Solving the neutrino parameter degeneracy by measuring the T2K off-axis beam in Korea

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Abstract

The T2K neutrino oscillation experiment will start in 2009. In this experiment the center of the neutrino beam from J-PARC at Tokai village will go through underground beneath Super-Kamiokande, reach the sea level east of Korean shore, and an off-axis beam at 0.5° to 1.0° can be observed in Korea. We study physics impacts of putting a 100 kt-level water Čerenkov detector in Korea during the T2K experimental period. For a combination of the 3° off-axis beam at SK with baseline length $L = 295$ km and the 0.5° off-axis beam in the east coast of Korea at $L = 1000$ km, we find that the neutrino mass hierarchy (the sign of $m_3^2 - m_1^2$) can be resolved and the CP phase of the MNS unitary matrix can be constrained uniquely at 3σ level when $\sin^2 2\theta_{\rm rct} \gtrsim 0.06$.
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The results of solar and atmospheric neutrino oscillation experiments are consistent with the 3 neutrino model, so are all the other observations except for the LSND experiments [1]. In this Letter we assume the 3 neutrino model, which has 6 observable parameters in neutrino oscillation experiments. They are 2 mass squared differences, 3 mixing angles and one CP phase. The atmospheric neutrino oscillation experiments which measure the ν_μ survival probability determine the absolute value of one of the two mass squared-differences and one mixing angle [2] as

$$1.5 \times 10^{-3} < \left| m_3^2 - m_1^2 \right| < 3.4 \times 10^{-3} \ {\rm eV}^2 \quad \text{and} \quad \sin^2 2\theta_{\rm atm} > 0.92 \tag{1}$$

at the 90% confidence level. The K2K experiment [3], which is the first accelerator based long baseline (LBL) neutrino oscillation experiment, confirms the above results. The solar neutrino experiments, which measure the ν_e survival probability in the sun [4], and the KamLAND experiment which measures the $\bar{\nu}_e$ survival probability from reactors [5], determine the other mass squared-difference and another mixing angle as

$$m_2^2 - m_1^2 \equiv 8.2^{+0.6}_{-0.5} \times 10^{-5} \ {\rm eV}^2 \quad \text{and} \quad \tan^2 \theta_{\rm sol} = 0.40^{+0.09}_{-0.07}. \tag{2}$$

The CHOOZ reactor experiment [6] gives the upper bound of the third mixing angle ($\theta_{\rm rct}$) as

$$\sin^2 2\theta_{\rm rct} < 0.20 \quad \text{for } \left| m_3^2 - m_1^2 \right| = 2.0 \times 10^{-3} \ {\rm eV}^2,$$
$$\sin^2 2\theta_{\rm rct} < 0.16 \quad \text{for } \left| m_3^2 - m_1^2 \right| = 2.5 \times 10^{-3} \ {\rm eV}^2,$$
$$\sin^2 2\theta_{\rm rct} < 0.14 \quad \text{for } \left| m_3^2 - m_1^2 \right| = 3.0 \times 10^{-3} \ {\rm eV}^2, \tag{3}$$

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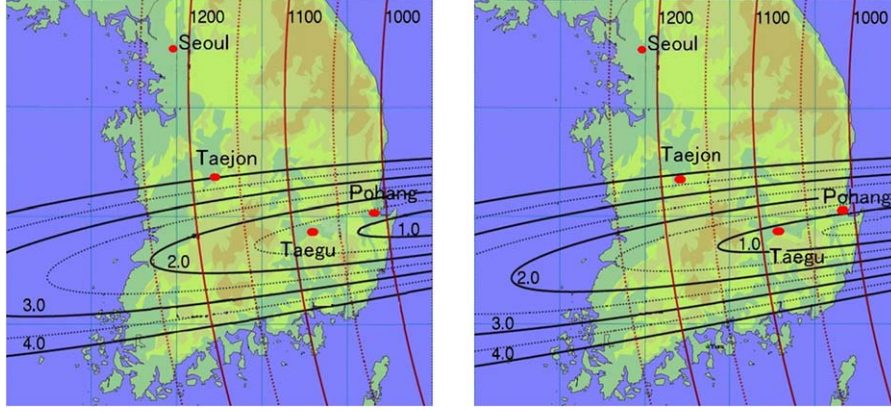
Fig. 1. The off-axis angle of the neutrino beam from J-PARC on the sea level in Korea, when the beam center is 2.5° (left) and 3.0° (right) off at SK. The baseline length for $L = 1000, 1100, 1200$ km are shown by vertical contours, and the off-axis angles are shown by elliptic contours between 0.5° and 4.0°.

at the 90% confidence level. The CP phase (δ_{MNS}) has not been constrained. In the future neutrino oscillation experiments, we should not only measure $\sin^2 2\theta_{\text{rct}}$ and δ_{MNS}, but also resolve the parameter degeneracies [7–9], such as the sign of $m_3^2 - m_1^2$.

