

Dark energy, induced gravity and broken scale invariance

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Abstract

We study the cosmological evolution of an induced gravity model with a self-interacting scalar field σ and in the presence of matter and radiation. Such model leads to Einstein gravity plus a cosmological constant as a stable attractor among homogeneous cosmologies and is therefore a viable dark-energy (DE) model for a wide range of scalar field initial conditions and values for its positive γ coupling to the Ricci curvature $\gamma\sigma^2 R$.

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1. Introduction

Several years ago a model for a varying gravitational coupling was introduced by Brans and Dicke [1]. The model consisted of a massless scalar field whose inverse was associated with the gravitational coupling. Such a field evolved dynamically in the presence of matter and led to cosmological predictions differing from Einstein gravity (EG) in that one generally obtained a power-law dependence on time for the gravitational coupling. In order to reduce such a strong time dependence in a cosmological setting, while retaining the Brans–Dicke results in the weak field limit, several years ago a simple model for induced gravity [2,3] involving a scalar field σ and a quartic $\lambda\sigma^4/4$ potential was introduced. This model was globally scale invariant (that is did not include any dimensional parameter) and the spontaneous breaking of scale invariance in such a context led to both the gravitational constant and inflation, through a non-zero cosmological constant [4]. The cosmological consequences of introducing matter as a perturbation were studied

leading to a time dependence for the scalar field and consistent results. Since the present observational status is compatible with an accelerating universe dominated by dark energy (DE) we feel that the model should be studied in more detail.

In an EG framework quadratic or quartic potentials for canonical scalar fields are consistent with DE only with an extremely fine tuning in the initial conditions leading to slow-roll evolution until the present time. Indeed, a massive or a self-interacting scalar field in EG, respectively behave as dust or radiation during the coherent oscillation regime. In contrast with this, induced gravity with a self-interacting $\lambda\sigma^4/4$ potential has as attractor EG plus a cosmological constant on breaking scale invariance. The simple model we consider illustrates how non-trivial and non-perturbative dynamics can be obtained in the context of induced gravity DE, and more generally within scalar-tensor DE.

2. The original model

Let us consider a system described by the Lagrangian

$$\mathcal{L} = \frac{1}{2} \sqrt{-g} \left(-g^{\mu\nu} \partial_\mu \sigma \partial_\nu \sigma + \gamma \sigma^2 R - \frac{\lambda}{2} \sigma^4 \right) + \sum_{j=R,M} \mathcal{L}_j \quad (1)$$

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where γ, λ are dimensionless, positive definite parameters and the \mathcal{L}_i 's are the contributions of cosmological fluids behaving as dust and radiation. If we consider the homogeneous mode for the scalar field evolving on a spatially flat Robertson–Walker background

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = -dt^2 + a^2(t) d\vec{x}^2 \quad (2)$$

from the above Lagrangian one obtains the following set of independent equations

$$\begin{cases} H^2 = \sum_{j=R,M} \frac{\rho_j}{3\gamma\sigma^2} + \frac{1}{6\gamma} \frac{\dot{\sigma}^2}{\sigma^2} - 2H \frac{\dot{\sigma}}{\sigma} + \frac{\lambda}{12\gamma} \sigma^2, \\ \frac{d}{dt}(a^3 \sigma \dot{\sigma}) = a^3 \sum_{j=R,M} \frac{(\rho_j - 3P_j)}{(1+6\gamma)}, \\ \dot{\rho}_j = -3H(\rho_j + P_j) \end{cases} \quad (3)$$

where $P_j = w_j \rho_j$, $w_R = 1/3$, $w_M = 0$, the dot denotes differentiation with respect to the cosmic time t and $H \equiv \dot{a}/a$. Note that the system (3) of equations are respectively the Friedmann, the Klein–Gordon and the continuity equations. In the above context, the critical densities for radiation, matter and σ can be taken to be:

$$\begin{aligned} \tilde{\Omega}_R &= \frac{\rho_R}{3\gamma\sigma^2 H^2}, & \tilde{\Omega}_M &= \frac{\rho_M}{3\gamma\sigma^2 H^2}, \\ \tilde{\Omega}_\sigma &= \frac{(\dot{\sigma}^2 - 12H\dot{\sigma}\sigma + \frac{\lambda\sigma^4}{2})}{(6\gamma\sigma^2 H^2)}, \end{aligned} \quad (4)$$

where $\tilde{\Omega}_M + \tilde{\Omega}_R + \tilde{\Omega}_\sigma = 1$.

