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On some of Jerzy Baksalary's contributions to the theory of block designs☆

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Abstract

A review of some results obtained by Jerzy Baksalary with regard to the theory of block designs is presented. Particular attention is drawn to his results concerning various concepts of balance, some methods of constructing block designs, the connectedness of PBIB designs, conditions for a kind of robustness of block designs, and certain criteria concerning Fisher's condition for block designs. The importance of his results is stressed. References to other relevant works in this field are also made. There is no doubt that Baksalary's contributions to experimental design are important both from a theoretical and a practical point of view.

Keywords: Block design; Connectedness; Orthogonality; Efficiency balance; Variance balance; Combinatorial balance; BIB design; PBIB design; Robustness; Fisher's condition

0. Introduction and preliminaries

Jerzy Baksalary became interested in the theory of block designs in the late 70s, when the Poznań school of mathematical statistics and biometry was already quite

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advanced in this field. He was trying to investigate the mathematical background of the various concepts related to the theory of experimental designs, particularly of block designs, a subject of intensive study in Poznań at that time.

It will be helpful first to recall that any block design can be described by its $v \times b$ incidence matrix $\boldsymbol{N} = [n_{ij}]$, with a row for each treatment and a column for each block, where n_{ij} is the number of experimental units in the jth block receiving the ith treatment ($i = 1, 2, \ldots, v$; $j = 1, 2, \ldots, b$). This matrix, together with the vector of block sizes, $\boldsymbol{k} = [k_1, k_2, \ldots, k_b]' = \boldsymbol{N}'\mathbf{1}_v$, the vector of treatment replications, $\boldsymbol{r} = [r_1, r_2, \ldots, r_v]' = \boldsymbol{N}\mathbf{1}_b$, and the total number of units ($n = \mathbf{1}_b'\boldsymbol{k} = \mathbf{1}_v'\boldsymbol{r} = \mathbf{1}_v'\boldsymbol{N}\mathbf{1}_b$, where $\mathbf{1}_a$ is an $a \times 1$ vector of 1's) is used in defining various matrices that help us to understand the statistical properties of the design. In particular, an important role in studying these properties is played by the $v \times v$ matrix

$$\boldsymbol{C} = \boldsymbol{r}^{\delta} - \boldsymbol{N}\boldsymbol{k}^{-\delta}\boldsymbol{N}',$$

where $\boldsymbol{r}^{\delta} = \mathrm{diag}[r_1, r_2, \ldots, r_v]$, $\boldsymbol{k}^{\delta} = \mathrm{diag}[k_1, k_2, \ldots, k_b]$ and $\boldsymbol{k}^{-\delta} = (\boldsymbol{k}^{\delta})^{-1}$. On it, the so-called intra-block analysis of the experimental data is based (see [14, Section 3.2.1]). The interest of Baksalary was at that time confined to this type of analysis.

1. Concepts of balance

In one of his earliest papers in this field [3], the concept of balance of a block design is considered. Two notions of balance are defined there, for connected and disconnected block designs. But first it is noted that the rank of $\boldsymbol{C}$ is strictly related to the concept of connectedness.

Definition 1 [3]. A block design is said to be connected if $\mathrm{rank}(\boldsymbol{C}) = v - 1$, and is said to be disconnected of degree $g - 1$, $g \geqslant 2$, if $\mathrm{rank}(\boldsymbol{C}) = v - g$.

Definition 2 [3]. A connected (disconnected of degree $g - 1$) block design is said to be V-balanced if all the nonzero eigenvalues of its matrix $\boldsymbol{C}$, $v - 1$ ($v - g$) in number, are equal.

Definition 3 [3]. A connected (disconnected of degree $g - 1$) block design is said to be J-balanced if all the nonzero eigenvalues of its matrix $\boldsymbol{C}$ with respect to the matrix $\boldsymbol{r}^{\delta}$, $v - 1$ ($v - g$) in number, are equal.

The notion of V-balance can be traced back to [33]. Now, it is more commonly termed "variance-balance (VB)" (see, e.g., [31, p. 54]). The notion of J-balance goes back to the concept of balance introduced by Jones [20], though implicitly already used by Nair and Rao [26]. Graf-Jaccottet [19] introduced the term J-balanced, or "balanced in the Jones sense". More frequently, this type of balance is called "efficiency-balance (EB)", due to Williams [34] and Puri and Nigam [29,30]. However, it can be shown that the introduction of the terms VB and EB has been to some extent arbitrary (see, e.g., [14, Section 4.1]). An extreme case of J-balance is the orthogonality of a block design.

