method of interpolating logarithms was similar but not identical to the method of his contemporary Henry Briggs, and his interpolation formula was rediscovered by Newton and James Gregory in the 1660s.

It is remarkable that it has taken nearly 400 years for this manuscript to appear in print. In the book's introduction, there is a discussion on how Harriot's new mathematical ideas were disseminated among a small group of his friends—none of whom found the time to write a commentary explaining his symbolic interpolation formulas or were capable of understanding this new interpretation. Undoubtedly, part of the problem was the lack of any explanation by Harriot of his mathematical accomplishments. Further delaying its publication, the manuscript was lost for over a century before it turned up in 1784 at Petworth House in Sussex, England. All these matters are discussed in detail in this edition. There is also valuable commentary with each manuscript page. The editors, building on their extensive knowledge of Harriot and his mathematics (see, for example, Stedall [2003] and Beery [2004, 2007]), have done exemplary work researching the history and evolution of the manuscript and explaining the mathematics involved. For lovers of the history of 17th-century mathematics and algebra in particular, this book is a highly recommended addition to the mathematical literature.

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Available online 25 June 2010

10.1016/j.hm.2010.05.006

Duel at Dawn: Heroes, Martyrs and the Rise of Modern Mathematics

By Amir Alexander. Cambridge, Mass. and London (Harvard University Press). 2010. ISBN: 978-0-674-04661-0. 307 pp. US$28.95.

Amir Alexander has a flair for proposing novel links of mathematics to the wider culture. His *Geometrical Landscapes* [Alexander, 2002] argued the influence of the great overseas explorations on late-Renaissance and early-modern mathematics, even on such ostensibly internal issues as the use of infinitesimals. Now he sets the career of mathematics against the background of Europe's perennially fascinating passage from the 18th century to the 19th, from the "Age of Reason" to the age of the Romantics.

That mathematics changed fundamentally in the 19th century is now surely a commonplace; and the chronological coincidence of that evolution's early phase with the heyday of the Romantic movement does catch the eye. Perhaps some mathematicians absorbed, perhaps their work reflected, the ambient mood? Increasing remoteness from immediate physical experience, an enlarged sense of freedom, greater scope for imagination and creativity – on the face of it these features of 19th-century mathematics resonate well with the mindset of a Keats or of a Byron, to invoke two of Alexander's well chosen examples. In the saga of non-Euclidean geometry the familiar anguish of Bolyai *père* ("I have traversed this bottomless night") and rapture of Bolyai *fils* ("I have created a new and different world out of nothing"), both duly quoted here (pp. 233, 234), have the authentic ring. No doubt there are counterexamples: who had the more "Romantic" attitude toward convergence of series, Euler in 1750 or Abel in 1825? But the general picture is suggestive.

Amir Alexander brings to this promising theme a characteristic historiographical stance, which informed also his earlier book. Every era's mathematics, he urges (p. 6 of this new one), is molded by "broader cultural trends", to which the "stories" that are told of mathematicians are an "indispensable guide". Such tales work indeed in two directions, at once "mirroring" the global background and helping to shape the "technical practice" of the professionals (p. 6). Thus the images of mathematicians that emerge from these stories are for him vital aids to interpretation and understanding. Their role in the present book is central.

The 18th-century accounts that he finds most revealing in this sense are the famous eulogies, tributes to recently dead scientists written by friends, colleagues or protégés. He cites three in particular: Fontenelle's of Varignon and Condorcet's of Euler and of D'Alembert. He gives most weight to the last of these, drawing from it a picture which in his eyes makes D'Alembert "iconic" (p. 8) and "paradigmatic" (p. 262) among all of that era's mathematicians. He admits (p. 24) that Condorcet's words may not capture the historical D'Alembert in every respect; indeed, he says, the eulogies really aim less to describe individuals than to "tell us what an *ideal* mathematician should be in the eyes of his contemporaries" (p. 43, my italics) – hence their great value as historical documents. But that model, he adds, was accepted by the mathematicians themselves, who "did their best to live up to the image" (p. 47).