On using the equations of motion one finds that the curvature scalar is given by

$$R = \frac{1}{\gamma\sigma^2} \left(\lambda\sigma^4 - \dot{\sigma}^2 + \sum_{j=R,M} \frac{\rho_j - 3P_j}{1+6\gamma} \right). \quad (5)$$

Even in the absence of the fluids ρ_j , the quantity (5) has an undefined sign, implying that the σ potential has one or two minima depending on the phase space trajectory of σ .

We also observe that the above model (1) corresponds to an interacting Brans–Dicke field ϕ . Indeed on redefining $\phi = \sigma^2$ one has

$$\mathcal{L} = -\frac{\omega}{2\phi} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{\lambda}{4\gamma^2} \phi^2 + \frac{\phi}{2} R + \sum_i \mathcal{L}_i \quad (6)$$

where $\omega = (4\gamma)^{-1}$. The above model (1) also admits a transformation in the Einstein frame in which the potential for the redefined field is just a cosmological constant ($= 16\pi^2 G^2 \lambda / \gamma^2$) and dark matter and the redefined (free) scalar field are coupled (by $(\gamma^2 \sigma^4)^{-1}$).

2.1. Super-acceleration

Let us note that scalar-tensor theories admit naturally super-acceleration, i.e. $\dot{H} > 0$ (see [5] for a generalization of the model in [2,3]). From the Friedmann equation and expression (5) one finds

$$\dot{H} = -\frac{1}{\gamma\sigma^2} \left[\sum_{j=R,M} \frac{(1+8\gamma+w_j)}{2(1+6\gamma)} \rho_j + \frac{\dot{\sigma}^2}{2} - 4\gamma H \sigma \dot{\sigma} \right]. \quad (7)$$

The possibility of super-acceleration is due to the $\gamma H \sigma \dot{\sigma}$ contribution in the brackets. Let us note that γ should be large enough in order to have a super-accelerated phase which can be distinguished from a de Sitter phase. This cannot be accomplished in EG with a scalar field having a standard kinetic term.

2.2. Scalar field plus radiation

When pressureless dust can be neglected with respect to radiation ($\rho_R \gg \rho_M$) a phase space analysis can be performed for the system. In this case, the Klein–Gordon equation in (3) can be integrated and leads to

$$H^2 = \frac{c_0}{a^8 \sigma^2 \sigma'^2} \quad (8)$$

where a prime denotes a derivative with respect to $N \equiv \ln a$ and c_0 is an integration constant. The Friedmann-like equation can then be cast in the form of an autonomous equation describing the possible solutions for the rescaled field $\varphi \equiv a\sigma$:

$$\begin{aligned} \gamma c_0^2 \left[\varphi^2 - \frac{1}{6\gamma} (\varphi' - \varphi)^2 + 2\varphi(\varphi' - \varphi) \right] \\ = \frac{\varphi^4}{3} \left(\frac{\rho_{R,0}}{\varphi^2} + \frac{\lambda}{4} \varphi^2 \right) (\varphi' - \varphi)^2 \end{aligned} \quad (9)$$

where we set $\rho_{R,0} = \rho_R|_{a=1}$ (similarly we shall set $\rho_{M,0} = \rho_M|_{a=1}$). The right-hand side of Eq. (9) is a semi-positive definite quantity and thus its left-hand side must satisfy the inequality

$$\varphi^2 \beta \equiv \varphi^2 - \frac{1}{6\gamma} (\varphi' - \varphi)^2 + 2\varphi(\varphi' - \varphi) \geq 0 \quad (10)$$

which determines the allowed region of phase space.