There are many next generation LBL neutrino oscillation experiments [10], which plan to measure the model parameters and to solve the parameter degeneracy. In this Letter, we investigate the possibility of detecting in Korea the neutrino beam from J-PARC (Japan Proton Accelerator Research Complex) at Tokai village [11], that will be available during the period of the T2K (Tokai-to-Kamioka) experiment [12]. In the T2K experiment, the center of the neutrino beam will go through underground beneath Super-Kamiokande, and reach the sea level near the Korean shore. At the baseline length $L = 295$ km away from J-PARC, the upper side of the beam at 2° to 3° off-axis angle is observed at Super-Kamiokande (SK), and the lower side of the same beam at 0.5° to 1.0° appears in the east coast of Korea [13], at about $L = 1000$ km; see Fig. 1. In order to quantify our investigation, we study physics impacts of putting a 100 kt water Čerenkov detector in Korea during the T2K experiment period, which is for 5 years with 10^{21} POT (protons on target) per year.

The merits of measuring the T2K beam in Korea can be summarized as follows:

1. Because 0.5°–1.0° off-axis beam has significantly harder energy spectrum than 2.5°–3.0° off-axis beam, one can measure the $\nu_\mu \to \nu_e$ transition probability near the oscillation maximum both at Korea and at SK, at the same time.
2. Because of the matter effect that grows with the baseline length, the difference in the $\nu_\mu \to \nu_e$ transition probability between Korea and SK can reveal the neutrino mass hierarchy pattern [14,15].
3. The ν_μ energy dependence of the oscillation probabilities measured by selecting the quasi-elastic events both at Korea and at SK allows us to constrain both cosine and sine of the CP phase δ_{MNS}.

The ν_μ survival probability and the $\nu_\mu \to \nu_e$ transition rate can be expressed as

$$P_{\nu_\mu \to \nu_\mu} = 1 - \sin^2 2\theta_{\text{atm}}(1 + A^\mu)\sin^2\left(\frac{\Delta_{13}}{2} + B^\mu\right), \tag{4}$$

$$P_{\nu_\mu \to \nu_e} = 4\sin^2\theta_{\text{atm}}\sin^2\theta_{\text{rct}}(1 + A^e)\sin^2\left(\frac{\Delta_{13}}{2} + B^e\right), \tag{5}$$

where $\Delta_{ij} = (m_j^2 - m_i^2)L/2E$. Here A^α and B^α are the corrections to the magnitude and the phase of the oscillation probabilities, respectively, from the matter effect and the smaller mass-squared difference. If we keep only those terms linear in Δ_{12} and $aL/2E$, we find

$$\begin{cases} A^\mu = -\frac{aL}{\Delta_{13}E}\frac{1-2\sin^2\theta_{\text{atm}}}{\cos^2\theta_{\text{atm}}}\sin^2\theta_{\text{rct}}, \\ B^\mu = \frac{aL}{4E}\frac{1-2\sin^2\theta_{\text{atm}}}{\cos^2\theta_{\text{atm}}}\sin^2\theta_{\text{rct}} - \frac{\Delta_{12}}{2}(\cos^2\theta_{\text{sol}} + \tan^2\theta_{\text{atm}}\sin^2\theta_{\text{sol}}\sin^2\theta_{\text{rct}} - \tan\theta_{\text{atm}}\sin 2\theta_{\text{sol}}\sin\theta_{\text{rct}}\cos\delta_{\text{MNS}}), \end{cases} \tag{6}$$

$$\begin{cases} A^e = \frac{aL}{\Delta_{13}E}(1 - 2\sin^2\theta_{\text{rct}}) - \frac{\Delta_{12}}{2}\frac{\sin 2\theta_{\text{sol}}}{\tan\theta_{\text{atm}}\sin\theta_{\text{rct}}}\sin\delta_{\text{MNS}}, \\ B^e = -\frac{aL}{4E}(1 - 2\sin^2\theta_{\text{rct}}) + \frac{\Delta_{12}}{2}\left(\frac{\sin 2\theta_{\text{sol}}}{2\tan\theta_{\text{atm}}\sin\theta_{\text{rct}}}\cos\delta_{\text{MNS}} - \sin^2\theta_{\text{sol}}\right). \end{cases} \tag{7}$$

The angles are expressed as the terms of the 3×3 MNS matrix [16] elements $\sin^2\theta_{\text{rct}} = |U_{e3}|^2$, $\sin^2\theta_{\text{atm}} = |U_{\mu3}|^2$, $\sin^2 2\theta_{\text{sol}} = 4|U_{e1}U_{e2}|^2$ as in Refs. [17,18]. $\Delta_{13} > 0$ for the normal hierarchy, $\Delta_{13} < 0$ for the inverted hierarchy, and $\Delta_{12} \approx |\Delta_{13}|/30$ from the

constraints Eqs. (1) and (2). The term a controles the matter effect [19],

$$a = 2\sqrt{2} G_F E n_e = 7.56 \times 10^{-5} \text{ eV}^2 \left(\frac{\rho}{\text{g/cm}^3}\right)\left(\frac{E}{\text{GeV}}\right), \tag{8}$$

where n_e is the number density of the electron and ρ is the matter density. The magnitude of $aL/2E$ is about 0.17 at SK, while it is about 0.57 at Korea with $\rho = 3.0$ g/cm^3. By inserting the typical values of the observed parameters Eqs. (1) and (2), we find

