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Numerical study of wings with wavy leading and trailing edges

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Abstract

The use of wavy leading edges is presented in the literature as a possible way of delaying stall, allowing a wing to have a better aerodynamic performance at high angles of attack. In this paper, we present results of direct numerical simulations of the flow around infinite wavy wings with a NACA0012 profile at a Reynolds number $Re = 1000$. The simulations were carried out using the Spectral/hp Element Method, with a coordinate system transformation being employed to treat the waviness of the wing. Several combinations of wavelength and amplitude were considered, showing that for this value of $Re$ the waviness leads to a reduction in the lift-to-drag ratio ($L/D$), associated with a suppression of the fluctuating lift coefficient. Also, flow visualization indicate that this behaviour is associated with a regime where the flow remains attached behind the peaks of the leading edge while there are distinct regions of flow separation behind the troughs.

1. Introduction

The possibility of using wavy wings as a way of obtaining improved aerodynamic performance started receiving attention after the work of Fish and Battle\textsuperscript{1}, where the morphology of the pectoral flipper of the humpback whale (\textit{Megaptera novaeangliae}) was analysed with a focus on its hydrodynamic performance. These flippers have protuberances on the leading edge, and this was suggested to act as a mechanism to delay the stall, allowing the flipper to maintain a high lift coefficient at high angles of attack, giving the whale a good maneuverability.

The idea that leading edge protuberances could delay stall gained support with the work of Miklosovic et al.\textsuperscript{2,3}. They presented experiments for full-span and half-span wings with a NACA0020 profile in configurations with and without leading edge waviness. For the half-span model, which had a planform similar to the flipper of the humpback whale, the Reynolds number was around $6 \times 10^5$ and the modified wing led to an increase in the stall angle. This increase in the stall angle contributed to an increase in the maximum lift coefficient of the wing. However, for the full-span model, for which the Reynolds number was around $2.7 \times 10^5$, their results show that the protuberances lead to a premature stall, being beneficial only in the post-stall regime. Also, the experiments for full-span wings presented in\textsuperscript{4} showed the same behaviour, with the modified leading edge causing a premature stall.

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Although the first studies about the effect of leading edge protuberances showed a strong distinction between the behaviour of full-span and half-span models, more recent works suggest that the main factor affecting the results is the Reynolds number. First, Stanway\textsuperscript{5} presented experiments for a model similar to the half-span wing of Miklosovic et al.\textsuperscript{3}, but considering different values of $Re$ between $4 \times 10^4$ and $1.2 \times 10^5$. Only for the highest value of $Re$ considered the waviness caused an increase in the maximum lift coefficient, indicating that the value of $Re$ has an important role in determining whether the use of wavy leading edges will improve aerodynamic performance. Another work which support the importance of the Reynolds number effect on this flow is that of Hansen et al.\textsuperscript{6}. They performed experiments with rectangular wing mounted in both full-span and half-span configurations, in an attempt to isolate the influence of the wing tip on the results. The effect of using a wavy leading edge was similar in both cases, indicating that three-dimensional effects related to the wing-tip have a secondary importance in the effectiveness of the wing waviness.

In this paper, we present results of direct numerical simulations of the flow around infinite wings with a NACA0012 profile and sinusoidal waviness along the span. All simulations were performed for $Re = 1000$, allowing for the clear identification of the three-dimensional structures that appear in the flow. The rest of this paper is organized as follows: section 2 contains a brief description of the numerical methods employed in the simulations, section 3 shows the results, and section 4 presents the main conclusions of the work.

2. Numerical methods

To account for the spanwise waviness of the geometries considered in this study, the approach presented in\textsuperscript{7} was employed, with consists in applying a coordinate system transformation that maps the geometry to a straight wing. In the physical Cartesian system $(x', y', z')$ we want to solve the Navier-Stokes equation for an incompressible fluid:

\[
\frac{\partial u'}{\partial t} = -(u' \nabla)u' - \nabla p' + \frac{1}{Re} \nabla^2 u' \\
\n \nabla \cdot u' = 0
\]  

(1)

where $u' = (u', v', w')$ is the velocity vector, $p'$ is the pressure and $Re = U_\infty c/\nu$ is the Reynolds number defined in terms of the chord $c$, the freestream velocity $U_\infty$ and the kinematic viscosity $\nu$.