Once γ and λ are fixed, one must choose the initial conditions encoded in c_0 and $\rho_{R,0}$ in order to determine completely a phase space trajectory. Note that if $\gamma = 0$ or $c_0 = 0$ there is only one possible trajectory given by $\varphi' = \varphi$ corresponding to the GR limit $\dot{\sigma} = 0$.

3. Numerical analysis

If a realistic cosmological evolution is considered without any approximations, the system (3) can only be solved numerically. On defining a rescaled Hubble parameter $\mathcal{H} = \lambda^{-1/4} H$ and introducing the dynamical variable $x = \lambda^{1/2} \sigma^2$ the relevant equations can be rewritten as

$$\begin{cases} \mathcal{H}^2 x'' + [2\mathcal{H}^2 + \frac{1}{2}(\mathcal{H}^2)'] x' = \frac{2\rho_M}{6\gamma+1}, \\ \mathcal{H}^2 \left[1 - \frac{1}{24\gamma} \left(\frac{x'}{x} \right)^2 + \frac{x'}{x} \right] = \frac{\rho_M + \rho_R}{3\gamma x} + \frac{x}{12\gamma} \end{cases} \quad (11)$$

with $\rho_R = \rho_{R,0}/a^4$ and $\rho_M = \rho_{M,0}/a^3$. In this form the λ dependence simplifies and one can find numerical solutions with a high precision even when λ is extremely small which is the case (note that this is crucial since in the $\lambda \rightarrow 0$ limit one would otherwise just recover the Brans–Dicke theory).

The initial kinetic energy-density of the σ field is rapidly dissipated during early stages when radiation dominates over dust, as can be deduced from phase space analysis and is confirmed

by Fig. 1. Consideration of high values for the time derivative of the scalar field (when it dominates the energy-density of the universe, see Fig. 2) lead to a “stiff” universe. If it existed, such an epoch should have occurred well before nucleosynthesis in order not to spoil the observed abundances of the light elements. Indeed this could be the case for very high densities arising from fermions (quarks) interacting with vector mesons (gauge fields) [2].

The value for σ remains constant until the onset of the matter dominated era and then its variation is well approximated by

$$\frac{d\sigma^2}{dN} \simeq \frac{4\gamma\sigma_0^2}{1+6\gamma}, \quad (12)$$

where we use $\sigma_0 = \sigma|_{z=0}$, and then it approaches the constant value finally leading to the de Sitter stage. The above relation (12) is obtained by assuming a matter (dust) dominated universe and is as expected within the context of induced gravity theories [1,2].

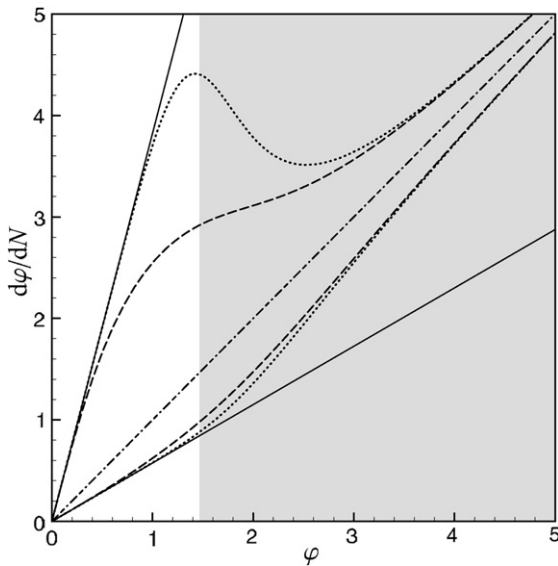


Fig. 1. Phase space plot of Eq. (9); on the horizontal axis the field φ is represented while on the vertical we plot its derivative $\frac{d\varphi}{dN}$ with $\gamma = 2 \times 10^{-1}$, $\lambda = 10^{-1}$, $c_0 = 1$ and $r_0 = 0$ (dotted trajectories) or $r_0 = 5 \times 10^{-1}$ (dashed trajectories). The solid lines are the boundaries of the accessible region of the phase space while the dotted-dashed line represents all the possible solutions with $\dot{\sigma} = 0$. The shaded area is the portion of phase space where the scalar field has a double well potential.

