:

$$G_e(s) = \sum_k p_2 s^k = \frac{\sum_k kp(k)s^k}{\langle k \rangle} = \frac{sG_0'(s)}{G_0'(1)} \quad (7)$$

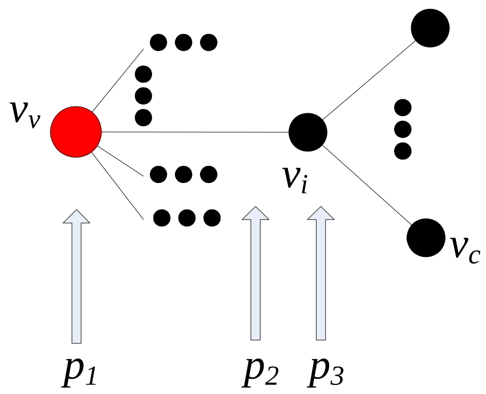

Fig. 4. Probability of a random node generate cascade congestion in traffic network .

In traffic network, one node come into congestion randomly, $p_1 = p(k)$ is the probability of the node which degree is $d = k$, k of congestion edges were generated after the node congested which the influence is Inf_1 , one congestion edges may cause other child congestion edges generated, the probability of generate child congestion edges is $p'(k) = p_2 p_3$. p_2 represents the probability of the congestion edge which the influence is Inf_1 connect to the next node which the remaining degree [10] is $d = k$ (the degree is $k+1$). p_3 represents the probability of the congestion edge that cause the connected node come into congestion state, if this node is in state of congestion, another new round of congestion propagation will be generated as shown in figure 4.

Select a congestion edge randomly which the influence is Inf_1 , the probability generating function of the new congestion edges that generated by this edge can be represented as:

$$g_1(s) = \sum_{k=0}^{\infty} p'(k)s^k = (\sum_{k=0}^{\infty} \frac{kp(k)}{< k >} p_3 s^k)/s \quad (8)$$

Analyze the problem of traffic congestion. $h_1(s)$ represents the probability generating function of the number of child congestion edge that generated by a congestion edge (within multi-cycle), giant component [10] is not included in this situation(giant component means the situation of massive traffic congestion), unless giant component appears in the process of phase transition. Then, analysis $h_1(s)$ in figure5:

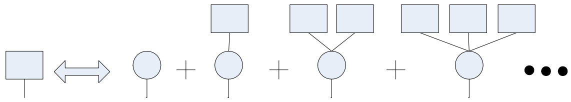

Fig. 5. Decomposition of probability generating function.

The left side of the equals sign represents the probability generating function of a congestion edge generate child congestion edges, the first item on the right side represents the congestion edge connect to a node which degree is 1, zero child congestion edge was generated when it is invalid, and the second item represents congestion edge connect to a node which degree is 2, 1 child congestion edges was generated when it is invalid etc. Then $h_1(s)$ can be described as:

$$\begin{aligned} h_1(s) &= sq(0)+sq(1)h_1(s)+sq(2)[h_1(s)]^2+\cdots \\ &= s\sum_{k=0}^{\infty} q(k)[h_1(s)]^k \end{aligned} \tag{9}$$

$q(k)$ represents the probability that the congestion edge connect to a node which the remaining degree is k, compare equation 8 and equation 9:

$$h_1(s) = s \times g_1(h_1(s)) \tag{10}$$

In the same way, $h_0(s)$, the probability generating function of a random congested node generate child congestion edges, can be described as:

$$h_0(s) = s \times g_0(h_1(s)) \tag{11}$$

Therefore, $<size>$, the average number of a random congested node generate child congestion edges, can be expressed as:

$$<size>= h_0'(1)=1+g_0'(1)h_1'(1) \tag{12}$$

Obtained from equation (10):

$$h_1'(1)=1+g_1'(1)h_1'(1) \tag{13}$$

Take equation (13) into equation (12):

$$<size> =1+\frac{g_0'(1)}{1-g_1'(1)} \tag{14}$$

It is clear that $<size>$ have phase transition [10] while $g_1'(1)=1$. When $g_1'(1)>1$, massive traffic congestion will be generated. That is to say, once a node congested randomly, the average number of child congestion edges is more than 1, and then it will cause massive traffic congestion. Therefore, to a random congested node, $g_1'(1)=1$ is the critical point of massive traffic congestion.