Instead of solving the problem given by equation 1, a different coordinate system $(x, y, z)$ is taken, which is related to the Cartesian system by $x = x' - \xi(z')$, $y = y'$, $z = z'$. In this new coordinate system we want to have a straight rectangular wing, so that the geometries that are consistent with this transformation are wings which are deformed along the span without variation in the cross-section. Throughout this work, we are concerned with sinusoidal waviness given by $\xi(z') = -\frac{h}{2} \cos\left(\frac{2\pi z'}{\lambda}\right)$, so that the parameters required to define the waviness are the wavelength $\lambda$ and the peak-to-peak amplitude $h$. An example of a wing that can be treated by this transformation is shown in Fig. 1, with the relevant geometric parameters indicated in the figure.

Applying the coordinate transformation to equation 1, the Navier-Stokes equations for the system $(x, y, z)$ is obtained. This equations can be represented as:

\[
\frac{\partial u}{\partial t} = -(u \nabla)u - \nabla p + \frac{1}{Re} \nabla^2 u + A(u, p, \xi) \\
\n \nabla \cdot u = 0
\]  

(2)

where $u = (u, v, w)$ with $u = u' - w' \frac{\partial \xi}{\partial z}, v = v'$ and $w = w'$ is the representation of the velocity after the transformation, $p = p'$ is again the pressure, and $A(u, p, \xi)$ is an acceleration term containing the effect of the coordinate system transformation. The full form of $A$ is available in\textsuperscript{7}.

The equations were discretized using the Spectral/hp Element Method described in\textsuperscript{8} and integrated in time using the stiffly stable splitting scheme presented in\textsuperscript{9}. Since the coordinate transformation makes the spanwise direction homogeneous, it was possible to use a Fourier expansion in this direction, as discussed in\textsuperscript{10}. This is one of the main advantages of using the coordinate transformation, since this way it is not necessary to perform fully three-dimensional simulations. Other advantages of this approach are the possibility of using a single two-dimensional mesh for different forms of waviness, and the fact that equation 2 is the same as equation 1 with an additional forcing term, which makes the implementation of the method simple. This forcing term was treated explicitly, together with the advection term.
The mesh used in all the simulations consisted of 449 quadrilateral elements extending from the $-10$ to $10$ in the $x$ direction and $-15$ to $15$ in the $y$ direction, with the NACA0012 profile having a unit chord aligned with the $x$ axis and with the leading edge located at the origin. The dimensions of the mesh were defined using convergence tests following a procedure similar to the one presented in\textsuperscript{11}. The same procedure was used to define that the spatial discretization in the $x$–$y$ plane should be done using $8^{th}$ order polynomials in each quadrilateral element, and to define that the spanwise direction, whose length was always equal to the airfoil chord, should be discretized using 16 Fourier modes.

3. Results

Simulations were performed for a straight wing, which served as a baseline for comparison, and for wavy wings with several combinations of wavelength and amplitude. Three wavelengths were considered, and for each one of them three amplitudes, totalling nine different geometries of wings with waviness. Table 1 summarizes the parameters for each of the cases, together with a naming convention to simplify the notation. Also, for each geometry simulations were performed for angles of attack $\alpha$ between 0 and 21 deg, with increments of 3 deg, allowing for the observation of the aerodynamic performance over a wide range of conditions.

Table 1: List of cases considered in the study, with the corresponding parameters of the waviness.