From here on, we shall consider different choices of parameters and initial conditions for our analysis (see Table 1). In the first two cases in Table 1 γ has been taken to be its maximum value allowed by solar system constraints [6–8], while initial conditions for σ describe two extreme situations: $\beta_{\text{ini},1} \simeq 0$ and $\beta_{\text{ini},2} \simeq 1$. The early stages of the evolution, when the matter contribution is negligible, are well approximated by the plots in Fig. 1. As expected, the field σ rapidly evolves toward the shaded region where $\beta \simeq 1$ (see Fig. 3). Since $\gamma \ll 1$, when $\beta \simeq 0$ the Hubble parameter scales as a^{-3} .

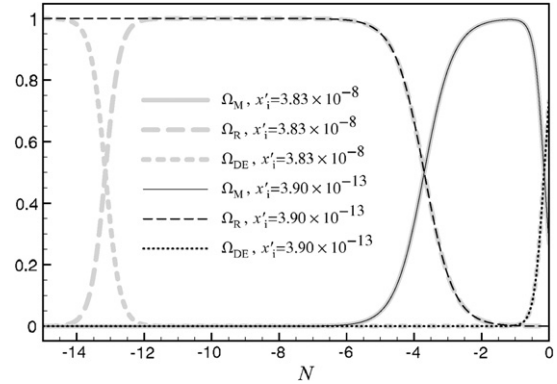


Fig. 2. Plots of Ω_R , Ω_M , Ω_{DE} for the sets of parameters and initial conditions 1 and 2 of Table 1.

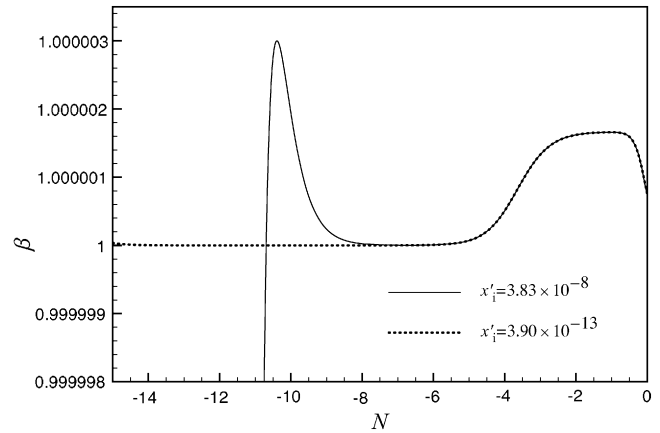


Fig. 3. Evolution of β defined in (10) plotted for the sets of initial conditions and parameters 1 and 2 of Table 1. Note that the solid and the dotted curves are indistinguishable between $N \sim -8$ and $N \sim 0$.

Table 1
Parameters, initial conditions and results for different cases. All these cases have $\Omega_M \simeq 0.27$, $\Omega_{\text{DE}} \simeq 0.73$, $H_0 = 73 \text{ km s}^{-1} \text{ Mpc}^{-1}$, $\gamma\sigma_0^2 \simeq (8\pi G)^{-1}$, $\gamma_M = 5 \times 10^{-7}$. Note that the solar system bound on γ is relaxed for # 3 to 8; nonetheless # 3 to 7 of the model are still satisfactory on comparing with cosmological data coming from SNIa and first Doppler peak of CMBR anisotropies. The smaller the value of γ the closer is our model to Λ CDM

