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In praise of Claude Berge

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Sometimes it seems to me that one mark of distinction shared by the better class of towns in parts of the United States is the Macy's department store: when you have got your Macy's, you have got it made. Where I grew up, which was Czechoslovakia, the better class of towns possessed a different status symbol. It was a store with a big red sign in the Cyrillic reading SOVIETSKAYA KNIGA, which translates to Soviet Book. In a way, these Soviet Book stores were like churches with their atmosphere of quiet and with their pervasive smells. They were quiet because most self-respecting citizens wouldn't be caught dead in them; they were full of smells because, while Russians were busy getting Gagarin into orbit, they had not quite come to grips with the challenge of manufacturing odorless book glue. But the Soviet Book stores had one excellent characteristic: they carried first-rate mathematical books at prices that high-school students could afford. So I made a habit of going there to browse. And that was how one day in 1964 I came across a book that aroused my curiosity for two reasons.

First, there was the author: K. BERZH. That was not the kind of a name one was used to seeing in SOVIETSKAYA KNIGA. Berzhovich, Berzhinsky, Berzhev, Berzhenko, all of these I would take in my stride. But Berzh? That author, I decided, must have been a Kalmyk. Or maybe a Chechen. In any case, one of those nationalities that enjoyed so much autonomy and freedom then and that enjoy even more of them now.

Second, there was the title: TEORIYA GRAFOV, meaning Theory of Graphs. How pompous can people get, I wondered. *Theory* of graphs? You draw your x axis, you draw your y axis, you plot the graph of your function, and K. BERZH calls that a theory already? I opened the book, leafed through a few pages, then leafed through a few more, and found no parabolas, no straight lines, no sinusoids. No x axis. No y axis. There were no graphs in *Theory of Graphs*. There were plenty of illustrations in the text, but most of them were alluring configurations of small circles connected by straight line segments that sometimes had arrows on them. And there were chess-boards filled with improbable arrangements of chess pieces. And there were theorems

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and there were proofs and there were exercises ... Definitely an intriguing book, even if its title was highly misleading.

Five years later I met the Chechen face to face. And he changed my life. He took me through the looking glass to enchanted worlds where I found myself. I owe him a lot. And, in a more restricted sense, the global tribe of combinatorists owes him a lot.

To begin, there are Claude's books. Looking through a few of them, I came across Gian-Carlo Rota's preface to *Principles of Combinatorics*. He writes

> "I am tempted to suggest that the title of this book be changed to 'Seduction into Combinatorics' "

and I suspect that the sight of myself at the age of eighteen in that foul-smelling store would tempt more than one person to suggest that the title of the book I was reading be changed to 'Seduction into Graph Theory'.

In certain rare cases, the seduction had harmful side effects: some people [6,10–12] were swept off their feet to the point of calling a 'Berge graph' what to the rest of us is simply a 'graph'. To me, that is unfortunate: unaware of the earlier usage, Najiba Sbihi and I [5] proposed to reserve the term *Berge graph* for a graph G such that neither G nor its complement $\overline{G}$ contains an induced subgraph isomorphic to a cycle whose length is odd and at least five. Our proposal allows crisp reformulations of statements such as

(SPGC) "a graph is perfect if and only if
neither it nor its complement
contains an induced subgraph isomorphic to
a cycle whose length is odd and at least five"

and, bearing in mind who made the Strong Perfect Graph Conjecture, it seems amply justified.

Claude's first three books appeared in rapid succession:

- *Théorie générale des jeux à n personnes*, Gauthier-Villars, Paris, 1957.
- *Théorie des graphes et ses applications*, Dunod, Paris, 1958.
- *Espaces topologiques: Fonctions multivoques*, Dunod, Paris, 1959.

I used to think of the book on game theory and the book on topology as a couple of false starts from the days before Claude found his true calling in graph theory and combinatorics. A computer search through the *Mathematical Reviews* CD-ROM changed my mind: with each of these two books, Claude left a lasting mark on the subject. I was pleased to learn that the notions of *Berge equilibrium* and *Berge strategies* were being studied by game theorists [7,8,16] thirty years after the publication of Claude's book; I was pleased to learn that the *maximum theorem of Berge* and *Berge upper semicontinuity* were being studied by economists [9,13,14] thirty years after the publication of Claude's book on topology. I was pleased to read Mark Walker's [18]

words

> "The maximum theorem and its generalizations have become one of, the most useful tools in economic theory. The theorem — first stated, and proved by C. Berge — gives conditions under which ...".

It is amusing to speculate that, just as Claude Berge is a combinatorist to many of us combinatorists, he may be a game theorist to some game theorists and he may be a topologist to some economists. It is entertaining to imagine how he must appear to those among them who are unaware of his combinatorial career: Claude Berge, like Arthur Rimbaud, accomplished great things in his youth and then he disappeared from view. Whatever happened to him after 1959?