$$\begin{cases} A^\mu \sim 0, \\ B^\mu \sim -\left[0.037 + 0.0004\left(\frac{\sin^2 2\theta_{\text{rct}}}{0.10}\right) - 0.008\left(\frac{\sin^2 2\theta_{\text{rct}}}{0.10}\right)^{1/2}\cos\delta_{\text{MNS}}\right]\frac{|\Delta_{13}|}{\pi}, \end{cases} \tag{9}$$

$$\begin{cases} A^e \sim 0.11\frac{\pi}{\Delta_{13}}\frac{L}{295 \text{ km}} - \left[0.49\left(\frac{0.10}{\sin^2 2\theta_{\text{rct}}}\right)^{1/2}\sin\delta_{\text{MNS}}\right]\frac{|\Delta_{13}|}{\pi}, \\ B^e \sim -0.08\left(\frac{L}{295 \text{ km}}\right) + \left[0.24\left(\frac{0.10}{\sin^2 2\theta_{\text{rct}}}\right)^{1/2}\cos\delta_{\text{MNS}} - 0.016\right]\frac{|\Delta_{13}|}{\pi}, \end{cases} \tag{10}$$

near the oscillation maximum, $|\Delta_{13}| \sim \pi$. Because the magnitude of A^μ and B^μ are very small, the ν_μ survival rate is rather insensitive to the matter effect and subleading terms of order Δ_{12}. Accordingly, measurements of the ν_μ survival probability improve the constraints on $|m_3^2 - m_1^2|$ and $\sin^2 2\theta_{\text{atm}}$. On the other hand, both A^e and B^e can affect the $\nu_\mu \to \nu_e$ oscillation probability significantly. Most importantly, the magnitude of the transition probability receives the correction term from the matter effect whose sign follows that of $m_3^2 - m_1^2$ and whose magnitude grows with L near the oscillation maximum, $|\Delta_{13}| \sim \pi$. If we define the difference of the $\nu_\mu \to \nu_e$ transition probability between at Korea and SK, it can be expressed near the oscillation maximum as

$$\begin{aligned} \Delta P_{\text{normal}} &= P_{\mu \to e}(L_{\text{far}}, \Delta_{13} = +\pi) - P_{\mu \to e}(L_{\text{near}}, \Delta_{13} = +\pi), \\ \Delta P_{\text{inverted}} &= P_{\mu \to e}(L_{\text{far}}, \Delta_{13} = -\pi) - P_{\mu \to e}(L_{\text{near}}, \Delta_{13} = -\pi), \end{aligned} \tag{11}$$

respectively, for the normal hierarchy ($\Delta_{13} = \pi$) and the inverted one ($\Delta_{13} = -\pi$). The difference is positive, and can be expressed as

$$\Delta P_{\text{normal}} - \Delta P_{\text{inverted}} \sim 8 \sin^2 \theta_{\text{atm}} \sin^2 \theta_{\text{rct}} \left(\frac{aL_{\text{far}}}{\pi E_{\text{far}}} - \frac{aL_{\text{near}}}{\pi E_{\text{near}}}\right) \sim 0.01 \left(\frac{\sin^2 2\theta_{\text{rct}}}{0.10}\right)\left(\frac{L_{\text{far}} - L_{\text{near}}}{295 \text{ km}}\right). \tag{12}$$

The difference grows linearly with the distance, L_{far}, as long as the oscillation maximum is covered by the flux. The ability of excluding the inverted hierarchy ($\Delta_{13} = -\pi$) is then determined by the error of the ΔP_{normal}, which can be estimated as

$$\delta(\Delta P) = \left[\left(\delta P_{\mu \to e}(L_{\text{near}})\right)^2 + \left(\delta P_{\mu \to e}(L_{\text{far}})\right)^2\right]^{1/2} = \left[\left(\frac{P_{\mu \to e}(L_{\text{near}})}{\sqrt{N_e^{\text{near}}}}\right)^2 + \left(\frac{P_{\mu \to e}(L_{\text{far}})}{\sqrt{N_e^{\text{far}}}}\right)^2\right]^{1/2}. \tag{13}$$

Here N_e is the number of ν_e appearance events. $N_e^{\text{far}}/N_e^{\text{near}}$ can be expressed as

$$\frac{N_e^{\text{far}}}{N_e^{\text{near}}} = \frac{V_{\text{far}}}{V_{\text{near}}} \frac{\Phi_{\text{far}}(E_\nu \text{ at } \Delta_{13} = \pi, L = L_{\text{far}})}{\Phi_{\text{near}}(E_\nu \text{ at } \Delta_{13} = \pi, L = L_{\text{near}})}, \tag{14}$$

where V denotes the fiducial volume of the detector and $\Phi(E_\nu, L)$ is the neutrino beam flux at L, which is proportional to $(1/L)^2$. The neutrino cross section of quasi-elastic events is almost the constant in the 0.7–10 GeV region. Typical event number, N_e^{near}, for $\sin^2 2\theta_{\text{rct}} = 0.1$ and $\delta_{\text{MNS}} = 0°$ during the 5 years running at SK is about 200; see a few bins around 0.5 GeV in Fig. 2. We therefore estimate significance of excluding the fake hierarchy as