#	γ/γ_M	x_i	x'_i	$w_{\text{DE}0} + 1$	χ^2_{SNIa}	$(d_{\text{L}}^{\text{LSS}} - d_{\text{L}\Lambda}^{\text{LSS}})/d_{\text{L}\Lambda}^{\text{LSS}}$
1	1	1.10×10^{-5}	3.83×10^{-8}	-10^{-6}	186	$+8.2 \times 10^{-6}$
2	1	1.12×10^{-5}	3.90×10^{-13}	-10^{-6}	186	-6.6×10^{-7}
3	$2^{-3} \times 10^2$	1.056×10^{-5}	1.30×10^{-7}	-10^{-5}	186	-1.1×10^{-5}
4	$2^{-3} \times 10^2$	1.112×10^{-5}	1.38×10^{-11}	-10^{-5}	186	-1.4×10^{-5}
5	2×10^2	1.12×10^{-5}	0	-2×10^{-4}	186	-1.8×10^{-4}
6	$2^{-3} \times 10^4$	2.01×10^{-5}	-2.32×10^{-6}	-10^{-3}	188	-1.0×10^{-3}
7	2×10^3	1.09×10^{-5}	0	-2×10^{-3}	189	-2.0×10^{-3}
8	2×10^4	8.32×10^{-6}	0	-0.02	234	-1.7×10^{-2}

Let us end this section by noting that one should not evolve the scalar field equations backwards in time—i.e. with redshift—since the correct physical trajectory on which the scalar field evolves cannot be identified.

3.1. Solar system Newtonian constraints

The parameters of a scalar-tensor theory can be constrained through solar system observations which test classical general relativity [9]. Since our model coincides with the Brans–Dicke case in the weak field limit [2,3], one has [9,10] $\beta_{PN} - 1 = 0$ and

$$\gamma_{PN} - 1 = -\frac{4\gamma}{1 + 8\gamma}. \tag{13}$$

Therefore, since our γ is positive and $\gamma_{PN} - 1 = (2.1 \pm 2.3) \times 10^{-5}$ [6,7], we shall take an upper bound for γ given by $\gamma_M = 5 \times 10^{-7}$ which we shall use. Let us note however that for other values of γ_{PN} reported in the body of Ref. [6] the resulting γ_M would be 5 to 10 times larger.

In scalar-tensor gravity the effective Newtonian constant, namely the gravitational coupling measured in Cavendish like experiments, is

$$G_{\text{eff},0} = \frac{1}{8\pi\gamma\sigma_0^2} \frac{8\gamma + 1}{6\gamma + 1}. \tag{14}$$

Possible variations of the effective Newtonian constant $G_{\text{eff},0}$, are allowed in the interval $-7 \times 10^{-14} \text{y}^{-1} < \dot{G}_{\text{eff},0}/G_{\text{eff},0} < 3 \times 10^{-14} \text{y}^{-1}$ [8]. The value G_N of Newton’s constant is known with a precision of $\sim 10^{-3}$ [11]; we shall thus restrict our analysis to a set of initial conditions for σ leading to values of $G_{\text{eff},0}$ within the bounds given by the above experimental uncertainty on the measurement of G_N .

4. Comparison with Einstein gravity

In order to compare a scalar-tensor theory with dynamical scalar field models in EG (also dubbed quintessence [12]), we first identify DE energy-density and pressure. A consistent definition of such quantities can be given [10] if one identifies the effective energy-density and pressure in the set of equations (3) in an EG framework with $8\pi G_N \equiv \gamma\sigma_0^2$:

$$\begin{cases} \rho_{\text{DE}} = 3\gamma\sigma_0^2 H^2 - \sum_{j=R,M} \rho_j, \\ \rho_{\text{DE}} + p_{\text{DE}} = -2\gamma\sigma_0^2 \dot{H} - \sum_{j=R,M} (\rho_j + p_j). \end{cases} \tag{15}$$