The "type" which D'Alembert is thus taken to embody is complex. Ideal mathematicians retained (according to Alexander) a childlike simplicity, a closeness to nature, uncorrupted by an "artificial and shallow" society (p. 41); he acutely sees behind this profile the seminal influence of Rousseau. But the representative mathematicians were *also* "worldly, successful men", indeed "scientific, cultural and even political leaders" (p. 3). Innocent yet sophisticated – the mix may seem uneasy; the unifying theme is that in both contexts, the private and the public, these favored individuals were at home in the world, "in tune with their surroundings" (p. 127). Again echoing Rousseau, Alexander adopts for them the tag "natural men".

Their special gift – the argument continues – was a unique suitability (p. 37) for both the study and the application of mathematics. For, says Alexander, 18th-century mathematics, like the "natural men" themselves, remained very close to ordinary experience. Abstracted from the sensible world, it "never lost touch with its roots" there – that would have been "unthinkable" (pp. 10–11); its relations "expressed actual physical realities" (p. 60). And that intimacy, that directness of access, guaranteed success for its use by the childlike "natural men" in science's quest to "unveil" (p. 51) the "hidden harmonies" (p. 73) of nature's workings. But as time passed that success was imperilled by the very progress of mathematics. During the 18th century the subject's focus moved increasingly from geometry to algebra, and hence toward greater abstraction – a development which (says Alexander) provoked widespread dismay. By 1800 "a growing anxiety about the field and its future was palpable among the leading mathematicians" (p. 72). For this abstraction threatened a "dissociation" (p. 72) from the physical world so complete that mathematics "could not find its way back" (p. 72) and so would lose its purpose and its power.

Soon there "lived and breathed" in Paris (p. 74) a young man whom posterity would regard as "paradigmatic" (p. 264) for a new kind of mathematics – Galois, of course. Galois's death, in the duel which gives the book its title, is for Alexander *the* watershed, "the birth of modern mathematics" (p. 1). Before long three other figures played crucial parts in the subject's remaking: Abel, Janos Bolyai and Cauchy. These revolutionaries, says Alexander, "proposed" (pp. 4, 269) a "wholly new way of viewing and practicing mathematics" (p. 4), which for them was a "pure realm of truth and beauty" (p. 182) "unsullied by physical reality" (p. 269). That made them kindred spirits with the great Romantic artists – he instances (pp. 160–61) Wordsworth, Keats, Beethoven, Chopin, Friedrich, Géricault. "The 'worldly geometer' of the Enlightenment had been transformed into a romantic 'mathematical poet'" (p. 164). But inevitably these pioneers aroused in the mathematical establishment incomprehension, hostility, even outright rejection, and all came eventually to be seen as martyrs to the "wondrous alternative reality" (p. 4) which they had glimpsed.

For – of course – after the lives, the legends. Later generations' accounts of Galois, Abel, Bolyai, and Cauchy are in this context the "stories" to which Alexander always gives such interpretive value. He urges (pp. 127, 164) that whether these posthumous tales distort the realities of their subjects' lives does not matter – the images take over. He weaves the legends into a striking portrait (p. 183):

> [A] conception of mathematics as insulated from any form of physical and human reality leads almost inevitably to the iconic view of the tragic mathematical genius, for in this view a mathematician is that blessed individual who is granted a special sight that allows him to observe, or even to reside within, the alternative mathematical universe. It is an inestimable gift, but it comes at a heavy price: striving for his true home in the realm of mathematics, but trapped in the harsh world of physical things and the petty company of lesser men, the mathematician is a tragic figure, doomed to a difficult and disastrous life.

Elsewhere (p. 267) are added the details that this unhappy soul is "misunderstood by his contemporaries and doomed to persecution by an uncaring world".

The cultural influence of this "tragic romantic hero" (p. 164), says Alexander, has been profound and lasting. Indeed his twofold identification of that legacy constitutes the core of his argument. His first claim is that even in our time the tales of Galois, Abel, Bolyai and Cauchy represent, "to mathematicians and the broader public alike … an image of what a true mathematician is" (p. 164). Moreover – this is the second of his contentions – those legends, and that image, have been "essential accompaniments" (p. 265) of modern mathematical development. He concedes (pp. 263–64) some validity to the usual internalist view of the subject's recent history, but he maintains that by itself that account remains incomplete. In keeping with the value that he attaches to "stories" he declares that technical advance has gone "hand in hand" (p. 2) with the formation and impact of those legends. Early in the 19th century "a new story of genius and martyrdom, drawn from the discourse of High Romanticism, legitimized and allowed for a new type of mathematical knowledge" (p. 5); and that influence persists in our own time – "[f]or the past two centuries the study of pure mathematics has been inseparable from the story of the mathematician as a tragic romantic hero" (p. 267).