4. Case Study

One traffic network with the attribute of uniform distribution, the max degree $d_{max} = 7$, the impact factor of the congestion state and the normal state are meeting as followed:

$$\begin{cases} Inf_1 = \alpha \\ Inf_2 = 1 \end{cases} \tag{15}$$

α represents the relative influence value of the congested node to other nodes, $\alpha = Inf_1 / Inf_2$, so, $\alpha > 1$. $\alpha > d_{max}$ (d_{max} represents the max degree of the node in network), and according to the former traffic congestion model, all of nodes will come into the state of congestion. Therefore, $1 \le \alpha \le d_{max}$. Set $1 < k' \le d_{max}$, $g_1(s)$ can be represents as:

$$\begin{aligned} g_1(s) &= (\sum_{k=0}^{k'} \frac{kp(k)}{<k>} s^k)/s \\ &= \frac{2}{d_{max}(d_{max}+1)} \sum_{k=1}^{k'} ks^{k-1} \quad k'-1 < \alpha \le k' \end{aligned} \tag{16}$$

$g_1'(1)$, the number of child congestion edge generated by a congestion edge, can be represented as:

$$\begin{aligned} g_1'(1) &= \frac{2}{d_{max}(d_{max}+1)} \sum_{k=0}^{k'} k(k-1)s^{k-2} |_{s=1} \\ &= \frac{2}{d_{max}(d_{max}+1)} \sum_{k=0}^{k'} k(k-1) \quad k'-1 < \alpha < k' \end{aligned} \tag{17}$$

Set $f(\alpha) = g_1'(1)$, $1 \le \alpha \le d_{max}$ as the function of α, it's easy to find that $f(\alpha)$ is an increasing function, and $f(1) = 0$, $f(d_{max}) > 1$. According to the nature of increasing function, $f^-(\alpha^*) < 1$, $f^+(\alpha^*) > 1$, and then α^* is the critical value.

$d_{max} = 7$, according to equation (17), find $f(4) < 1$, $f(5) > 1$, therefore $\alpha^* = 5$. List the analytical results and the simulation results [11] in figure 6 separately.

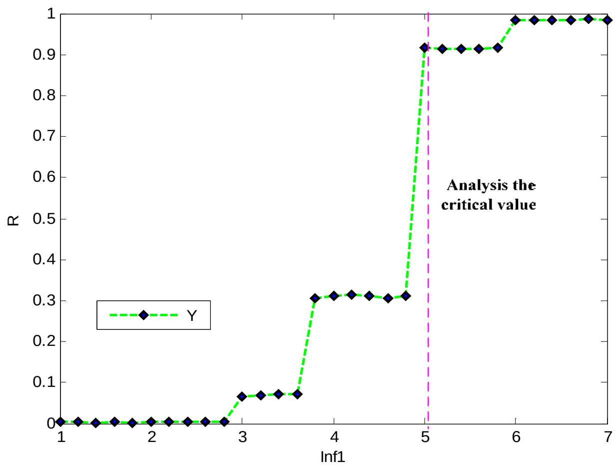

Fig. 6. Simulation results of traffic congestion.

The abscissa Inf_1 represents the influence of congestion edge, the ordinate R represents the ratio of the number of congested nodes n_0 and the network size (that is the total number of nodes N), $R = n_0 / N$ .Curve Y in figure 6 represents the simulation results, and dotted line represents the analytical results which obtained in the process of analysis critical value. The reason that simulation result is not coincide with analytical results are as followed： On the one hand, assume not form a ring structure is not realistic. On the other hand, both of them have some errors, analytical method used uniform random network as project, and simulation method used uniform random network as project.

In figure 6, it is easy to find that: 1) In traffic network, a random traffic congestion can cause the whole traffic network come into the state of congestion. 2) As the relative influence of random traffic congestion increased (Inf_1 represents the ratio between the influence of congested traffic and the influence of normal traffic), the scale of traffic congestion continue to expand (which means R value continue to increase). 3) When the influences of random traffic congestion have reached the critical point, the entire transportation system will come into the state of massive traffic congestion.

5 Conclusions

A model of traffic congestion based on the network coordinate game is built in this paper, Inf_1 represents the relative impact of traffic congested node on its adjacent nodes. Probability generation function method is introduced to analysis the functional relationship between local traffic congestion and massive traffic congestion, the critical value of Inf_1 is derived though case study, correctness of the derivation results is also verified by using simulation method, and then some appropriate control strategies can be developed according to these conclusion. When Inf_1 value is low, let the traffic congestion diffuse is fine, because the network can neutralize the impact of traffic congestion; when Inf_1 value is high, some appropriate measures must be taken to control the diffusion of local traffic congestion, and then large-scale traffic congestion won't be happened.

Acknowledgements

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