Such an identification maintains the continuity equation for DE and leads explicitly to

$$\begin{cases} \rho_{\text{DE}} \equiv \frac{\sigma_0^2}{\sigma^2} \left(\frac{\dot{\sigma}^2}{2} - 6\gamma H \dot{\sigma} \sigma + \frac{\lambda}{4} \sigma^4 \right) + \sum_{j=R,M} \rho_j \left(\frac{\sigma_0^2}{\sigma^2} - 1 \right), \\ p_{\text{DE}} \equiv \frac{\sigma_0^2}{\sigma^2} \left[\frac{\dot{\sigma}^2}{2} - 2\gamma H \dot{\sigma} \sigma - \frac{\lambda}{4} \sigma^4 + \sum_{j=R,M} \frac{2\gamma\rho_j + p_j}{1+6\gamma} \right] - p_R. \end{cases} \tag{16}$$

We can as well identify the relative densities for the fluids involved in the evolution

$$\Omega_R = \frac{\rho_R}{3\gamma\sigma_0^2 H^2}, \quad \Omega_M = \frac{\rho_M}{3\gamma\sigma_0^2 H^2},$$

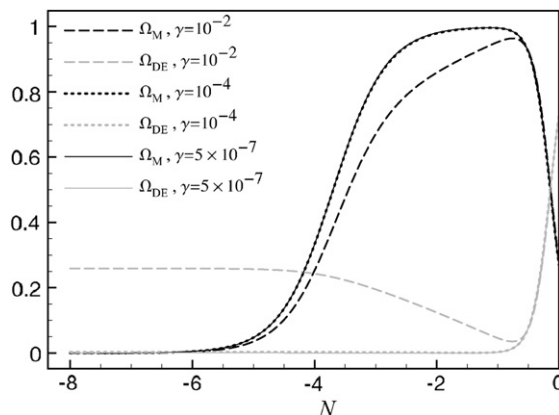


Fig. 4. Evolution of Ω_M and Ω_{DE} as defined in (16) for different choices of γ .

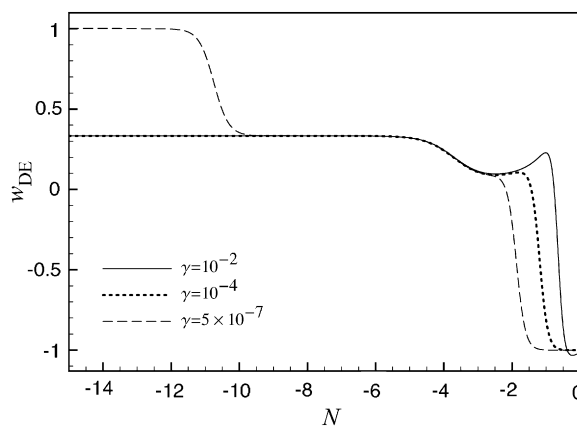


Fig. 5. Evolution of w_{DE} as defined in (16) for different choices of γ .

$$\Omega_{\text{DE}} = \frac{\rho_{\text{DE}}}{3\gamma\sigma_0^2 H^2} \tag{17}$$

where $\Omega_M + \Omega_R + \Omega_{\text{DE}} = 1$. We stress that these critical densities based on this fictitious EG model differ from the previous ones (4) wherein the gravitational coupling varies. For $\gamma \ll 1$ differences between the two definitions are negligible. The matter and DE critical densities are plotted in Fig. 4. The state parameter w_{DE} defined in Eq. (16) is displayed in Fig. 5 for $\gamma = 5 \times 10^{-7}, 10^{-4}, 10^{-2}$ (cases #1, 5, 8 in Table 1). Note that interpreting this self-interacting induced gravity as an EG model, the parameter of state of DE is 1/3 during the radiation era, is positive during the matter dominated stage (with a non-negligible bump for $\gamma = 10^{-2}$) before setting to $-1 + \mathcal{O}(\gamma)$ at present, as can be seen in Table 1, where we have reported few examples of the simulations we have performed. Let us also note how the EG model is not completely dominated by matter, in particular for $\gamma \sim 10^{-2}$.