Definition 4 [3]. A connected (disconnected of degree $g-1$) block design is said to be orthogonal if all the nonzero eigenvalues of its matrix $\boldsymbol{C}$ with respect to the matrix $\boldsymbol{r}^{\delta}$, $v-1$ $(v-g)$ in number, are equal to 1.

See also Corollary 2.3.3 and Remark 2.4.2 in [14]. An equivalent condition is given in the following theorem.

Theorem 1 [3]. *If a block design is orthogonal, then the rank of its incidence matrix* $\boldsymbol{N}$ *is equal to* 1 *when the design is connected, and is equal to* g *when the design is disconnected of degree* $g-1$.

2. Constructional methods

Other characterizations of EB and VB designs are given in [4], as follows.

Lemma 1 [4]. *A block design is connected and EB if and only if, for some positive scalar* p, $\boldsymbol{N}\boldsymbol{k}^{-\delta}\boldsymbol{N}' - p\boldsymbol{r}\boldsymbol{r}'$ *is a diagonal matrix. If this is the case, the efficiency factor of the design equals* $\varepsilon = np$.

Lemma 2 [4]. *A block design is connected and VB if and only if, for some positive scalar* q, $\boldsymbol{N}\boldsymbol{k}^{-\delta}\boldsymbol{N}' - q\mathbf{1}_v\mathbf{1}_v'$ *is a diagonal matrix.*

Note that this way of defining balance is related to the early definitions based on the off-diagonal elements of the matrix $\boldsymbol{N}\boldsymbol{k}^{-\delta}\boldsymbol{N}'$, called "weighted concurrences" by Pearce [28]. Thus, Lemma 2 is equivalent to the concept of total balance (Type T_0) introduced by Pearce [28, Section 4.A] for the case when the weighted concurrences are all equal. On the other hand, Lemma 1 is equivalent to the concept of total balance in the sense of Jones [20], introduced for the case when the weighted concurrences are equally proportional to the products of the relevant treatment replications (see Definitions 2.4.3 and 2.4.5 in [14]).

Using these characterizations of balance, Baksalary et al. [4] gave several theorems useful for constructing connected EB designs (Theorems 1, 4, and 5 in [4]) and connected VB designs (Theorems 2 and 3 in [4]). Of particular interest is a corollary following from their Theorem 4, which can be written as follows.

Corollary 1 [4]. *If* $\boldsymbol{N}_h$, $h = 1, 2, \ldots, a$, *are the incidence matrices of connected EB designs with a common number of treatments and with the replications of treatments* *mutually proportional among the designs, then their juxtaposition (assemblage)* $[N_1 : N_2 : \cdots : N_a]$ *is the incidence matrix of a connected EB design, with its efficiency factor equal to the weighted average of the efficiency factors of the initial designs.*

For some applications of this result, see, e.g., [15, Section 8.2.2]. Further characterizations of connected designs as well as some constructions of these designs are considered in another of Baksalary's papers [11]. In particular, of interest is the following result.

Lemma 3 [11]. *A block design is connected if and only if it is not isomorphic, with respect to permutations of blocks and/or treatments, to a design with the incidence matrix of the form* $\mathrm{diag}[N_1 : N_2 : \cdots : N_g]$, *where* $2 \leqslant g \leqslant v$ *and* $N_\ell,\ \ell = 1, 2, \ldots, g$, *are all incidence matrices of connected block designs.*

This result rephrases Theorem 3.1 of [16]. Evidently, if the design is not connected (in the above sense), it is disconnected of degree $g - 1$ (see Definition 2.2.6a in [14]).

From both the theoretical and practical points of view, connectedness is a desirable property of a block design. In fact, the most frequent block designs used in practice are binary (i.e., with $n_{ij} = 0$ or $n_{ij} = 1$ for every $i = 1, 2, \ldots, v$ and $j = 1, 2, \ldots, b$) and connected designs.