Naturally then Alexander's *description* of modern mathematics mirrors exactly the vision that he ascribes to its ill-starred pioneers: it is a "wondrous alternative reality", governed only by its own "eternal laws" and "unsullied by the crass realities of the world around us" (p. 4). He singles out one of its aspects as especially important: he speaks twice (pp. 9, 268) of "the new *rigorous* mathematics" (my italics), as if that adjective captures its essence better than any other. The choice may surprise, but the motive is made clear: a science that shuns "worldly contamination" (p. 269) can have no "standard of truth", no guarantee of validity, except "strict internal rigor" (p. 266). (It is from this perspective that (p. 12) he places Cauchy among his tragic heroes: Cauchy earned martyrdom by resisting colleagues who balked at his insistence on *teaching* (my emphasis) his rigorized version of the calculus.) For this remarkable characterization of modern mathematics Alexander draws textual support from Jacobi's passionate declaration (p. 179) that "the only aim of science is the honor of the human spirit", and (pp. 171–72) from Hardy's *Apology*, with its well known celebration of the subject's pure and nonutilitarian strains.

He makes of all this a smoothly flowing narrative. A chapter ("The Eternal Child") exhibiting D'Alembert as the quintessential "natural man" is followed by well expounded examples of "natural mathematics", from Johann Bernoulli's geometrical study of the catenary to Lagrange's algebraization of mechanics. (Readers wary of technicalities are assured that they can skip these sections without losing the main argument.) Then in the book's second part (which is much the longer) the author gives, for each of his four protagonists, a substantial biographical sketch, an account of his posthumous reputation, and (except for Abel) a quite detailed description of one aspect of his work. A final chapter recasts the argument through portraits, which of course the book reproduces. Galois and Abel, Alexander says (p. 257), *look* like Keats and Byron – "intense young men with blazing eyes, focused not on us but on greater truths beyond the horizon" (p. 15). In contrast, he argues, 19th-century *scientists* (here represented pictorially by Kelvin and Helmholtz) resemble those "natural men" who made *18th*-century mathematics (here D'Alembert and Johann Bernoulli).

Alexander's theories are ingenious, and he has told his tale well. His account has many solid virtues, to which I will return below. But I want first to voice some reservations.

These begin with his key idea, in the Enlightenment context, of a "natural man", whose blend of childlike simplicity and social ease fit him uniquely for success in mathematics and science. Alexander's goal here is transparent: he wants to maximize the conceptual distance between this figure and the lonely misfits who for him represent Romantic mathematics, and indeed he declares in due course the starkness of the contrast (p. 44). Unfortunately the very relevance of this "natural man" is (I think) deeply problematic. His scientific aptitude is said to stem from the fact that like mathematics itself he retains especially close contact with the physical world; but oddly enough one can challenge that badge of distinction on two exactly opposite counts. On the one hand, grounding mathematics in experience is an obvious, common-sense approach which dates from antiquity, a perspective which would not seem to require in its devotees any particular personality. And on the other hand, much more significantly, in the 18th century mathematics had long since been moving *away from* sole reliance on those physical roots – and thus, one would think, leaving the comfort zone of a child of nature – by adding concepts born of its own needs and serving its own purposes – perhaps it is enough to cite "imaginary" (!) numbers. Thus a more nuanced view than Alexander's of Enlightenment mathematics seems to render the allegedly special qualifications of the "natural men" neither necessary nor sufficient.

Moreover that same perspective undermines some of his key ascriptions of mathematical newness to the 19th century. He is wrong in saying (p. 5) that not until then was the subject "self-contained": already in the age of D'Alembert it wrestled with its own internal problems, for their own sake and without thought or chance of application. Enlightenment number theory rebuts by itself his contention that pure, beautiful mathematics, disconnected from the physical, was invented by his Romantics. I shall return below to this critical issue of novelty.