$$\frac{\Delta P_{\text{normal}} - \Delta P_{\text{inverted}}}{\delta(\Delta P)} = 2.8 \left(\frac{\sin^2 2\theta_{\text{rct}}}{0.10}\right)^{1/2}\left(\frac{L_{\text{far}} - L_{\text{near}}}{295 \text{ km}}\right)\left[1 + 0.225\left(\frac{L_{\text{far}}}{295 \text{ km}}\right)^2 \frac{100 \text{ kt}}{V_{\text{far}}}\right]^{-1/2}. \tag{15}$$

We find that when we put a 100 kt detector at $L = 1000$ km, the significance can exceed 3.5 in this rough estimate.

As of March 2005, there is no proposal to construct a huge neutrino detector in Korea. In our case study, we consider a 100 kt level detector, in order to compensate for the decrease in the neutrino flux which is about $(300 \text{ km}/1000 \text{ km})^2 \sim 1/10$ of that at SK. We adopt a water Čerenkov detector because it allows us to distinguish clearly the $e^\pm$ events from $\mu^\pm$ events. We use the charged-current-quasi-elastic (CCQE) events in our analysis, because they allow us to reconstruct the neutrino energy by measuring the strength and the orientation of the Čerenkov lights. Since the Fermi-motion effect of the target dominates the uncertainty of the neutrino energy reconstruction, which is about 80 MeV, in the following analysis we take the width of the energy bin as $\delta E_\nu = 200$ MeV for $E_\nu > 400$ MeV. The signals in the ith energy bin, $E_\nu^i = 200 \text{ MeV} \times (i + 1) < E_\nu < E_\nu^i + \delta E_\nu$, are then calculated as

$$N_\alpha^i(\nu_\mu) = M N_A \int_{E_\nu^i}^{E_\nu^i + \delta E_\nu} \Phi_{\nu_\mu}(E) P_{\nu_\mu \to \nu_\alpha}(E) \sigma_\alpha^{\text{QE}}(E)\, dE, \tag{16}$$

where $P_{\nu_\mu \to \nu_\alpha}$ is the neutrino oscillation probability including the matter effect, M is the mass of detector, $N_A = 6.017 \times 10^{23}$ is Avogadro constant, Φ_{ν_μ} is the ν_μ flux from J-PARC, and $\sigma_\alpha^{\mathrm{QE}}$ is the CCQE cross section per nucleon in water. In this study, we assume that the fiducial volume of Super-Kamiokande is 22.5 kt, and that of a detector in Korea is 100 kt. We also assume that the detection efficiencies of both detectors for the CCQE events is 100%. The typical event numbers with $\sin^2 2\theta_{\mathrm{rct}} = 0.1$ and $\delta_{\mathrm{MNS}} = 0°$ is shown by Fig. 2.

We consider the following background events for the signal e- and μ-like events

$$N_\alpha^{i,\mathrm{BG}} = N_\alpha^i(\nu_e) + N_{\bar{\alpha}}^i(\bar{\nu}_e) + N_{\bar{\alpha}}^i(\bar{\nu}_\mu) \quad (\alpha = e, \mu). \tag{17}$$

The three terms represent the contribution from the secondary neutrino flux of the ν_μ primary beam, which are calculated as in Eq. (16) where $\Phi_{\nu_\mu}(E)$ is replaced by $\Phi_{\nu_\beta}(E)$ for $\nu_\beta = \nu_e, \bar{\nu}_e, \bar{\nu}_\mu$. After summing up these background events, the e-like and μ-like events for the ith bin are obtained as

$$N_\alpha^i = N_\alpha^i(\nu_\mu) + N_\alpha^{i,\mathrm{BG}} \quad (\alpha = e, \mu). \tag{18}$$

Since our concern is the possibility to distinguish the neutrino mass hierarchy and to measure $\sin^2 2\theta_{\mathrm{rct}}$ and the CP phase, we study how the above 'data' can constrain the model parameters by using the χ^2 function

$$\Delta\chi^2 = \chi_{\mathrm{SK}}^2 + \chi_{\mathrm{Kr}}^2 + \chi_{\mathrm{sys}}^2 + \chi_{\mathrm{para}}^2. \tag{19}$$