When designing an experiment, the research project and the experimental material available determine the treatment replications and the block sizes, i.e., the vectors $\boldsymbol{r}$ and $\boldsymbol{k}$, of a block design to be used. In [11], three theorems are proved that allow one to construct binary and connected block designs for given $\boldsymbol{r}$ and $\boldsymbol{k}$, starting from a known binary block design, not necessarily connected. The first two theorems show that although disconnected designs are not desirable in general, under certain conditions they can be transformed into connected binary block designs with desired treatment replications and block sizes. The third theorem provides a sequential procedure for transforming a connected binary block design with the minimal number of experimental units into a connected binary block design with desired vectors $\boldsymbol{r}$ and $\boldsymbol{k}$, preserving in each step the property of connectedness.

3. Connectedness of PBIB designs

Another paper written by Baksalary and Tabis [12] concerns the connectedness of partially balanced incomplete block (PBIB) designs. These binary designs are often used when balanced incomplete block (BIB) designs with required treatment replications and block sizes are not available. The properties of a PBIB design are determined by a relevant so-called association scheme with m classes (see, e.g., [31, Chapter 8] and [15, Section 6.0.2]). Usually, the association schemes provide connected PBIB designs, but there may be cases where the connectedness is not preserved. In this paper a theorem is proved which gives a suitable criterion for examining the connectedness of PBIB designs based on various association schemes. Its applicability is shown in the context of the group-divisible m-associate-class PBIB designs introduced by Roy [32].

In such a design there are $v = s_1 s_2 \cdots s_m$ treatments, each denoted by m indices $(i_1, i_2, \ldots, i_m)$, where $i_1 = 1, 2, \ldots, s_1$, $i_2 = 1, 2, \ldots, s_2$, $\ldots$, $i_m = 1, 2, \ldots, s_m$. Two treatments $(i_1, i_2, \ldots, i_m)$ and $(j_1, j_2, \ldots, j_m)$ are the uth associates if only their first $m - u$ indices are the same. They then occur together in exactly λ_u blocks, this number being independent of the particular pair of uth associates chosen, $u = 1, 2, \ldots, m$ (see also [31, Section 8.12.6]). In practice, PBIB designs of this type of association scheme are used mainly for $m = 2$ or $m = 3$. But the established criterion (Corollary 2, below) can be applied for any m, thus extending the previously known results (see [21] and [27]).

Corollary 2 [12]. *A group-divisible m-associate-class PBIB design is connected if and only if* $\lambda_m > 0$ (*where* λ_m *is the number of blocks in which any two treatments being the mth associates occur together*).

This will be illustrated by an example (Example 6.0.7 in [15]). The following incidence matrix shows a group-divisible 3-associate-class PBIB design with parameters $v = b = 8$, $r = k = 4$, $s_1 = s_2 = s_3 = 2$, $\lambda_1 = 2$, $\lambda_2 = 1$, $\lambda_3 = 2$, with eight treatments as (1, 1, 1), (1, 1, 2), (1, 2, 1), (1, 2, 2), (2, 1, 1), (2, 1, 2), (2, 2, 1), (2, 2, 2):

$$\begin{array}{c} (1,1,1) \\ (1,1,2) \\ (1,2,1) \\ (1,2,2) \\ (2,1,1) \\ (2,1,2) \\ (2,2,1) \\ (2,2,2) \end{array} \left[\begin{array}{cccc|cccc} 1 & 1 & 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 & 0 \end{array}\right].$$

Evidently, this design (which is a 2-resolvable design) could well be used for a 2^3 factorial experiment, which would allow the contrast between main effects of one of the factors to be estimated in the intra-block analysis with full efficiency.

4. Robustness of block designs

Another subject of interest studied by Baksalary was related to the robustness of block designs against the unavailability of data. Three sufficient conditions for a block design to be maximally robust have been derived by Baksalary and Tabis [13]. They have used the following definition.

Definition 5 [13]. Let a block design $\mathscr{D}$ be binary and connected, let $r_{[v]}$ denote the smallest treatment replication of $\mathscr{D}$, and let $\mathscr{D}_{\#}$ denote a design obtained from $\mathscr{D}$ by deleting any $r_{[v]} - 1$ blocks. Then $\mathscr{D}$ is said to be maximally robust against the unavailability of data and with respect to the estimability of treatment contrasts if $\mathscr{D}_{\#}$ is connected irrespective of the choice of the blocks deleted.

Their main results are as follows.