Incidentally, the CD-ROM was crawling with Berges, and not all of them were Claude. The other Berges are

Anne-Marie Berge	Bordeaux	Algebraic number theory
		Geometry of numbers
John Berge	Austin	Manifolds and cell complexes
Luc Berge	Villeneuve	Partial differential equations
		Quantum theory
Pierre Berge	Gif-sur-Yvette	Fluid mechanics
		Global analysis

A Berge who came just a little too early to have his publications reviewed by *Mathematical Reviews* is P.O. Berge from Stockholm [3]. Stretching the point, we could also include

Jos M.F. ten-Berge	Groningen	Statistics
		Matrix Theory

And there is the enigma of Berge Tatian [17], whose given name is Claude's family name. But let us not engage in unseemly speculations.

Unlike our hypothetical game theorists, we graph theorists *do* know what happened to Claude Berge after 1959. Claude did not disappear from *our* view. We all know his work in graph theory and combinatorics; I do not propose reviewing it here. But I cannot help making one exception: among the things that happened to Claude shortly after 1959, there was the Halle-am-Salle conference in April 1960. That was where he first announced his Strong Perfect Graph Conjecture. Since then, quite a few of us have been playing with it, turning it around, trying to find a chink in its armor. By now, nearly three hundred papers on the subject of perfect graphs have been published; at the 1993 DIMACS Workshop on Perfect Graphs in Princeton, there were over seventy participants. The global tribe of combinatorists owes Claude Berge a lot indeed.

What I do propose is pointing out a few of Claude's publications that some of us may *not* know about. One of these is

- *Sculptures multipètres* (Introduced by Phillipe Soupault and followed by "Des lithomorphites aux pierres velues" by Noël Arnaud), Lanord, Paris, 1962.

It is hard to search for words that would capture the magic beauty of Claude Berge's sculptures. It is tempting to look for a link between these objects and Claude's interest documented in his

- *L'Art Asmat*, Maison de Sciences de l'Homme, 1994 and its shorter version [2].

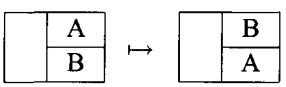
And then there are Claude's contributions to OuLiPo (Ouvroir de Littérature Potentielle). What is OuLiPo? An association of some twenty people working in a loosely defined area between literature and mathematics. For example, one oulipien notion is the *snowball*, defined as a sentence where, for all applicable values of k, the kth word has precisely k letters. I do not know where OuLiPo gathers material. Its works sometimes overlap with pataphysics: as a member of OuLiPo, Raymond Queneau [15] developed an axiomatic system of literature

(Axiom I.1) Every two words are contained in a sentence

(Axiom I.2) Every two words are contained in at most one sentence

and so on, with enlightening examples. As a member of OuLiPo, Italo Calvino [4] proposed to use computers in designing plots of mystery stories. As a member of OuLiPo, Claude Berge [1] wrote the classical sonnet shown in Fig. 1.

To see how this creation fits into the framework of OuLiPo, think of the lines of the sonnet as numbered $1, 2, \ldots, 15$ from top to bottom, line 9 being the blank line that separates the two quatrains from the two tercets; then add a blank line 16 at the bottom. When Fig. 1 is printed on a sheet of paper, cut this sheet vertically into two parts and then cut the right part horizontally into two parts: part A from line 1 to line 9 and part B from line 10 to line 16. The transformation



maps the sonnet of 14 alexandrines into the poem of 15 alexandrines shown in Fig. 2. This transformation is not Claude's invention: it had been used in strips of paper sold in novelty stores with instructions for transforming k leprechauns into $k+1$. It is the transition from leprechauns to alexandrines that constitutes an impressive *tour de force*: the blank line 9 of Fig. 1 absorbs the additional alexandrine of Fig. 2, whose twelve syllables are created by turning silent *e*'s into voiced ones and by skillful exploitation of *diérèse*.

To complete my selection from the works of Claude Berge, here is an outline of

- "Who Killed the Duke of Densmore?", in: *Oulipo Laboratory*, Atlas Press, 1995, pp. 27–37.

The shadow of a grisly murder is hanging over the otherwise idyllic coast near Craymouth: carbonized bodies of two men and a crocodile have been found in a tower of Densmore Castle on the Isle of White. All three died a year before in the explosion of a charge skilfully connected to a light switch. The two men are Sir Jeremy Morse and his butler Stewart; the crocodile was just a pet.

Tandis qu'en frissonnant elle égrenait des vers
L' Aztèque imperturbable à la touque imprécise
Serrait sa souveraine une blonde aux yeux verts
D'un lien libidineux que la froideur attise
Dans l'Ouest enfoui dit-elle à son amant pervers,
C'est là que l'art jaillit, que l'Inca prosaïse,
Et que la pyramide abolit l'univers !
Nul n'entend le muet qui tout doucement s'enlise...

Comme le perspicace inoui conjecturait,
Jeune ami présomptueux plus fou qu'il ne parait,
N'offre pas de pactole à ton gardien farouche
Si le verbe à la fois oppresseur et charmant
D'un tel triomphateur ne trouble le diamant ...
Même Xipe Totec fuit et détruit sa souche

Fig. 1. A sonnet of 14 alexandrines.

