such as the "Mathematics Genealogy Project" and "The Mac Tutor History of Mathematics Archive" will be updated on the basis of Tobies' dictionary. The book is solidly bound and contains several little known pictures of biographees.

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The Architecture of Modern Mathematics: Essays in History and Philosophy
By José Ferreirós and Jeremy Gray. Oxford (Oxford University Press). 2006. ISBN 978-0-19-856793-6. 472 pp. $69.50

Traditionally, above all within analytical circles, the history and the philosophy of mathematics have interacted only occasionally, and almost accidentally, on specific issues. These interactions did not affect the way the two subjects were developed. Such a state of affairs, far from being the consequence of carelessness or neglect, was explicitly theorized by Frege, the founding father of analytical philosophy, who famously said,

> Do the concepts, as we approach their supposed sources, reveal themselves in peculiar purity? [Frege, 1884, Introduction, vii]

and replied,

> Not at all; we see everything as through a fog, blurred and undifferentiated. It is as though everyone who wished to know about America were to try to put himself back in the position of Columbus, at the time when he caught the first dubious glimpse of his supposed India. Of course, a comparison like this proves nothing; but it should, I hope, make my point clear. It may well be that in many cases the history of earlier discoveries is a useful study, as a preparation for further researches; but it should not set up to usurp their place. [Frege, 1884, Introduction, vii–viii]

This negative attitude toward the contribution that the history of mathematics can give to the philosophy of mathematics is found not only in Frege and among analytical philosophers, but is something that also colors the thought of Brouwer and Hilbert.

It seems to me that, in contrast with what is usually said about this, such an attitude is not the consequence of the fact that these authors and their programs are concerned with the foundations of mathematics. Such an attitude has rather to do, on the one hand, with Brouwer's and Hilbert's philosophies of mathematics, for which mathematics is not a science of matters of fact (in Hume's sense), and, on the other, with their overly narrow view of the history of mathematics. This is seen by them as a mere temporal ordering of past events bound to develop either into a purely descriptive history of ideas or into an occasionally entertaining, but philosophically unhelpful, production of biographies.

It must be emphasized that such a way of thinking about the relationship between the history and the philosophy of mathematics has not been a strict monopoly of philosophers, but has also been shared by many historians of mathematics, as witnessed, even recently, by interesting contributions to the subject, such as [Moore, 1982] and [Avellone et al., 2002].

What I have described above as the traditional separation between the history and the philosophy of mathematics began to be challenged by I. Lakatos in *Proofs and Refutations* [Lakatos, 1963–1964]. In much of Lakatos's work in the philosophy of mathematics, the history of mathematics is no longer seen as a mere descriptive enterprise, but becomes a rational reconstruction guided by explicit philosophical paradigms—such as the Popperian one of conjectures and refutations—that provide explicit schemata of interpretation.

It is important to notice that what we might call "Lakatos's provocation" did not remain an isolated cry. Some time after the publication of *Proofs and Refutations*, and other contributions by Lakatos to the philosophy of mathematics, a string of books—mainly collections of essays—and journal articles started to appear in which the traditional demarcation line between the history and the philosophy of mathematics was crossed from each side of the divide. Examples of such collections are [Aspray and Kitcher, 1988, Gillies, 1995, Grosholz and Breger, 2000, Tymoczko, 1986].

The Architecture of Modern Mathematics (*AMM*), the object of the present review, is one of the latest, very stimulating, products of this relatively recent tradition, in which the synergy between the history and the philosophy of mathematics is exploited with great benefit for the topics discussed. Ferreirós and Gray's edited volume is divided into three parts: "Reinterpretations in the history and philosophy of foundations"; "Explorations into the emergence of modern mathematics"; and "Alternative views and programmes in the philosophy of mathematics." These three parts are preceded by an ample introduction written by both editors, and are followed by a "Coda" by Gray, entitled "Modern mathematics as a cultural phenomenon."

In the introduction—one of the most interesting things to be found in the book—after an explanation of what they mean by "The architectural metaphor" (p. 2), namely mathematics as a finished building with solid foundations—Ferreirós and Gray engage in a detailed and useful discussion of the relationship between the history and the philosophy of mathematics. Their aim is that of contributing to the development of a philosophy of mathematical practice that is supported by, and harmonizes with, the history of mathematics.