Here the first two terms, χ_{SK}^2 and χ_{Kr}^2, measure the parameter dependence of the fit to the SK and the Korean detector data,

$$\chi_{\mathrm{SK,Kr}}^2 = \sum_i \left\{ \left(\frac{(N_e^i)^{\mathrm{fit}} - N_e^i}{\sqrt{N_e^i}} \right)^2 + \left(\frac{(N_\mu^i)^{\mathrm{fit}} - N_\mu^i}{\sqrt{N_\mu^i}} \right)^2 \right\}, \tag{20}$$

where the summation is over all bins from 0.4 GeV up to 5.0 GeV for N_μ, 1.2 GeV for N_e at SK, and 2.8 GeV for N_e at Korea. These upper bounds are chosen such that most of the bins used in our analysis contain more than 10 events. Here $N_{\mu,e}^i$ is the calculated number of events in the ith bin, and its square root gives the statistical error. In our analysis, we calculate $N_{\mu,e}^i$ by assuming the following input ('true') parameters:

$$\left.\begin{aligned}
(m_3^2 - m_1^2)^{\mathrm{true}} &= 2.5 \times 10^{-3} \ \mathrm{eV}^2 \ (>0), \\
(m_2^2 - m_1^2)^{\mathrm{true}} &= 8.3 \times 10^{-5} \ \mathrm{eV}^2, \\
\sin^2 \theta_{\mathrm{atm}}^{\mathrm{true}} &= 0.5, \\
\sin^2 2\theta_{\mathrm{sol}}^{\mathrm{true}} &= 0.84, \\
\sin^2 2\theta_{\mathrm{rct}}^{\mathrm{true}} &= 0.1, 0.06, \\
\delta_{\mathrm{MNS}}^{\mathrm{true}} &= 0°, 90°, 180°, 270°,
\end{aligned}\right\} \tag{21}$$

with the constant matter density, $\rho^{\mathrm{true}} = 2.8 \ \mathrm{g/cm}^3$ for T2K and $\rho^{\mathrm{true}} = 3.0 \ \mathrm{g/cm}^3$ for the Tokai-to-Korea experiments. Note that in Eq. (21), we assume the normal hierarchy ($m_3^2 - m_1^2 > 0$) and examine several input values of $\sin^2 2\theta_{\mathrm{rct}}$ and δ_{MNS}.

N_i^{fit} is calculated by allowing the model parameters to vary freely and by allowing for systematic errors. In our analysis, we consider 4 types of systematic errors. The first ones are for the overall normalization of each neutrino flux, for which we assign 3% errors,
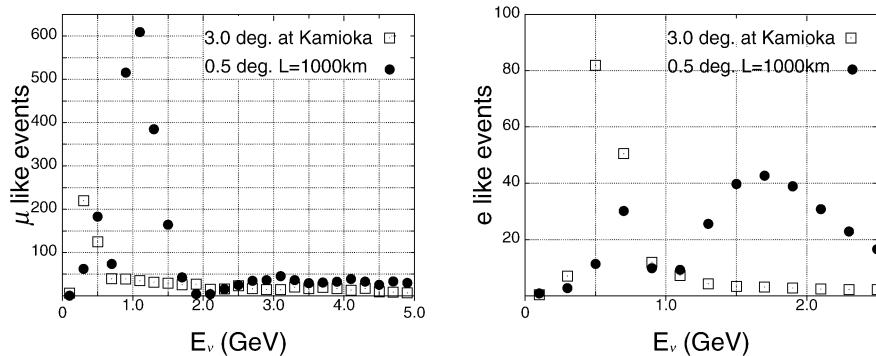
$$f_{\nu_\beta} = 1 \pm 0.03, \tag{22}$$

Fig. 2. The typical numbers of the μ events (left), and those of the e events (right), for the exposure time of 5 years, for the 3.0° OAB at SK (open square), and for the 0.5° OAB at $L = 1000$ km with 100 kt detector (solid circles). The inputs are $\sin^2 2\theta_{\mathrm{rct}} = 0.1$ and $\delta_{\mathrm{MNS}} = 0°$.

for $\nu_\beta = \nu_e, \bar{\nu}_e, \nu_\mu, \bar{\nu}_\mu$, which are taken common for T2K and the Tokai-to-Korea experiments. The second systematic error is for the uncertainty in the matter density, for which we allow 3% overall uncertainty along the baseline, independently for T2K (f_ρ^{SK}) and the Tokai-to-Korea experiment (f_ρ^{Kr}):

$$\rho_i^{fit} = f_\rho^i \rho_i^{true} \quad (i = SK, Kr). \tag{23}$$

The third uncertainty is for the CCQE cross section,

$$\sigma_\alpha^{QE,fit} = f_\alpha^{QE} \sigma_\alpha^{QE,true}. \tag{24}$$

Since ν_e and ν_μ QE cross sections are expected to be very similar theoretically, we assign a common overall error of 3% for ν_e and ν_μ ($f_e^{QE} = f_\mu^{QE} \equiv f_\ell^{QE}$), and an independent 3% error for $\bar{\nu}_e$ and $\bar{\nu}_\mu$ QE cross sections ($f_{\bar{e}}^{QE} = f_{\bar{\mu}}^{QE} \equiv f_{\bar{\ell}}^{QE}$). The last one is the uncertainty of the fiducial volume, for which we assign 3% error independently for T2K (f_V^{SK}) and the Tokai-to-Korea experiment (f_V^{Kr}). $N_\alpha^{i,fit}$ is then calculated as

$$N_\alpha^{i,fit}(\nu_\beta) = f_{\nu_\beta} f_\alpha^{QE} f_V^{SK,Kr} N_\alpha^i(\nu_\beta), \tag{25}$$

and χ^2_{sys} has four terms;

$$\chi^2_{sys} = \sum_{\alpha=e,\bar{e},\mu,\bar{\mu}} \left(\frac{f_{\nu_\alpha} - 1}{0.03}\right)^2 + \sum_{\alpha=l,\bar{l}} \left(\frac{f_\alpha^{QE} - 1}{0.03}\right)^2 + \sum_{i=SK,Kr} \left\{ \left(\frac{f_\rho^i - 1}{0.03}\right)^2 + \left(\frac{f_V^i - 1}{0.03}\right)^2 \right\}. \tag{26}$$