Tandis qu'en frissonnant elle conjecturait,
L' ami présomptueux plus fou qu'il ne parait,
Serrait sa souveraine une blonde farouche
D'un lien à la fois oppresseur et charmant
Dans l'Ouest enfoui dit-elle à son amant ...
C'est là que l'art fuit et détruit sa souche
Et que la pyramide abolit l'univers
Nul n'entend le muet qui égrenait des vers
Aztèque imperturbable à la touque imprécise
Comme le perspicace inoui aux yeux verts
Jeune libidineux que la froideur attise
N'offre pas de pactole à ton gardien pervers,
Si le verbe jaillit, que l'Inca prosaïse
D'un tel triomphateur ne trouble le diamant !
Même Xipe Totec tout doucement s'enlise...

Fig. 2. A poem of 15 alexandrines.

During his last year, the duke had received eight visitors at his castle; each of them was brought first to the island and then back to the mainland by a motor boat. None of them recalls the precise dates or duration of her stay, but each remembers with certainty whom she met there:

Felicia Wynn	met	Emily Healey and Ann Laybourn;
Cynthia Mansfield	met	Diana MacLeod, Emily Healey, Ann Laybourn, Betty Townsend, and Helen Grimshaw;
Georgia Blake	met	Ann Laybourn and Helen Grimshaw;
Diana MacLeod	met	Cynthia Mansfield and Emily Healey;
Emily Healey	met	Felicia Wynn, Cynthia Mansfield, Diana MacLeod, and Ann Laybourn;

Ann Laybourn	met	Felicia Wynn, Cynthia Mansfield, Georgia Blake, Emily Healey, and Betty Townsend;
Betty Townsend	met	Cynthia Mansfield, Ann Laybourn, and Helen Grimshaw;
Helen Grimshaw	met	Cynthia Mansfield, Georgia Blake, and Betty Townsend.

The testimony of each woman is confirmed by the others, and collusion between any two is out of the question: this is the work of a serious writer, not one of your Agatha Christies. The butler is declared the killer and his own death is supposed to have been accidental. The case is closed.

But then Detective Ralston pays a visit to his friend Cedric Turner-Smith, a professor of Mathematics at Merton College. The brilliant mathematician suggests that one of the eight visitors is the murderer: during her stay on the island, she had hidden in the vast cellars once, or perhaps several times, in order to prepare her evil deed.

And TEORIYA GRAFOV comes into play ...

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References

[1] C. Berge, La reine aztèque ou contraintes pour un sonnet à longueur variable, Bibliothèque Oulipienne, No. 22 (1983).
[2] C. Berge, Asmat ancestor art, Tribal Art 1 (1994) 35–47.
[3] P.O. Berge, A note on a form of Tchebycheff's theorem for two variables, Biometrika 29 (1937) 405–406.
[4] I. Calvino, Prose et anticombinatoire, in: Atlas de littérature potentielle, Gallimard (1981) 319–331.
[5] V. Chvátal and N. Sbihi, Recognizing claw-free perfect graphs, J. Combin. Theory Ser. B 44 (1988) 154–176.
[6] I.I. Dzegelenok, Berge graphs and the interpretation of logical patterns, Trudy Moskov. Orden. Lenin. Energet. Inst. Vyp. 121 (1972) 120–123 (in Russian).
[7] S.D. Gaidov, Optimal strategies in two-player stochastic differential games, C. R. Acad. Bulgare Sci. 39 (1986) 33–36.
[8] S.D. Gaidov, Berge-equilibrium in stochastic differential games, Math. Balkanica (N.S.) 1 (1987) 25–32.
[9] P. Kall, On approximations and stability in stochastic programming, in: Parametric Optimization and Related Topics, Plaue, 1985 (Akademie-Verlag, Berlin, 1987) 387–407.
[10] A.M. Kalmanovich, Semiheaps of pairs of homomorphisms of Berge graphs, in: Modern Algebra, Leningrad. Gos. Ped. Inst., Leningrad (1980) 37–49 (in Russian).
[11] A.M. Kalmanovich, Isomorphisms of semigroups of homomorphisms of some classes of pairs of Berge graphs, in: Studies of semigroups, Leningrad. Gos. Ped. Inst., Leningrad (1990) 33–39 (in Russian).
[12] W. Kornacki, A method for determining the reliability core in a multiprocessor system, Arch. Automat. Telemech. 25 (1981) 519–525 (in Polish).
[13] D.M. Mandy, Continuity of optima, J. Econom. Theory 54 (1991) 460–467.

[14] M. Nermuth, Sensitivity of optimal growth paths with respect to a change in target stocks or in the length of the planning horizon in a multisector model, J. Math. Econom. 5 (1978) 289–301.
[15] R. Queneau, Fondements de la littérature d'après David Hilbert, Bibliothèque Oulipienne, No. 5.
[16] V.V. Rozen, Equilibrium situations in games on graphs, in: Theory of Semigroups and its Applications (Saratov. Gos. Univ., Saratov, 1991) 88–95 (in Russian).
[17] B. Tatian, Method for obtaining the transfer function from the edge response function, J. Opt. Soc. Amer. 55 (1965) 1014–1019.
[18] M. Walker, A generalization of the maximum theorem, Internat. Econom. Rev. 20 (1979) 267–272.