The account here of the 18th century's mathematized science also evokes qualms. That a child of nature would have the best prospects in this line would seem to require that the physical world be, and be thought to be, immediately intelligible, and Alexander asserts just that (p. 264). But again history's testimony begs to differ. Many in that age – including D'Alembert! – concluded reluctantly that the reality behind sensory appearances is in fact unknowable. It is ironic that for the physical "mysteries" (p. 51) supposedly uncovered by mathematics Alexander's favorite adjective is "hidden"; for a common positivist methodology *settled for* mathematical description of phenomena whose underlying essence was assumed to *remain* beyond our grasp. Geometers could compute – so ran the justification – even where they lacked understanding.

Shaky too, in my opinion, is Alexander's idea (p. 72) that toward 1800 increasing abstraction in mathematics was widely seen to pose a grave threat to mathematized science. For this he offers neither spokesman nor evidence except a remark by Lagrange: "The mine [of mathematics] is almost too deep already, and unless new seams are discovered, it will be necessary to abandon it sooner or later" (quoted on p. 73). But this says nothing whatever about abstraction; it says exactly what it *seems* to say, that mathematical progress has apparently left not much of great interest or consequence to discover. Actually the *application* of mathematics in those years, so far from breeding angst, was flourishing. The 18th century's last decades saw a massive upsurge of quantification in many fields, thanks above all to dramatic improvements in measuring instruments, an aspect of the landscape which Alexander never mentions.

But it is his conclusions about the Romantic legacy, the heart of his argument, which evoke my greatest unease. Take first his insistence that even now the legends of Galois, Abel, Bolyai and Cauchy determine the prevailing "image" of mathematicians, both for professionals and "in the popular imagination" (p. 3). Actually his position is not quite clear: pages 164 and 266–67 say respectively that those stories convey what "a true mathematician *is*" and what he or she "*should be*" (my italics in both cases). But since elsewhere (p. 3) Alexander concedes that in fact "the majority of practicing mathematicians are undoubtedly normal individuals", I shall assume in what follows the second option, that the Romantic heritage points to an *ideal*. So then – do mathematicians think that they, or their colleagues, *should be* "tragic romantic heroes"? Like (I presume) many readers of *Historia Mathematica*, I have known quite a few professional mathematicians, and I have read about many more, and for me the absurdity of that idea is an elementary empirical fact. And the general public – how does *it* view mathematicians? For his own answer Alexander offers, so far as I can see, no grounds. What would in fact count as evidence? Pollster surveys? Try a thought experiment: imagine putting the question to (say) 50 people met randomly in the street. "What, in your opinion, should a mathematician be like?" How many respondents would use *any* of the words "tragic", "romantic" and "hero", let alone all of them? How many would point to Galois, Abel, Bolyai or Cauchy as a prototype?

How many, come to that, would have the foggiest idea who any of those people were? To my mind this supposed popular "image" is the least plausible of all the conclusions that Alexander draws.

But presumably the more interesting issue for readers of this journal lies elsewhere, in his other major claim, that the legends of his foursome – as opposed, of course, to their technical contributions – went "hand in hand" (p. 248) with the birth of a radically new mathematics, whose modern career those legends continue to sustain. Again justification seems elusive: his main statement on the matter (p. 264), read carefully, turns out to *assume* that influence, without proof. Perhaps though the impact of the "tragic romantic heroes" speaks, so to say, for itself, and is conspicuous in a mere overview? Here is another of the capsule characterizations which Alexander provides (pp. 269–70):

> [T]he fundamental assumptions that govern the practice of mathematics have ... persisted from their [his four iconic "heroes"] day to ours. The notion that mathematics is its own self-contained world, separate from physical reality, governed by pure reason, and safeguarded by strict standards of rigor, seemed radical when it was first proposed by Galois, Abel and their contemporaries. But nearly two centuries later these tenets appear so obvious to professional mathematicians that they are hardly ever stated.