In Part I, we find four contributions, by Michael Beaney, José Ferreirós, Jamie Tappenden, and Leo Corry. In his paper, Beaney argues in favor of the compatibility between foundational studies and history-based work on mathematical methodology, while Ferreirós defends the philosophical relevance of part of Riemann's work, showing that this comes about also as a consequence of Riemann's epistemological concerns. In the third contribution to this part of the book we find that, in Tappenden's opinion,

> … we have misunderstood much of what Frege was trying to do, and missed the intended significance of much of what he wrote, because our received stories underestimate the complexity of nineteenth-century mathematics and mislocate Frege's work within that context. (p. 97)

Tappenden tries to put us right on Frege, showing the relevance of Riemann's thought on complex analysis to Frege's foundational concerns, in particular, with regard to the concept of function. Last, Corry's paper revolves around the idea that, in contrast with the received view, Hilbert's conception of geometry changed somewhat during his career, even after the publication of the *Grundlagen*, as a consequence, among other things, of his reflections on the origin of geometrical knowledge.

Part II of the book opens with an article in which Jeremy Avigad discusses Dedekind's development of his theory of ideals as a contribution to a philosophy of mathematical practice. This is in recognition of the unproductive character of discussions on whether Dedekind's philosophy of mathematics ought to be used to explain his methodological innovations or vice versa—because these two objects of Dedekind's concern are not separable.

Then, in a very interesting article, Colin McLarty investigates the emergence of the structuralist viewpoint in algebra, topology, and, ultimately, category theory in relation to Emmy Noether's work. We next find Paolo Mancosu's paper, in which he argues that "Tarski upheld a fixed-domain conception of a model in his 1936 paper and that he was still propounding it in 1940" (p. 209).

Finally, the main thesis of Jean-Pierre Marquis's contribution is that the correct representation of mathematical knowledge differs from the traditional "axioms–definitions–theorems–proofs of truth picture" (p. 240). He argues this while engaging in a truly exciting discussion of the history of homotopy theory that reveals "the emergence, proliferation and establishment of *systematic mathematical technologies*" (p. 240), which, for Marquis, ought to be the object of study of an epistemology of mathematical instrumentation. According to Marquis, the need for such an epistemology shows that, in contrast with the received view on these matters, "parts of mathematical knowledge should be thought of as a form of conceptual engineering" (p. 240).

The third, and final, part of *The Architecture of Modern Mathematics*, like Parts I and II, consists of four articles. The object of investigation of the first, by Moritz Epple, is Felix Hausdorff's view of epistemology. I must say that I was very surprised in reading this paper. Not only had I ignored the fact that Hausdorff had contributed, under the pseudonym of Paul Mongré, to "philosophy, lyrics, and drama" (p. 264), but I did not even suspect a philosophical interest on the part of a mathematician who, after having assigned an object $\mathfrak{a}$—a cardinal number—to each set A in such a way that "equivalent sets, and equivalent sets only, have the same object corresponding to them" [Hausdorff, 1962, 28], and having explained what cardinal numbers are supposed to do, concluded, in a sarcastic tone, that "we must leave the determination of the 'essence' of the cardinal number to philosophy" [Hausdorff, 1962, 29].

Erhard Scholz defends the view according to which Weyl, in his later contributions to the philosophy of mathematics, became a supporter of what Scholz calls "symbolic realism" (p. 292); and Hourya Sinaceur accompanies the reader on a quick but intense tour of French philosophy of mathematics at the beginning of the 20th century, a tour in which the contributions of Cavaillès occupy center stage.

Lastly, we find a well known paper by Wilfried Sieg, which was originally published in *Synthese* in 1990. Starting from the acknowledgment of the failure of Hilbert's program, Sieg is impressed by the philosophical relevance of some of the results obtained in proof theory. These are the results which establish the consistency of classical theories, e.g., Peano arithmetic, relative to constructive ones, e.g., Heyting arithmetic. For Sieg, much has to be learned concerning the nature of mathematics from knowing that

> A considerable portion of classical mathematical practice, including all of analysis, can be carried out in a small corner of Cantor's paradise that is consistent relative to the constructive principles formalized in intuitionistic number theory (p. 360)

and from a reflection on the mathematical principles used in the relative consistency proofs. In more general terms, these results and their philosophical importance confirm, for Sieg, the correctness of the thesis according to which

> ... we shall advance our understanding of mathematics only if we continue to develop the dialectic of mathematical investigation and philosophical reflection. (p. 368)

In contrast with the internalist approach to the history of mathematics followed by most of the contributors to *AMM*, Gray's Coda is written having in mind an externalist viewpoint whereby modern mathematics becomes "a particular case of Modernism" (p. 371). I should point out that Gray's paper is a novel exercise in cultural history rather than in the more beaten track of the sociology of knowledge.

As should be clear by now, I think that *The Architecture of Modern Mathematics* gives a valuable contribution to the history and the philosophy of mathematics, both by gesturing toward what Ferreirós and Gray call "philosophy of mathematical practice" (p. 12), and by providing particularly interesting historic–philosophical studies of salient episodes in the history of mathematics. I therefore have no hesitation in recommending this book to philosophers and historians of mathematics alike.

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