In short, we assign 3% errors for the normalization of each neutrino flux, the ν_e and $\bar{\nu}_e$ CCQE cross sections, the effective matter density along the base line, and for the fiducial volume of SK and the Korean detector. Finally, χ^2_{para} accounts for the present constraints on the model parameters, summarized in Eqs. (1) and (2):

$$\chi^2_{para} = \left(\frac{|(m_3^2 - m_1^2)^{fit}| - |(m_3^2 - m_1^2)^{true}|}{0.5 \times 10^{-3}}\right)^2 + \left(\frac{(m_2^2 - m_1^2)^{fit} - (m_2^2 - m_1^2)^{true}}{0.6 \times 10^{-5}}\right)^2 + \left(\frac{\sin^2 2\theta_{atm}^{fit} - \sin^2 2\theta_{atm}^{true}}{0.04}\right)^2$$
$$+ \left(\frac{\sin^2 2\theta_{sol}^{fit} - \sin^2 2\theta_{sol}^{true}}{0.07}\right)^2. \tag{27}$$

Here we interpret the 90% CL lower bound on $\sin^2 2\theta_{atm}$ in Eq. (1) as the 1.96σ constraint from $\sin^2 2\theta_{atm}$ is greater than 0.96, and the asymmetric error for $\tan^2 \theta_{sol}$ in Eq. (2) has been made more symmetric for $\sin^2 2\theta_{sol}$. We do not include the bounds on $\sin^2 2\theta_{rct}$ in Eq. (3) in our $\Delta\chi^2$ function. In total, our $\Delta\chi^2$ function depends on 16 parameters, the 6 model parameters and the 10 normalization factors.

First, we search for the best place for the detector in Korea and the best combination of the off-axis angle for SK and the Korean detector to determine the sign of $m_3^2 - m_1^2$. We show in Fig. 3 the minimum $\Delta\chi^2$ as functions of the off-axis angle and the base-line length in Korea, when the data, N_α^i, are generated for the normal hierarchy, $m_3^2 - m_1^2 = 2.5 \times 10^{-3}$ eV$^2 > 0$, Eq. (21), and the fit has been performed by assuming the inverted hierarchy, $m_3^2 - m_1^2 < 0$. We set $\sin^2 2\theta_{rct}^{true} = 0.10$ and $\delta_{MNS}^{true} = 0°$ in this analysis. The left-hand figure shows the minimum $\Delta\chi^2$ for the 2.5° off-axis beam at SK, and the right-hand one is for the 3.0° off-axis beam at SK. The four symbols, solid circle, open circle, triangle, and square are for $L = 1000$ km, 1050 km, 1100 km, and 1150 km, respectively. When the off-axis angle at SK is 2.5°, the 0.5° beam does not reach the Korean coast; see Fig. 1. It is clear from these figures that the best discriminating power is obtained for the combination $L = 1000$ km and 0.5°, which is available only when
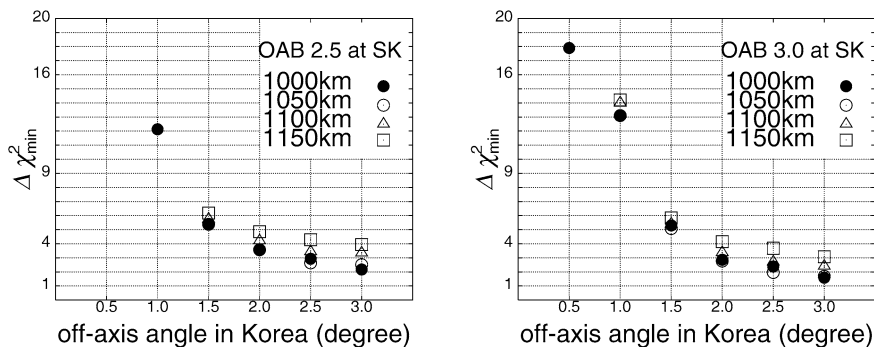