Theorem 2 [13]. *Let a block design $\mathscr{D}$ be binary and connected, and let $r_{[1]} \geqslant r_{[2]} \geqslant \cdots \geqslant r_{[v]}$ and $k_{[1]} \geqslant k_{[2]} \geqslant \cdots \geqslant k_{[b]}$ be its treatment replications and block sizes. Then the condition*

$$k_{[r_{[v]}]} + k_{[b]} > v$$

is sufficient for $\mathscr{D}$ to be maximally robust against the unavailability of data and with respect to the estimability of treatment contrasts.

Theorem 3 [13]. *Let a block design $\mathscr{D}$ be binary and connected, and let $r_{[1]} \geqslant r_{[2]} \geqslant \cdots \geqslant r_{[v]}$ and $k_{[1]} \geqslant k_{[2]} \geqslant \cdots \geqslant k_{[b]}$ be its treatment replications and block sizes. Further, let κ_* and λ_* denote the smallest off-diagonal elements of $\boldsymbol{N}\boldsymbol{k}^{-\delta}\boldsymbol{N}'$ and $\boldsymbol{N}\boldsymbol{N}'$, respectively, and let*

$$K = \sum_{j=1}^{r_{[v]}-1} k_{[j]} \quad \text{and} \quad L = \sum_{j=1}^{r_{[v]}-1} k_{[j]}^2.$$

Then each of the conditions

$$\kappa_* > K/[4k_{[b]}(v - k_{[b]})]$$

and

$$\lambda_* > L/[4k_{[b]}(v - k_{[b]})]$$

is sufficient for $\mathscr{D}$ to be maximally robust against the unavailability of data and with respect to the estimability of treatment contrasts.

An immediate consequence of Theorem 3 is the following result.

Corollary 3 [13]. *Let a block design $\mathscr{D}$ be binary and connected, let $r_{[1]} \geqslant r_{[2]} \geqslant \cdots \geqslant r_{[v]}$ and $k_{[1]} \geqslant k_{[2]} \geqslant \cdots \geqslant k_{[b]}$ be its treatment replications and block sizes, and let K be as defined in Theorem 3. If $\mathscr{D}$ is VB and*

$$\frac{n-b}{v(v-1)} > \frac{K}{4k_{[b]}(v - k_{[b]})},$$

or if $\mathscr{D}$ is EB and

$$\frac{(n-b)r_{[v-1]}r_{[v]}}{n^2 - \boldsymbol{r}'\boldsymbol{r}} > \frac{K}{4k_{[b]}(v - k_{[b]})},$$

then $\mathscr{D}$ is maximally robust against the unavailability of data and with respect to the estimability of treatment contrasts.

This is due to the fact that a connected and binary block design is VB if and only if

$$\boldsymbol{C} = \boldsymbol{r}^{\delta} - \boldsymbol{N}\boldsymbol{k}^{-\delta}\boldsymbol{N}' = \frac{n-b}{v-1}\left(\boldsymbol{I}_v - v^{-1}\mathbf{1}_v\mathbf{1}_v'\right)$$

and is EB if and only if

$$\boldsymbol{C} = \boldsymbol{r}^{\delta} - \boldsymbol{N}\boldsymbol{k}^{-\delta}\boldsymbol{N}' = \frac{n(n-b)}{n^2 - \boldsymbol{r}'\boldsymbol{r}}\left(\boldsymbol{r}^{\delta} - n^{-1}\boldsymbol{r}\boldsymbol{r}'\right)$$

(see, e.g., [14, Section 2.4]).

Another consequence of Theorem 3 is the following result originally given by Ghosh [18].

Corollary 4 [13]. *Every BIB design is maximally robust against the unavailability of data and with respect to the estimability of treatment contrasts.*

Further results on this topic are given by Kageyama and Saha [23], Kageyama [22], Baksalary and Puri [8], and Baksalary and Hauke [6]. For other references see [15, Section 10.2].

5. Fisher's condition

Attention should also be paid to an interesting paper by Baksalary and Puri [7] concerning Fisher's [17] condition for BIB designs. The paper extends some earlier result obtained by Baksalary et al. [3] with regard to a direct relationship between EB of a block design and the rank of its incidence matrix $\boldsymbol{N}$. They have replaced the so-called Fisher's inequality, $v \leqslant b$, by *Fisher's condition*, defined as follows.

Definition 6 [7]. A block design is said to satisfy Fisher's condition if the rows of its incidence matrix are linearly independent.