<table>
<thead>
<tr>
<th>Case name</th>
<th>$\lambda/c$</th>
<th>$h/c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>baseline</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>L025h0125</td>
<td>0.25</td>
<td>0.0125</td>
</tr>
<tr>
<td>L025h025</td>
<td>0.25</td>
<td>0.025</td>
</tr>
<tr>
<td>L025h05</td>
<td>0.25</td>
<td>0.05</td>
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<tr>
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<tr>
<td>L05h10</td>
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</tr>
<tr>
<td>L10h05</td>
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</tr>
<tr>
<td>L10h20</td>
<td>1.0</td>
<td>0.20</td>
</tr>
</tbody>
</table>
Fig. 2: Comparison between results of $L/D$ obtained from wings with waviness of different wavelengths and the baseline wing.

Fig. 2 shows the results of the mean lift-to-drag ratio for all the cases, with each plot containing results for a different wavelength, with the baseline results presented in all plots as a reference. It is clear that for the shortest wavelength $\lambda/c = 0.25$ the waviness has minor influence on the results, with the flow being very similar to the straight wing case. For the other values of wavelength, the waviness tends to reduce the value of $L/D$ in a wide range of angles of attack. As the amplitude is increased, this reduction becomes more intense. Also, for low amplitudes this effect is present only for intermediate angles of attack, but as the amplitude is increased, the range of angles of attack where the loss of $L/D$ is observed also increases.

To understand in more detail how the waviness of the wing affects the flow, we will focus now on the cases with intermediate wavelength $\lambda/c = 0.5$. For these cases, Fig. 3 presents the results of mean drag coefficient, mean lift coefficient, and oscillating lift coefficient measured by its root mean square (rms) value. From this results, it is clear that the reduction of $L/D$ is the consequence of a combination of reductions in both lift and drag forces. Also, the waviness leads to a significant suppression in the lift fluctuations, with its value remaining close to zero until $\alpha = 12$ deg for the case L05h05 and until $\alpha = 15$ deg for the case L05h10. Comparison of Fig. 3 with Fig. 2 shows that the range of angle of attack where the lift fluctuations remain low is the same as the range where there is a significant reduction in $L/D$.

Fig. 4 and Fig. 5 show skin friction lines in the surface of the wing for selected cases with $\lambda/c = 0.5$ for angles of attack $\alpha = 12$ deg and $\alpha = 15$ deg, respectively. In these pictures, the flow is from left to right, and the white lines represent the surface skin friction vectors. Also, the colors correspond to the orientation of the skin friction in the stream direction, with blue being opposite to the freestream; therefore, the regions in blue in the pictures are the recirculation zones that appear as a consequence of flow separation. It can be seen that for these angles of attack, the flow in the baseline case is almost two-dimensional, with separation occurring over a large extension of the wing. As the waviness amplitude is increased, there is a tendency for the flow behind the peaks of the leading edge to become
attached, leading to a flow configuration characterized by distinct separation regions behind the troughs. Also, it can be noted that for the case L05h05 this structure breaks down when the angle of attack is increased from 12 to 15 degrees, what from Fig. 2 and Fig. 3 coincides with the sudden increase of the lift coefficient fluctuations and to the value of $L/D$ returning to a level close to the baseline result.

4. Conclusions

The flow around wings with spanwise waviness was investigated numerically for a Reynolds number $Re = 1000$. Several combinations of wavelength and amplitude were considered. For the shortest wavelength $\lambda/c = 0.25$, the modifications had no significant effect on the results. For $\lambda/c = 0.5$ and $\lambda/c = 1.0$, there is a reduction of the lift-to-drag ratio, caused by the combination of reductions in both the lift and the drag. This reduction of $L/D$ is accompanied by a suppression in the lift coefficient fluctuations. Also, flow visualizations showed that this behaviour is caused by a flow regime where there is a tendency for the flow to remain attached behind the waviness peaks, leading to the formation of distinct separation regions behind the troughs.

Acknowledgements

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Fig. 4: Skin friction lines on the wall of the wing (in white) for different cases with angle of attack 12 deg. The flow is from left to right and the colors represent the orientation of the skin friction in the stream direction, with blue corresponding to recirculation zones.

References

Fig. 5: Skin friction lines on the wall of the wing (in white) for different cases with angle of attack 15 deg. The flow is from left to right and the colors represent the orientation of the skin friction in the stream direction, with blue corresponding to recirculation zones.