Fig. 3. Minimum $\Delta\chi^2$ as functions of the off-axis angle and the base-line length from J-PARC at Tokai, when the normal hierarchy ($m_3^2 - m_1^2 = 2.5 \times 10^{-3}$ eV$^2 > 0$) is assumed in generating the events, and the inverted hierarchy ($m_3^2 - m_1^2 < 0$) is assumed in the fit. The left-hand figure is for the 2.5° off-axis beam at SK, and the right-hand one is for the 3.0° beam.
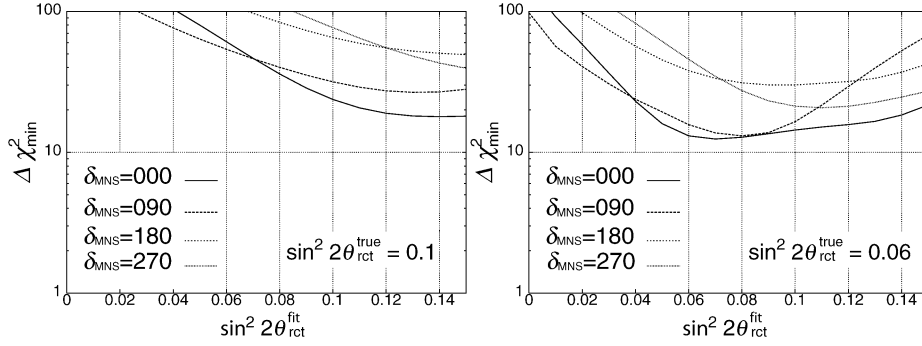
Fig. 4. Minimum $\Delta\chi^2$ as a function of $\sin^2 2\theta_{\text{rct}}^{\text{fit}}$ when the normal hierarchy ($m_3^2 - m_1^2 = 2.5 \times 10^{-3}$ eV$^2 > 0$) is assumed in generating the events, and the inverted hierarchy ($m_3^2 - m_1^2 < 0$) is assumed in the fit. The 4 lines are for 4 input CP phase values, $\delta_{\text{MNS}}^{\text{true}} = 0.0°$ (solid), 90° (long-dashed), 180° (short-dashed) and 270° (dotted). The left figure is for $\sin^2 2\theta_{\text{rct}}^{\text{true}} = 0.10$ and the right one is for $\sin^2 2\theta_{\text{rct}}^{\text{true}} = 0.06$.
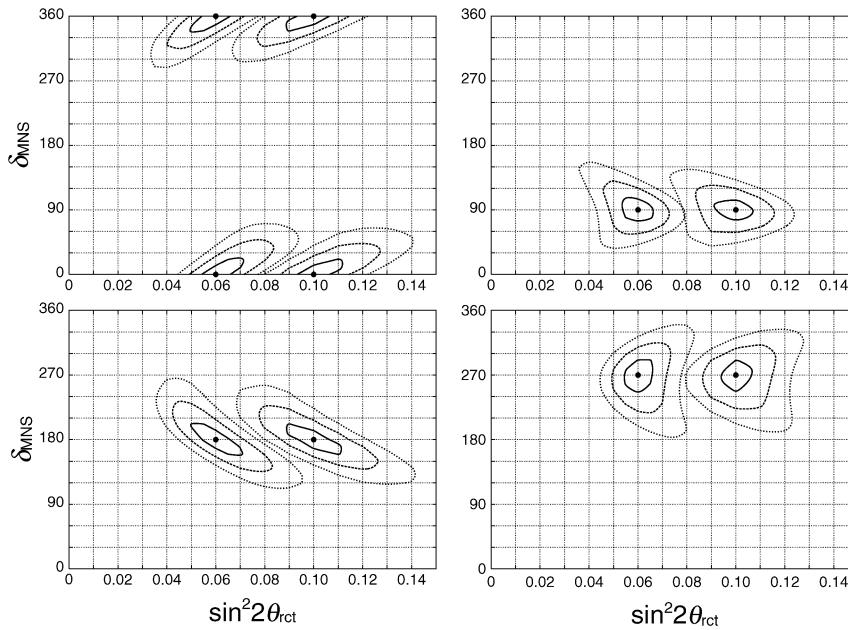

Fig. 5. Allowed region in the plane of $\sin^2 2\theta_{\text{rct}}^{\text{fit}}$ and $\delta_{\text{MNS}}^{\text{fit}}$, when the event numbers at SK and Korea are calculated for the parameters of Eq. (21). In each figure, the input points ($\sin^2 2\theta_{\text{rct}}^{\text{true}}, \delta_{\text{MNS}}^{\text{true}}$) are shown by solid-circles, and the regions where the minimum $\Delta\chi^2$ is less than 1, 4, 9 are depicted by solid, dashed and dotted boundaries, respectively.

the off-axis angle at SK is 3.0° (right figure). For this combination, we can distinguish the inverted hierarchy from the normal one at more than 4σ level. These figures show that longer base-line gives larger minimum $\Delta\chi^2$ for the same off-axis angle. This is because of the increase in the matter effect. For the same base-line length, lower off axis angle beams give better discriminating power. This is because the neutrino flux with smaller off-axis angle is harder [12,20], and the stronger matter effect to help us to distinguish the neutrino mass hierarchy [9,18,21].