Baksalary and Puri [7] have obtained necessary and sufficient conditions that give complete characterizations of all combinatorially-balanced (also called pairwise-balanced) and VB designs which satisfy Fisher's condition (and, consequently, Fisher's inequality). Their main results are as follows.

Theorem 4 [7]. *A combinatorially-balanced* (*not necessarily binary*) *block design satisfies Fisher's condition if and only if*

$$r_1^* > \lambda - \frac{\lambda}{1+\lambda\xi} \quad \textit{and} \quad r_2^* > \lambda,$$

where r_1^* *and* r_2^*, $r_1^* \leqslant r_2^*$, *are the two smallest numbers among* r_i^*, $i = 1, 2, \ldots, v$, *the diagonal elements of the concurrence matrix*, $\boldsymbol{NN}'$, *of the design*, *and where* λ *is the constant off-diagonal element of that matrix and* $\xi = \sum_{i=2}^{v} 1/(r_i^* - \lambda)$. (Recall that a block design is combinatorially-balanced if the off-diagonal elements of its matrix $\boldsymbol{NN}'$ are all equal.)

Theorem 5 [7]. *A connected VB* (*not necessarily binary*) *block design satisfies Fisher's condition if and only if*

$$r_1 > \theta - \frac{\theta}{v + \theta\zeta} \quad \text{and} \quad r_2 > \theta,$$

where r_1 *and* r_2, $r_1 \leqslant r_2$, *are the smallest treatment replications*, *and where* $(v - 1)\theta = n - \mathrm{tr}(\boldsymbol{N}\boldsymbol{k}^{-\delta}\boldsymbol{N}')$ *and* $\zeta = \sum_{i=2}^{v} 1/(r_i - \theta)$.

These results strengthen those given by Kageyama and Tsuji [24,25]. Certainly, they also complete the result of [3] for EB designs, which now may be written as follows.

Theorem 6 [3]. *An EB but not orthogonal block design satisfies Fisher's condition*, *irrespective of the connectedness or disconnectedness of the design*.

It may be mentioned here, that a more general result can be stated as follows.

Theorem 7. *A block design satisfies Fisher's condition if and only if the following two equivalent conditions hold*:

(a) *the matrix* $\boldsymbol{N}k^{-\delta}\boldsymbol{N}'$ *has no zero eigenvalues*,
(b) *the matrix* $\boldsymbol{C} = \boldsymbol{r}^{\delta} - \boldsymbol{N}k^{-\delta}\boldsymbol{N}'$ *has no unit eigenvalue with respect to* $\boldsymbol{r}^{\delta}$.

For a proof, see Corollary 2.3.1 in [14]. Note, finally, that the latter result corresponds to the following result given in [1]. It can be written as follows.

Corollary 5 [1]. *A block design with a* $v \times b$ *incidence matrix* $\boldsymbol{N}$ *satisfies the condition* $\mathrm{rank}(\boldsymbol{N}) = v - \rho$ *if and only if its matrix* $\boldsymbol{C}$ *has the unit eigenvalue with respect to* $\boldsymbol{r}^{\delta}$ *of multiplicity* ρ. *In particular*, *the design satisfies Fisher's condition if and only if all the eigenvalues of* $\boldsymbol{C}$ *with respect to* $\boldsymbol{r}^{\delta}$ *are strictly less than one*, *i.e.*, $\rho = 0$.

This result can also be expressed in terms of the intra-block estimation of some treatment contrasts, because the unit eigenvalue of $\boldsymbol{C}$ with respect to $\boldsymbol{r}^{\delta}$ implies that certain of these contrasts can be estimated intra-block with full efficiency. (For more on this, see [14, Sections 2.3 and 3.2].)

6. Conclusions

Concluding, it can be said that several results of Baksalary, obtained usually with some co-authors, have clarified certain important aspects of the theory of block designs, particularly those related to

(a) conditions for various concepts of balance,
(b) constructional methods for EB and VB block designs,
(c) conditions for constructing desirable connected designs, PBIB designs in particular,
(d) conditions for a kind of robustness of block designs,
(e) criteria concerning the validity of Fisher's condition for block designs.

Further results of Baksalary, useful for the theory of block designs, concern the estimation of variance components under a mixed model approach, as can be seen, e.g., in [2], or in [5]. This line of research is, however, beyond the scope of the present paper.

It should also be mentioned that Baksalary later extended his interest from block designs to the two-way elimination of heterogeneity designs, giving further interesting results, e.g., in the papers [9,10].

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