Since the state parameter at present differs only slightly from -1 it is therefore appropriate to compare this self-interacting induced gravity model with a Λ CDM model with $\Omega_{\text{DE},0}$ as the fractional energy-density stored in the cosmological constant. The relative deviation of such an Einstein Λ CDM model from the original model for the comoving distance

$$d_L(z) = (1+z) \int_0^z \frac{dz'}{H(z')}, \tag{18}$$

evaluated at the last scattering surface ($z = 1089$) is shown in the last column of Table 1. The comoving distance to the last scattering surface enters in the acoustic scale, which is related to the characteristic angular scale of the peaks of the CMB angular power spectrum. The current uncertainty on the acoustic scale is of the order of one percent; on the basis of the numbers reported in Table 1, this implies that models with $\gamma \lesssim \mathcal{O}(10^{-2})$ may be constrained by the position of the peaks of the CMB angular power spectrum.

We now compare the model with data from Supernovae Ia, which constrain the evolution of the scale factor for $0 < z < 2$, as done in other works studying scalar-tensor DE [13]. The supernova data we use are the “gold” set of 182 SNe Ia by Riess et al. [14]. In scalar-tensor theories, since the Chandrasekhar mass ($\sim x^{3/2}$) varies with redshift, the modulus-redshift relation is:

$$\mu(z) = 5 \log_{10} \frac{d_L(z)}{10 \text{ pc}} + \frac{15}{4} \log_{10} \frac{x_0}{x(z)}. \quad (19)$$

In Table 1 the χ^2 values are listed showing that $\gamma \sim 10^{-2}$ is strongly disfavoured by SNe Ia. An accurate statistical analysis is clearly beyond the scope of this work, but we expect that SNe Ia may constrain $\gamma \sim \mathcal{O}(10^{-3})$.

5. Conclusions

We have shown how a simple model of self-interacting induced gravity [2,3] is a viable DE model, for tiny values of the self-coupling ($\lambda \sim \mathcal{O}(10^{-128})$). The model has a stable attractor towards EG plus Λ and can be very similar to the Λ CDM model for the homogeneous mode, on taking into account the Solar system constraints quoted in [6–8]. At the cosmological level, in the presence of radiation and dust, it is interesting that for such a simple potential (quartic for induced gravity or interacting for the equivalent Brans–Dicke model) the model has an attractor towards EG plus Λ , very differently from the case of a massless scalar—i.e. $\lambda = 0$ —for which there is no mechanism of attraction towards EG. The attractor mechanism towards GR and the onset to acceleration are both inevitably triggered by the same mechanism, i.e. scale symmetry breaking in this induced gravity model. In contrast with EG, the choice of a runaway potential [15] for quintessence is not mandatory (see however [16] for a study of these potential in the context of scalar-tensor theories). If the final attractor is an accelerated universe, the only constraints on parameters and initial conditions come from observations. We have shown that late time cosmology—after

recombination, for instance—is mostly insensitive with respect to the initial time derivative of σ . For this reason, the full set of parameters and initial conditions of the model are fully specified on taking the observed values for G , H_0 , Ω_Λ .

We have discussed in detail such model in the context of Einstein gravity, i.e. keeping the Newton constant (approximately) fixed at its actual value. The model predicts an equation of state of the equivalent Einstein model with w_{DE} slightly less than -1 at present and homogeneous cosmology by itself seems able to constrain $\gamma \lesssim \mathcal{O}(10^{-3})$, although a full statistical analysis is clearly beyond the scope of this work.

It is interesting to also study also what other potentials, besides the simple potential $\lambda\sigma^4/4$ employed in this Letter, will also be compatible with the observed evolution of the universe.

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