Here after, we study the prospect for measuring the sign of $m_3^2 - m_1^2$ in more detail for the best combination, $L = 1000$ km and 0.5° in Korea, and 3.0° for SK. Fig. 4 shows the minimum $\Delta\chi^2$ as functions of the fitting parameter $\sin^2 2\theta_{\text{rct}}^{\text{fit}}$ by assuming the inverted hierarchy, when the normal hierarchy ($m_3^2 - m_1^2 = 2.5 \times 10^{-3}$ eV$^2 > 0$) is assumed in calculating the event numbers. There are 4 lines in each figure, which correspond to 4 input values of the CP phase, $\delta_{\text{MNS}}^{\text{true}} = 0°$ (solid), 90° (long-dashed), 180° (short-dashed) and 270° (dotted), respectively. The left figure is for $\sin^2 2\theta_{\text{rct}}^{\text{true}} = 0.1$ and the right one is for $\sin^2 2\theta_{\text{rct}}^{\text{true}} = 0.06$. $\Delta\chi^2$ is mainly controled by the difference of N_e^i between the normal hierarchy and the inverted hierarchy which is proportional to $\sin^2 \theta_{\text{rct}}$; see Eq. (12). Because $P_{\nu_\mu \to \nu_e}$ in the inverted hierarchy is smaller than that in the normal hierarchy due to the matter effect, the fitting parameter, $\sin^2 2\theta_{\text{rct}}^{\text{fit}}$, tends to be larger than the input value, $\sin^2 2\theta_{\text{rct}}^{\text{true}}$. Since large $\sin^2 2\theta_{\text{rct}}$ will be constrained more strongly in the future reactor experiments [22,23], we can conclude that the neutrino mass hierarchy will be determined at even higher confidence level once results from these reactor experiments are available.

We also examine the capability of the Tokai-to-Korea LBL experiments for measuring the CP phase. We show in Fig. 5 regions allowed by this experiment in the plane of $\sin^2 2\theta_{\text{rct}}$ and δ_{MNS}. The mean values of the inputs are calculated for the parameters of

Eq. (21). In each figure, the input points ($\sin^2 2\theta_{\mathrm{rct}}^{\mathrm{true}}$, $\delta_{\mathrm{MNS}}^{\mathrm{true}}$) are shown by solid-circles for $\sin^2 2\theta_{\mathrm{rct}}^{\mathrm{true}} = 0.10$, and 0.06. The regions where the minimum $\Delta\chi^2$ is less than 1, 4, 9 are depicted by solid, dashed and dotted boundaries, respectively. Even though we allow the sign of $m_3^2 - m_1^2$ to vary in the fit, no solution with the inverted hierarchy that satisfy $\Delta\chi^2_{\mathrm{min}} < 9$ appear in the figure.

From these figures, we learn that δ_{MNS} can be constrained to $\pm 30°$ at 1σ level, when $\sin^2 2\theta_{\mathrm{rct}}^{\mathrm{true}} > 0.06$. It is remarkable that we can constrain both $\sin\delta_{\mathrm{MNS}}$ and $\cos\delta_{\mathrm{MNS}}$ without using anti-neutrino experiments. We can determine $\sin\delta_{\mathrm{MNS}}$ uniquely by measuring the $\nu_\mu \rightarrow \nu_e$ oscillation probability near the oscillation maximum both at SK and Korea. This is because the significant difference in the matter effect term in Eq. (7) between SK and Korea allows us to resolve the correlation between $\sin^2\theta_{\mathrm{rct}}$ and $\sin\delta_{\mathrm{MNS}}$ [24–26]. As for $\cos\delta_{\mathrm{MNS}}$, it appears only in the phase shift of the $\nu_\mu \rightarrow \nu_e$ oscillation probability; see the term B^e in Eq. (7). It is therefore important to measure the neutrino energy by CCQE events, in order to constrain $\cos\delta_{\mathrm{MNS}}$.

In this Letter, we study the possibility of solving the degeneracy of the neutrino mass hierarchy and constraining $\sin^2 2\theta_{\mathrm{rct}}$ and δ_{MNS} by measuring the T2K off-axis beam in Korea. We find that by placing a 100 kt level water Čerenkov detector in the east coast of Korea, we can determine the sign of $m_3^2 - m_1^2$ and constrain $\sin^2 2\theta_{\mathrm{rct}}$ and δ_{MNS} uniquely, if $\sin^2 2\theta_{\mathrm{rct}} \gtrsim 0.06$.

Our results are based upon a very simple treatment of the systematic errors where 3% overall errors are assigned for all the 10 normalization factors of Eq. (26). Even if we enlarge all the systematic errors to 10% except for the matter density uncertainties, the significance of the mass hierarchy determination is not affected much. For instance, the $\Delta\chi^2_{\mathrm{min}}$ for the combination of the 3° OAB at SK and the 0.5° OAB at $L = 1000$ km in Fig. 3 is found to reduce from 18 to 16. Among the potentially serious background which we could not estimate in this Letter are;

- possible miss-identification of NC π^0 production as ν_e CCQE events,
- possible miss-identification of soft π emission events as ν_e CCQE events.

Although the above uncertainties were found to be rather small at K2K experiments [27], we should expect them to be more serious at high energies. Dedicated studies of their effects on the neutrino-energy reconstruction efficiency are mandatory. In addition, careful studies including possible energy dependence of the flux and cross section uncertainties, location dependence of the matter density may be needed to justify the physics case of our proposal.

Note added

When we were finalizing the manuscript for publication, we learned that a similar study has been performed by Ishituka et al. [28].

Acknowledgements

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