Possibly, one wants to reply, that is because *all* of those supposedly novel features actually *predate* the 19th century – the idea that mathematics represents a "higher" plane of "reality" than physical experience goes back to Plato – and so are *not* legacies of the Romantic age. Alexander is certainly right to sing their praises, but certainly wrong to credit their origins to his "heroes". The transformations that truly define the last two centuries lie deeper, and they have been wholly internal – see for example Howard Stein's splendid survey in William Aspray and Philip Kitcher's *History and Philosophy of Modern Mathematics* [Stein, 1988]. Only rarely does Alexander hint at those genuinely modern innovations – and then the results are telling. Thus he quotes (p. 209) Dieudonné's praise of Galois for shifting emphasis from calculations to deeper concepts and structures. To Dieudonné that policy seemed "modern" and "familiar", for of course after Galois others (Dirichlet, Riemann) reprised the theme, and it has passed into the mainstream. But it is a *technical* advance, a move *within* mathematics, and on Alexander's own showing the role that Dieudonné assigns in this context to Galois's life and legend is ... zero.

Of course all such criticisms must be set against the many strengths of Alexander's case and of its exposition. His main thesis is forcefully and (for the most part) clearly argued. The mini-biographies of his protagonists are outstanding, and the tracing of their respective posthumous reputations, which often taps out-of-the-way sources, is especially valuable. His prose, barring only some patches of purple, is admirable, and typos are nearly nonexistent. A bibliography would be welcome, and sterner editing could have tightened a text which can seem extremely repetitive, but on points Harvard has done an exemplary job of book-making.

I regret then that in a final verdict the misgivings which I have been trying to express must loom large. Alexander starts well. His sketch of the Enlightenment scene, for all its deficiencies, offers much – his portrait of D'Alembert, his mathematical samples – that is interesting and useful. He rightly identifies features shared by Romanticism and contemporary mathematics, and at one point (p. 268) he makes a moderate statement of influence which tempts agreement (remember again the younger Bolyai): "the new romantic sensibilities of the early nineteenth century allowed mathematics to develop in certain directions that were previously considered illegitimate". But I fear that in building on that slender foundation Alexander has seriously overplayed his hand. Throughout his book he displays a strong (Romantic!) tendency to exaggerate, to push potentially valid points "over the top", in substance or in rhetoric or both – his declaration that modern mathematics flees "worldly contamination" (p. 269) typifies *many* cases. And as in such small details, so also in his larger purpose. He has erected the few suggestive links between mathematics and the Romantic *Zeitgeist* into a theory whose vast reach and implication neither the historical record nor everyday experience will support. He misidentifies the truly novel features of modern mathematics, and he greatly underestimates its autonomous internal dynamic; and, correspondingly, in ascribing to the legends of Galois, Abel, Bolyai and Cauchy a determining influence on its real innovations, its technical progress and its popular image he makes claims which he does not, and in my view cannot, substantiate. He has written an original and attractive book, and I suspect that some readers will find his heroes and martyrs Romantically appealing. But he has also given new proof, were proof still needed, that history tends to be subtler than historians' cherished theories.

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Available online 3 July 2010

doi:10.1016/j.hm.2010.05.005

Flatland by Edwin A. Abbott: An Edition with Notes and Commentary
By William F. Lindgren and Thomas F. Banchoff. New York (Cambridge University Press). 2010. ISBN 978-0-521-75994-6. ix + 294. $14.00.

Edwin Abbott's *Flatland* (1884) is a deceptive little book. Masquerading as a fanciful exploration of multi-dimensional geometry, it is, as its author tells us 'A Romance of Many Dimensions'. Its hero is A Square (a pun on Abbott's name: Edwin Abbott Abbott) who lives in a two-dimensional world where all the men are geometric figures and all the women are straight lines. One day a Sphere arrives and takes him to a world of three dimensions, and the Square's mind is laid open to the possibility that worlds may exist which he can never fully understand – worlds of three or even four or more dimensions. The story is divided with near-mathematical precision into two halves: Part I is a stinging satire on contemporary society while Part II deals with the topic Abbott later told his readers was the primary subject of the text [Abbott, 1897, 28–29]. *Flatland* it appears is a spiritual exercise – an allegorical odyssey through the scientific and philosophical challenges facing liberal Christianity at the end of the 19th century. Woe betide any editor, then, who tries to produce the definitive edition of this slippery tale.